MATHEMATICS

Real Numbers 1

Real Numbers:

- The numbers which can be represented in the form of $\frac{p}{q}$, of where p and q are integers and $q \neq 0$ are called **Rational numbers.**
- Any number that cannot be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called **Irrational numbers.**
- Rational and Irrational numbers together constitute **Real numbers.**

Properties of Irrational numbers:

i. The **Sum, Difference, Product** and **Division** of two irrational numbers need not always be an irrational number.

ii. **Negative** of an irrational number is an irrational number.

iii. **Sum** of a **rational** and an **irrational** number is irrational.

iv. **Product** and **Division** of a non-zero rational and irrational number is always irrational.

Fractions:

- **Terminating fractions** are the fractions which leaves remainder 0 on normal division.
- **Recurring fractions** are the fractions which never leave a remainder 0 on normal division.

Note: (i) Rational numbers have terminating or non-terminating decimal expansion.

(ii) Irrational numbers have neither terminating nor-repeating expansion.

Properties related to prime numbers:

- If p is a prime and divides a^2, then p divides a, where 'a' is a positive integer.
- If p is a prime, then $\sqrt{p}$ is an irrational number.
- A number ends with the digit zero if and only if it has 2 and 5 as two of its prime factors.

Decimal Expansion:

- The decimal expansion of rational number is either **terminating** or **non-terminating recurring (repeating).**
- If the decimal expansion of rational number **terminates**, then we can express the number in the form of $\frac{p}{q}$, where p and q are co-prime, and the prime factorization of **q is of the form 2^n5^m**, where n and m are non negative integers.
- If $x = \frac{p}{q}$ is a rational number, such that the prime factorization of q is of the form 2^n5^m, where n, m are non-negative integers. Then, x has a decimal expansion which **terminates.**

- If the denominator of a rational number is of the form 2^n5^m, then it will terminate after n places if n > m or after m places if m > n.
- The decimal expansion of an irrational number is **non-terminating, non-recurring.**

Fundamental Theorem of Arithmetic:

Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

The procedure of finding **HCF (Highest Common Factor)** and **LCM (Lowest Common Multiple)** of given two positive integers a and b:

i. Find the prime factorization of given numbers.

ii. HCF (a, b) = Product of the smallest power of each common prime factors in the numbers.

iii. LCM (a, b) = Product of the greatest power of each prime factors, involved in the numbers.

Fundamental Theorem of Arithmetic states that every integer greater than 1 is either a prime number or can be expressed in the form of primes. In other words, all the natural numbers can be expressed in the form of the product of its prime factors. To recall, prime factors are the numbers which are divisible by 1 and itself only. For example, the number 35 can be written in the form of its prime factors as:

$$35 = 7 \times 5$$

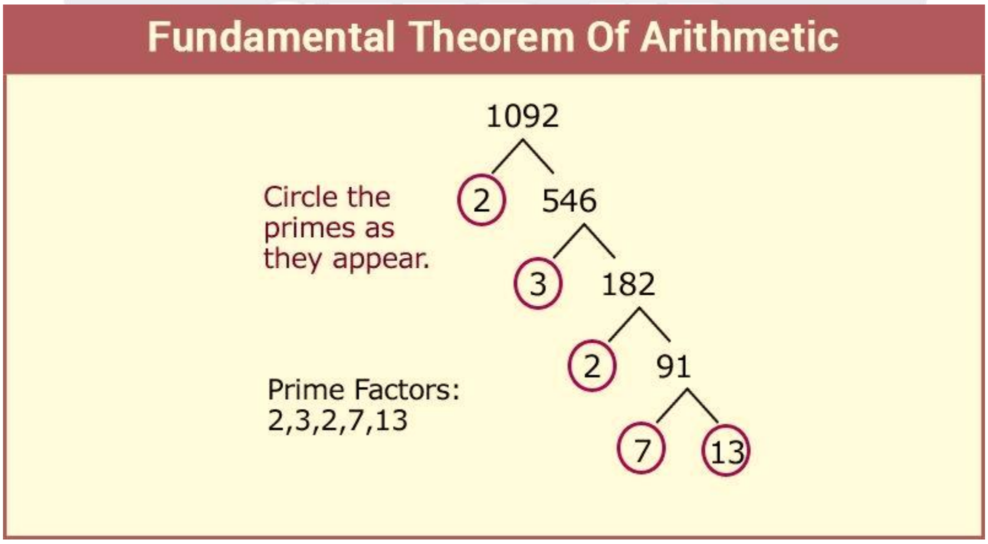
Here, 7 and 5 are the prime factors of 35

Similarly, another number 114560 can be represented as the product of its prime factors by using prime factorization method,

$$114560 = 27 \times 5 \times 179$$

So, we have factorized 114560 as the product of the power of its primes.

Therefore, every natural number can be expressed in the form of the product of the power of its primes. This statement is known as the Fundamental Theorem of Arithmetic, unique factorization theorem or the unique-prime-factorization theorem.



Proof for Fundamental Theorem of Arithmetic : In number theory, a composite number is expressed in the form of the product of primes and this factorization is unique apart from the order in which the prime factor occurs.

From this theorem we can also see that not only a composite number can be factorized as the product of their primes but also for each composite number the factorization is unique, not taking into consideration order of occurrence of the prime factors.

In simple words, there exists only a single way to represent a natural number by the product of prime factors. This fact can also be stated as:

The prime factorization of any natural number is said to be unique for except the order of their factors.

In general, a composite number “a” can be expressed as,

$a = p_1\ p_2\ p_3\ \ldots\ldots\ldots\ p_n$, where $p_1, p_2, p_3\ \ldots\ldots\ldots\ p_n$ are the prime factors of a written in ascending order i.e. $p_1 \leq p_2 \leq p_3\ \ldots\ldots\ldots \leq p_n$.

Writing the primes in ascending order makes the factorization unique in nature.

Relationship between HCF and LCM of two numbers:

If a and b are two positive integers, then HCF (a, b) × LCM (a, b) = a × b

Relationship between HCF and LCM of three numbers:

$$\text{LCM}(p, q, r) = \frac{p.\,q.\,r\,.\,\text{HCF}(p, q, r)}{\text{HCF}(p, q).\,\text{HCF}(q, r).\,\text{HCF}(p, r)}$$

$$\text{HCF}(p, q, r) = \frac{p.\,q.\,r\,.\,\text{LCM}(p, q, r)}{\text{LCM}(p, q).\,\text{LCM}(q, r).\,\text{LCM}(p, r)}$$

Method of Finding LCM

In Mathematics, the LCM of any two is the value that is evenly divisible by the two given numbers. The full form of LCM is Least Common Multiple. It is also called the Least Common Divisor (LCD). For example, LCM (4, 5) = 20. Here, the LCM 20 is divisible by both 4 and 5 such that 4 and 5 are called the divisors of 20.

LCM is also used to add or subtract any two fractions when the denominators of the fractions are different. While performing any arithmetic operations such as addition, subtraction with fractions, LCM is used to make the denominators common. This process makes the simplification process easier.

Least Common Multiple (LCM) is a method to find the smallest common multiple between any two or more numbers. A common multiple is a number which is a multiple of two or more numbers.

Properties of LCM

Properties	Description
Associative property	LCM (a, b) = LCM (b, a)
Commutative property	LCM (a, b, c) = LCM (LCM(a, b), c) = LCM(a, LCM(b, c))
Distributive property	LCM (da, db, dc) = d LCM (a, b, c)

LCM Formula

Let a and b are two given integers. The formula to find the LCM of a & b is given by:

LCM (a, b) = (a x b)/GCD (a, b)

Where GCD (a, b) means Greatest Common Divisor or Highest Common Factor of a & b.

LCM Formula for Fractions

The formula to find the LCM of fractions is given by:

L.C.M. = L.C.M Of Numerator/H.C.F Of Denominator

Different Methods of LCM

There are three important methods by which we can find the LCM of two or more numbers. They are:

- Listing the Multiples
- Prime Factorization Method
- Division Method

Listing the Multiples : The method to find the least common multiple of any given numbers is first to list down the multiples of specific numbers and then find the first common multiple between them.

Suppose there are two numbers 11 and 33. Then by listing the multiples of 11 and 33, we get;

Multiples of 11 = 11, 22, 33, 44, 55, ...

Multiples of 33 = 33, 66, 99, ...

We can see, the first common multiple or the least common multiple of both the numbers is 33. Hence, the LCM (11, 33) = 33.

LCM By Prime Factorization :

Another method to find the LCM of the given numbers is prime factorization. Suppose there are three numbers 12, 16 and 24. Let us write the prime factors of all three numbers individually.

$$12 = 2 \times 2 \times 3$$

$$16 = 2 \times 2 \times 2 \times 2$$

$$24 = 2 \times 2 \times 2 \times 3$$

Now writing the prime factors of all the three numbers together, we get;

$$12 \times 16 \times 24 = 2 \times 2 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

Now pairing the common prime factors we get the LCM. Hence, there are four 2's and one 3. So the LCM of 12, 16 and 24 will be;

$$\text{LCM}(12, 16, 24) = 2 \times 2 \times 2 \times 2 \times 3 = 48$$

LCM By Division Method

- Finding LCM of two numbers by division method is an easy method. Below are the steps to find the LCM by division method:
- First, write the numbers, separated by commas
- Now divide the numbers, by the smallest prime number.
- If any number is not divisible, then write down that number and proceed further
- Keep on dividing the row of numbers by prime numbers, unless we get the results as 1 in the complete row
- Now LCM of the numbers will be equal to the product of all the prime numbers we obtained in the division method

Example : To find the Least Common Multiple (L.C.M) of 36 and 56,

$36 = 2 \times 2 \times 3 \times 3$

$56 = 2 \times 2 \times 2 \times 7$

The common prime factors are 2×2

The uncommon prime factors are 3×3 for 36 and 2×7 for 56.

LCM of 36 and 56 = $2 \times 2 \times 3 \times 3 \times 2 \times 7$ which is 504

Method of Finding HCF

H.C.F can be found using two methods – Prime factorisation and Euclid's division algorithm.

Prime Factorisation : Given two numbers, we express both of them as products of their respective prime factors. Then, we select the prime factors that are common to both the numbers

Example : To find the H.C.F of 20 and 24

$20 = 2 \times 2 \times 5$ and $24 = 2 \times 2 \times 2 \times 3$

The factor common to 20 and 24 is 2×2, which is 4, which in turn is the H.C.F of 20 and 24.

HCF by Shortcut Method

Steps to find the HCF of any given numbers.

Step 1: Divide larger number by smaller number first, such as;
Larger Number/Smaller Number

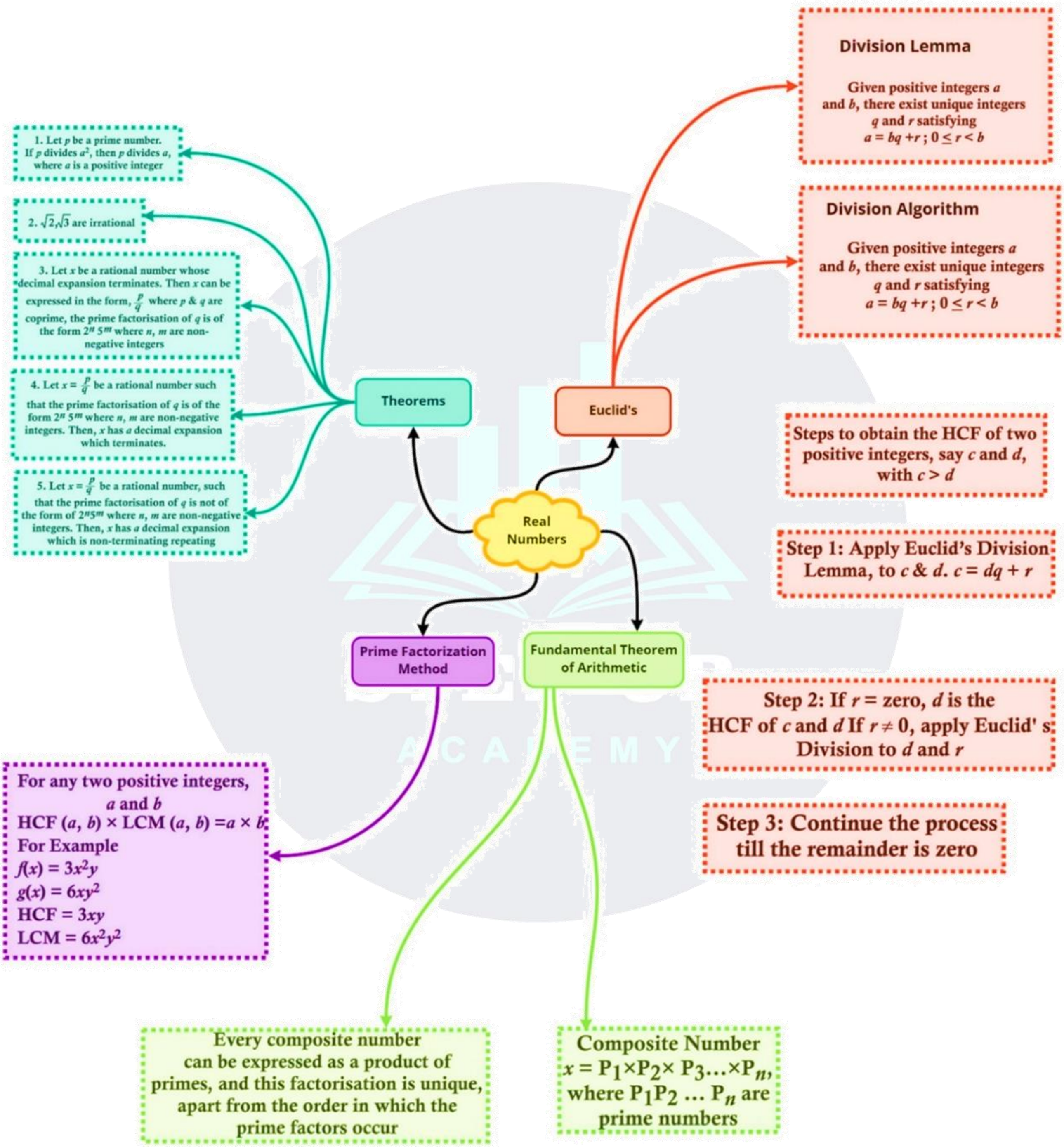
Step 2: Divide the divisor of step 1 by the remainder left.
Divisor of step 1/Remainder

Step 3: Again divide the divisor of step 2 by the remainder.
Divisor of step 2/Remainder

Step 4: Repeat the process until the remainder is zero.

Step 5: The divisor of the last step is the HCF.

Class : 10th mathematics
Chapter-1 : Real Numbers



Important Questions

Multiple Choice Questions:

1. HCF of 8, 9, 25 is

(a) 8

(b) 9

(c) 25

(d) 1

2. Which of the following is not irrational?

(a) $(2 - \sqrt{3})^2$

(b) $(\sqrt{2} + \sqrt{3})^2$

(c) $(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})$

(d) $\frac{2\sqrt{7}}{7}$

3. The product of a rational and irrational number is

(a) rational

(b) irrational

(c) both of above

(d) none of above

4. The sum of a rational and irrational number is

(a) rational

(b) irrational

(c) both of above

(d) none of above

5. The product of two different irrational numbers is always

(a) rational

(b) irrational

(c) both of above

(d) none of above

6. The sum of two irrational numbers is always

(a) irrational

(b) rational

(c) rational or irrational

(d) one

7. The product of three consecutive positive integers is divisible by

(a) 4

(b) 6

(c) no common factor

(d) only 1

8. The set A = {0,1, 2, 3, 4, …} represents the set of

(a) whole numbers

(b) integers

(c) natural numbers

(d) even numbers

9. Which number is divisible by 11?

(a) 1516

(b) 1452

(c) 1011

(d) 1121

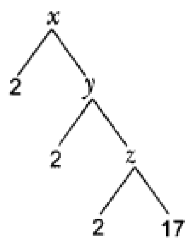
Very Short Questions:

1. What is the HCF of the smallest composite number and the smallest prime number?
2. The decimal representation of $\frac{6}{1250}$ will terminate after how many places of decimal?
3. If HCF of a and b is 12 and product of these numbers is 1800. Then what is LCM of these numbers?
4. What is the HCF of $3^3 \times 5$ and $3^2 \times 5^2$?
5. if a is an odd number, b is not divisible by 3 and LCM of a and b is P, what is the LCM of 3a and 2b?
6. If P is prime number then, what is the LCM of P, P^2, P^3?
7. Two positive integers p and q can be expressed as $p = ab^2$ and $q = a^2b$, a and b are prime numbers. What is the LCM of p and q?
8. A number N when divided by 14 gives the remainder 5. What is the remainder when the same number is divided by 7?
9. Examine whether $\frac{17}{30}$ is a terminating decimal or not.
10. What are the possible values of remainder r, when a positive integer a is divided by 3?
11. A rational number in its decimal expansion is 1.7351. What can you say about the prime factors of q when this number is expressed in the form $\frac{p}{q}$? Give reason.

12. Without actually performing the long division, find $\frac{987}{10500}$ will have terminating or non. terminating repeating decimal expansion. Give reason for your answer.

Short Questions:

1. Can the number 4^n, n be a natural number, end with the digit 0? Give reason.
2. Can two numbers have 18 as their HCF and 380 as their LCM? Give reason.
3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?
4. Find the LCM and HCF of 12, 15 and 21 by applying the prime factorisation method.
5. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.
   (1) 26 and 91 (ii) 198 and 144
6. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start from the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?
7. Write down the decimal expansions of the following numbers:
   i. $\frac{35}{50}$ ii. $\frac{15}{1600}$
8. Express the number $\overline{0.3178}$ in the form of rational number $\frac{a}{b}$.
9. If n is an odd positive integer, show that $(n^2 - 1)$ is divisible by 8.
10. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number.
11. Find the value of x, y and z in the given factor tree. Can the value of 'x' be found without finding the value of 'y' and 'z'? If yes, explain.



12. The decimal expansions of some real numbers are given below. In each case, decide whether they are rational or not. If they are rational, write it in the form $\frac{p}{q}$. What can you say about the prime factors of q?
   (i) 0.140140014000140000... (ii) $\overline{0.16}$

Long Questions:

1. 960 and 432
2. 4052 and 12576.
3. Using prime factorisation method, find the HCF and LCM of 30, 72 and 432. Also show that HCF × LCM ≠ Product of the three numbers.
4. Prove that $\sqrt{7}$ is an irrational number.
5. Show that $5 - \sqrt{3}$ is an irrational number.
6. Check whether 6^n can end with the digit 0 for any natural number n.
7. Show that there is iw positive integer n for which $\sqrt{n-1} + \sqrt{n+1}$ is rational.
8. Find the largest positive integer that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.

Case Study Questions:

1. Real numbers are extremely useful in everyday life. That is probably one of the main reasons we all learn how to count and add and subtract from a very young age. Real numbers help us to count and to measure out quantities of different items in various fields like retail, buying, catering, publishing etc. Every normal person uses real numbers in his daily life. After knowing the importance of real numbers, try and improve your knowledge about them by answering the following questions on real life based situations.
   i. Three people go for a morning walk together from the same place. Their steps measure 80cm, 85cm and 90cm respectively. What is the minimum distance travelled when they meet at first time after starting the walk assuming that their walking speed is same?
      a. 6120cm
      b. 12240cm
      c. 4080cm
      d. None of these

ii. In a school Independence Day parade, a group of 594 students need to march behind a band of 189 members. The two groups have to march in the same number of columns. What is the maximum number of columns in which they can march?

a. 9

b. 6

c. 27

d. 29

iii. Two tankers contain 768 litres and 420 litres of fuel respectively. Find the maximum capacity of the container which can measure the fuel of either tanker exactly.

a. 4 litres

b. 7 litres

c. 12 litres

d. 18 litres

iv. The dimensions of a room are 8m, 25cm, 6m, 75cm and 4m, 50cm. Find the length of the largest measuring rod which can measure the dimensions of room exactly.

a. 1m, 25cm

b. 75cm

c. 90cm

d. 1m, 35cm

v. Pens are sold in pack of 8 and notepads are sold in pack of 12. Find the least number of pack of each type that one should buy so that there are equal number of pens and notepads.

a. 3 and 2

b. 2 and 5

c. 3 and 4

d. 4 and 5

Assertion Reason Questions:

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

a. Both A and R are true and R is the correct explanation of A.

b. Both A and R are true and R is not the correct explanation of A.

c. A is true but R is false.

d. A is false but R is true.

Assertion: $11 \times 4 \times 3 \times 2 + 4$ is a composite number.

Reason: Every composite number can be expressed as product of primes.

2. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

a. Both A and R are true and R is the correct explanation of A.

b. Both A and R are true and R is not the correct explanation of A.

c. A is true but R is false.

d. A is false but R is true.

Assertion: If LCM = 350, product of two numbers is 25×70, then their HCF = 5

Reason: LCM × product of numbers = HCF

Answer Key

Multiple Choice Questions:

1. (d) 1
2. (c) $(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})$
3. (b) irrational
4. (b) irrational
5. (b) irrational
6. (a) irrational
7. (b) 6
8. (a) whole numbers
9. (b) 1452

Very Short Answer :

1. Smallest composite number = 4

Smallest prime number = 2

So, HCF (4, 2) = 2

2. $\frac{6}{1250}=\frac{3}{625}=\frac{3}{5^4}\times\frac{2^4}{2^4}=\frac{48}{(5\times2)^4}=\frac{48}{104}=0.0048$

This representation will terminate after 4 decimal places.

3. Product of two numbers = Product of their LCM and HCF

$\Rightarrow 1800 = 12 \times \text{LCM}$

$\Rightarrow \text{LCM} = \frac{1800}{12} = 150.$

4. HCF of $3^3 \times 5$ and $3^2 \times 5^2 = 3^2 \times 5 = 45$

5. 6P

6. P^3

7. a^2h^2

8. 5, because 14 is multiple of 7.

Therefore, remainder in both cases are same.

9. $\frac{17}{30}=\frac{17}{2\times3\times5}$

10. Since the denominator has 3 as its factor.

$\therefore \frac{17}{30}$ is a non terminating decimal.

11. According to Euclid's division lemma

a = 3q + r, where 0 r < 3 and r is an integer.

Therefore, the values of r can be 0, 1 or 2.

12. As 1.7351 is a terminating decimal number, so q must be of the form $2^m 5^n$, where in, n are natural numbers.

13. $\frac{987}{10500}=\frac{47}{500}$ and 500 = 22 × 53, so it has terminating decimal expansion.

Short Answer :

1. If 4^n ends with 0, then it must have 5 as a factor. But, $(4)^n = (2^2)^n = 2^{2n}$ i.e., the only prime factor of 4^n is 2. Also, we know from the fundamental theorem of arithmetic that the prime factorization of each number is unique.

$\therefore 4^n$ can never end with 0.

2. No, because here HCF (18) does not divide LCM (380).

3. For the maximum number of columns, we have to find the HCF of 616 and 32.

Now, since 616 > 32, we apply division lemma to 616 and 32.

We have, 616 = 32 × 19 + 8

Here, remainder 8 ≠ 0. So, we again apply division lemma to 32 and 8.

We have, 32 = 8 × 4 + 0

Here, remainder is zero. So, HCF (616, 32) = 8

Hence, maximum number of columns is 8.

4. The prime factors of 12, 15 and 21 are

12 $2^2 \times 3$, 15 = 3 × 5 and 21 = 3 × 7

Therefore, the HCF of these integers is 3.

2^2, 3^1, 5^1 and 7^1 and are the greatest powers involved in the prime factors of 12, 15 and 21.

So, LCM (12, 15, 21) = $2^2 \times 3^1 \times 5^1 \times 7^1 = 420$.

5. (i) We have, 26 = 2 × 13 and 91 = 7 × 13

Thus, LCM (26, 91) = 2 × 7 × 13 = 182

HCF (26, 91) = 13

Now, LCM (26, 91) × HCF (26, 91) = 182 × 13 = 2366 and Product of the two

numbers = 26 × 91 = 2366

Hence, LCM × HCF = Product of two numbers.

(ii) 144 = 24 × 32 and 198 = 2 × 32 × 11

∴ LCM(198,144)24 × 32 × 11 = 1584

HCF(198, 144) = 2 × 32 = 18

Now, LCM (198, 144) × HCF (198, 144) = 1584 × 18 = 28512 and product of 198 and 144 = 28512

Thus, product of LCM (198, 144) and HCF (198, 144) = Product of 198 and 144.

6. To find the time after which they meet again at the starting point, we have to find LCM of 18 and 12 minutes. We have

2	18
3	9
3	3
	1

2	12
2	6
3	3
	1

Therefore, LCM of 18 and 12 = $2^2 \times 3^2 = 36$

So, they will meet again at the starting point after 36 minutes.

7. (i) We have, $\frac{35}{50}=\frac{35}{5^2\times2}=\frac{35\times2}{5^2\times2\times2}=\frac{70}{5^2\times2^2}$

$=\frac{70}{10^2}=\frac{70}{100}=0.70$

(ii) We have, $\frac{15}{1600}=\frac{15}{2^6\times5^2}=\frac{15\times5^4}{2^4\times2^2\times5^2\times5^4}=\frac{15\times625}{2^6\times5^6}$

$=\frac{9375}{10^6}=\frac{9375}{1000000}=0.009375$

8. Let x = $\overline{0.3178}$

then x = 0.3178178178 … … (i)

10x = 3.178178178 … …(ii)

10000x = 3178.178178… …(iii)

On subtracting (ii) from (iii), we get

$$9990x = 3175 \Rightarrow x = \frac{3175}{9990} = \frac{635}{1998}$$

$$\therefore\ 0.3\overline{178} = \frac{635}{1998}$$

9. We know that an odd positive integer n is of the form (4q + 1) or (4 + 3) for some integer q.

Case – I When n = (4q + 1)

In this case $n^2 - 1 = (4q + 1)^2 1 = 16q^2 + 8q = 8q(2q + 1)$

which is clearly divisible by 8.

Case – II When n = (4q + 3)

In this case, we have

$n2^2 = (4q + 3)^2 - 1 = 16q^2 + 24q + 8 = 8(2q^2 + 3q + 1)$

which is clearly divisible by 8.

Hence $(n^2 - 1)$ is divisible by 8.

10. Let HCF of the numbers be x then according to question LCM of the number will be 14x

And x + 14 = 600 ⇒ 15x = 600 ⇒ x = 40

Then HCF = 40 and LCM = 14 × 40 = 560

∵ LCM × HCF = Product of the numbers

560 × 40 = 280 × Second number Second number $= \frac{560\times40}{280} = 80$

Then other number is 80.

11. z = 2 × 17 = 34; y = 34 × 2 = 68 and x = 2 × 68 = 136

Yes, value of x can be found without finding value of y or z as

x = 2 × 2 × 2 × 17 which arc prime 1tctors of x.

12. (i) We have, 0.140140014000140000… a non-terminating and non-repeating decimal expansion. So it is irrational. It cannot be written in the form of $\frac{p}{q}$

(ii) We have, $\overline{0.16}$ a non-terminating but repeating decimal expansion. So it is rational.

Let x = $\overline{0.16}$

Then, x = 0.1616… (i)

100x = 100x16.1616… ..(ii)

On subtracting (i) from (ii), we get

100x – x = 16.1616 – 0.1616

$\Rightarrow 99x = 16 \Rightarrow x = \frac{16}{99} = \frac{p}{q}$

The denominator (q) has factors other than 2 or 5.

Long Answer :

1. Given members = 30, 72, 432 .

30 = 2 × 3 × 5; 72 = 23 × 32 and 432 = 24 × 33

Here, 2' and 31 are the smallest powers of the common factors 2 and 3 respectively.

So, HCF (30, 72, 432) = 21 × 31 = 2× 3 = 6

Again, 2, 33 and 51 arc the greatest powers of the prime factors 2, 3 and 5 respectively.

So, LCM (30, 72, 432) = 24 × 33 × 51 = 2160

HCF × LCM = 6 × 2160 = 12960

Product of numbers = 30 × 72 × 432 = 933120 .

Therefore, HCF × LCM ≠ Product of the numbers.

2. Let us assume, to the contrary, that √7 is a rational number.

Then, there exist co-prime positive integers and such that

$\sqrt{7} = \frac{a}{b}$, b ≠ 0

So, a = √7 b

Squaring both sides, we have

$a^2 = 7b^2$ …… (i)

⇒ 7 divides a^2 ⇒ 7 divides a

So, we can write

a = 7c (where c is an integer)

Putting the value of a = 7c in (i), we have

$49c^2 = 7b^2\ 7^2 = b^2$

It means 7 divides b^2 and so 7 divides b.

So, 7 is a common factor of both a and b which is a contradiction.

So, our assumption that √7 is rational is wrong.

Hence, we conclude that √7 is an irrational number.

3. Let us assume that 5 – √3 is rational.

So, 5 – √3 may be written as

$5 - \sqrt{3} = \frac{p}{q}$, where p and q are integers, having no common factor except 1 and q ≠ 0.

$\Rightarrow 5 - \frac{p}{q} = \sqrt{3} \Rightarrow \sqrt{3} = \frac{5q-p}{q}$

Since $\frac{5q-p}{q}$ is a rational number as p and q are integers.

∴ √3 is also a rational number which is a contradiction.

Thus, our assumption is wrong.

Hence, 5 – √3 is an irrational number.

4. Since 2160 > 847 we apply the division lemma to 2160 and 847

we have, 2160 = 847 × 2 + 466

Since remainder 466 ≠ 0. So, we apply the division lemma to 847 and 466

847 = 466 × 1+ 381

Again remainder 381 ≠ 0. So, we again apply the division lemma to 466 and 381.

466 = 381 × 1 + 85

Again remainder 85 ≠ 0. So, we again apply the division lemma to 381 and 85

381 = 85 × 4 + 41

Again remainder 41 ≠ 0. So, we again apply the division lemma to 85 and 41.

85 = 41 × 2 + 3

Again remainder 3 ≠ 0. So, we again apply the division lemma to 4 1 and 3.

41 = 3 × 13 + 2

Again remainder 2 ≠ 0. So, we again apply the division lemma to 3 and 2.

3 = 2 × 1 + 1

Again remainder 1 ≠ 0. So, we apply division lemma to 2 and 1

2 = 1 × 2 + 0

The remainder now becomes O. So, our procedure stops.

Since the divisor at this stage is 1.

Hence, HCF of 847 and 2160 is 1 and numbers are co-prime.

5. If the number 6^n, for any n, were to end with the digit zero, then h would bc divisible by 5. That is, the prime factorisation of 6^n would contain the prime 5. But $6^n = (2 \times 3)^n = 2^n \times 3n$ So the primes in factorisation of 6n are 2 and 3. So the uniqueness of the Fundamental Theorem of Arithmetic guarantees that (here are no other primes except 2 and 3 in the factorisation of 6n.

So, there is no natural number n for which 6" ends with digit zero.

6. It is given that on dividing 398 by the required number, there is a remainder of 7. This means that 398 – 7 = 391 is exactly divisible by the required timber in other words, required number is a factor of 391.

Similarly, required positive integer is a Íctor of 436 – 11 = 425 and 542 – 15 = 527

Clearly, the required number is the HCF of 391, 425 and 527.

Using the factor tree, we get the prime factorisations of 391, 425 and 527 as follows:

391 = 17 × 23, 425 52 × 17 and 527 17 × 31

∴ HCF of 391, 425, and 527 is 17.

Hence, the required number = 17.

Case Study Answers:

1. **Answer :**

i. (b) 12240cm

Solution:

Here 80 = 24 × 5, 85 = 17 × 5 and 90 = 2 × 32 × 5

L.C.M of 80, 85 and 90 = 24 × 3 × 3 × 5 × 17 = 12240

Hence, the minimum distance each should walk when they at first time is 12240cm.

ii. (c) 27

Solution:

Here 594 = 2 × 33 × 11 and 189 = 33 × 7

HCF of 594 and 189 = 3^3 = 27

Hence, the maximum number of columns in which they can march is 27.

iii. (c) 12 litres

Solution:

Here 768 = 28 × 3 and 420 = 22 × 3 × 5 × 7

HCF of 768 and 420 = 22 × 3 = 12

So, the container which can measure fuel of either tanker exactly must be of 12 litres.

iv. (b) 75cm

Solution:

Here, Length = 825cm, Breadth = 675cm and Height = 450cm

Also, 825 = 5 × 5 × 3 × 11, 675 = 5 × 5 × 3 × 3 × 3 and 450 = 2 × 3 × 3 × 5 × 5

HCF = 5 × 5 × 3 = 75

Therefore, the length of the longest rod which can measure the three dimensions of the room exactly is 75cm.

v. (a) 3 and 2

Solution:

LCM of 8 and 12 is 24.

∴ The least number of pack of pens

$$=\frac{24}{8}=3$$

∴ The least number of pack of note pads

$$=\frac{24}{12}=2$$

Assertion Reason Answers :

1. (a) Both A and R are true and R is the correct explanation of A.
2. (c) A is true but R is false.

Polynomials 2

1. **What is a polynomial?**

   A **polynomial** p(x) in one variable x is an algebraic expression in x of the form $P(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_2x^2 + a_1x + a_0$, where x is a variable

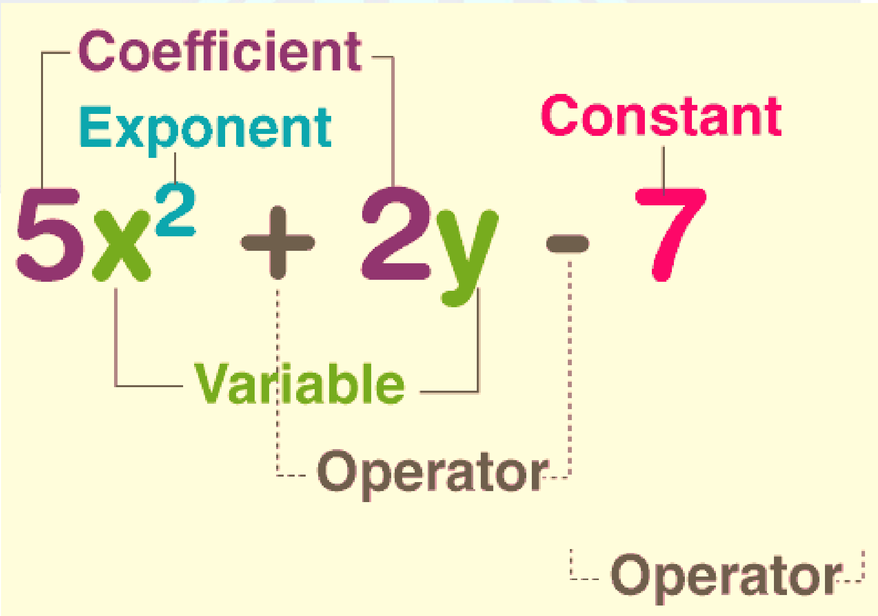
   i. $a_0, a_1, a_2 ......a_n$ are respectively the coefficients of $x^0, x^1, x^2, x^3, ... x^n$.

   ii. Each of a_nx^n, $a_{n-1}x^{n-1}$, $a_{n-2}x^{n-2}$, ... a_2x^2, a_1x, a_0, with $a_n \neq 0$, is called the term of a polynomial.

2. The highest exponent of the variable in a polynomial determines the **degree** of the polynomial.

3. Polynomials are algebraic expressions that consist of variables and coefficients. Variables are also sometimes called indeterminates. We can perform arithmetic operations such as addition, subtraction, multiplication and also positive integer exponents for polynomial expressions but not division by variable. An example of a polynomial with one variable is $x^2 + x - 12$. In this example, there are three terms: x^2, x and -12.

   The word polynomial is derived from the Greek words 'poly' means 'many' and 'nominal' means 'terms', so altogether it said "many terms". A polynomial can have any number of terms but not infinite. Learn about degree, terms, types, properties, polynomial functions in this article.



4. **Types of polynomials**

   i. A polynomial of degree zero is called a **constant polynomial**. Examples: $-9x^0, \frac{8}{14}$.

   ii. A polynomial of degree one is called a **linear polynomial**. It is of the form ax +b.

   Examples: x – 2, 4y + 89, 3x – z.

   iii. A polynomial of degree two is called a **quadratic polynomial**. It is of the form $ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$.

   Examples: $x^2 - 2x + 5$, $x^2 - 3x$ etc.

iv. A polynomial of degree 3 is called a **cubic polynomial** and has the general form $ax^3 + bx^2 + cx + d$.

For example: $x^3 + 2x^2 - 2x + 5$ etc.

Monomial: A monomial is an expression which contains only one term. For an expression to be a monomial, the single term should be a non-zero term. A few examples of monomials are:

- $5x$
- 3
- $6a^4$
- $-3xy$

Binomial: A binomial is a polynomial expression which contains exactly two terms. A binomial can be considered as a sum or difference between two or more monomials. A few examples of binomials are:

- $- 5x + 3$,
- $6a^4 + 17x$
- $xy^2 + xy$

Trinomial: A trinomial is an expression which is composed of exactly three terms. A few examples of trinomial expressions are:

- $- 8a^4 + 2x + 7$
- $4x^2 + 9x + 7$

5. **Value of the polynomial**

If $p(x)$ is a polynomial in x, and k is a real number then the value obtained after replacing x by k in $p(x)$ is called the value of $p(x)$ at $x = k$ which is denoted by $p(k)$.

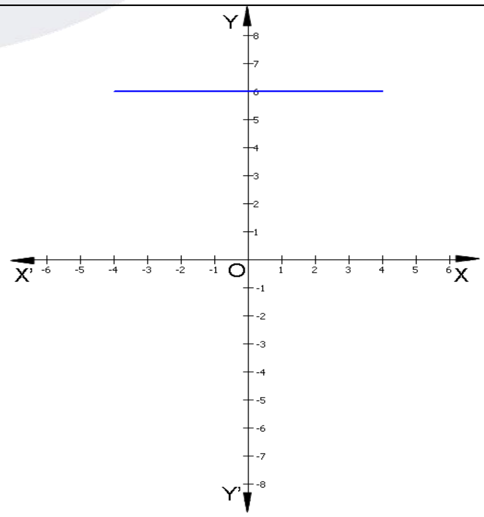
6. **Zero of a polynomial**

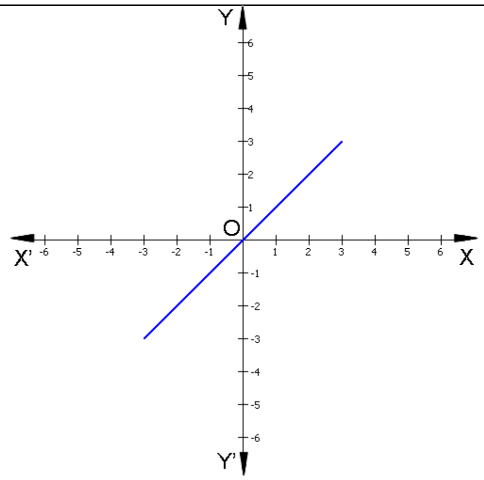
- A real number k is said to be the **zero** of the polynomial $p(x)$, if $p(k) = 0$.
- Zeroes of the polynomial can be obtained by solving the equation $p(x) = 0$.
- It is possible that a polynomial may not have a real zero atall.
- For any linear polynomial $ax + b$, the zero is given by the expression $(-b/a)$ = -(constant term)/(Coefficient of x).

7. **Number of zeroes of a polynomial**

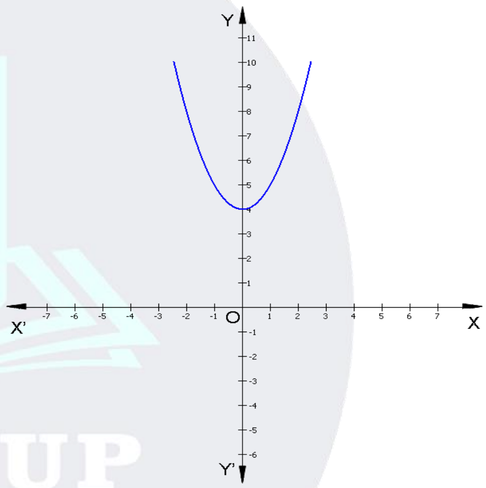
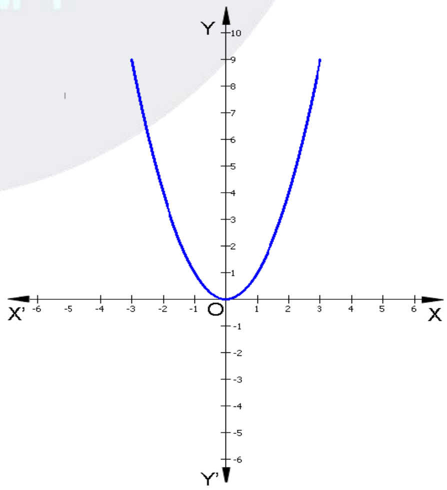
- The number of real zeros of the polynomial is the number of times its graph touches or intersects x-axis.
- The graph of a polynomial $p(x)$ of degree n intersects or touches the x-axis at at most n points.
- A polynomial of degree n has at most **n distinct real zeroes.**

8. **A linear polynomial has at most one real zero.**

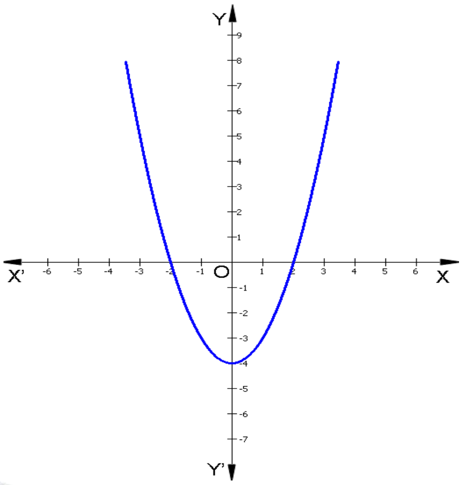
Linear Polynomial having **no zero**.	

Linear Polynomial having **one zero**.	

9. **A quadratic polynomial has at most two real zeroes.**

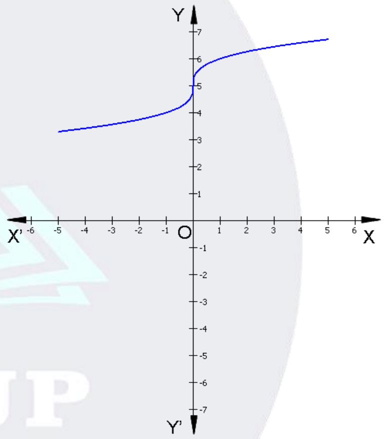
Quadratic Polynomial having no zeroes.	
Quadratic Polynomial having one zero.	

Quadratic Polynomial having two zeroes.

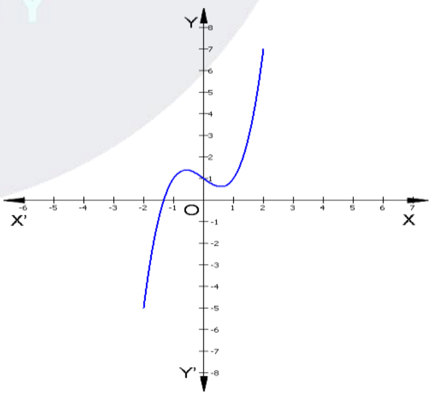


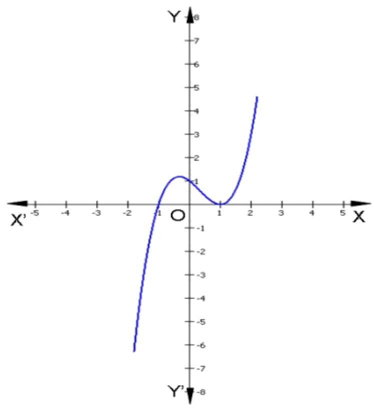
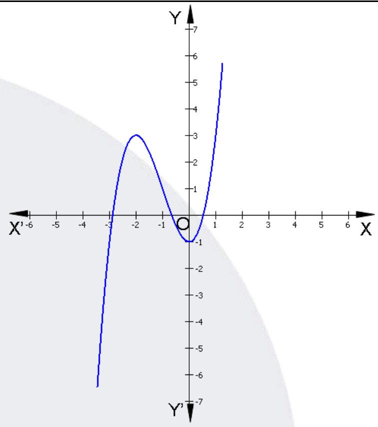
10. **A cubic polynomial has at most three real zeroes.**

Cubic Polynomial having no zeroes.



Cubic Polynomial having one zero.



Cubic Polynomial having one zeroes.	
Cubic Polynomial having three zeroes.	

11. **Relationship between zeroes and coefficients** of a polynomial:

i. For a linear polynomial ax + b, a ≠ 0, the zero is $x = \frac{-b}{a}$. It can be observed that:

$$\frac{-b}{a} = -\frac{\text{constant term}}{\text{Coefficient of x}}$$

ii. For a **quadratic polynomial** $ax^2 + bx + c$, a ≠ 0,

$$\text{Sum of the zeroes} = -\frac{b}{a} = -\frac{\text{Coefficient of x}}{\text{Coefficient of x}^2}$$

$$\text{Product of the zeroes} = -\frac{c}{a} = \frac{\text{constant term}}{\text{Coefficient of x}^2}$$

iii. For a **cubic polynomial** $ax^3 + bx^2 + cx + d = 0$, a ≠ 0,

$$\text{Sum of zeroes} = \frac{-b}{a} = -\frac{\text{Coefficient of x}^2}{\text{Coefficient of x}^3}$$

$$\text{Sum of the product of zeroes taken two at a time} = \frac{c}{a} = \frac{\text{Coefficient of x}}{\text{Coefficient of x}^3}$$

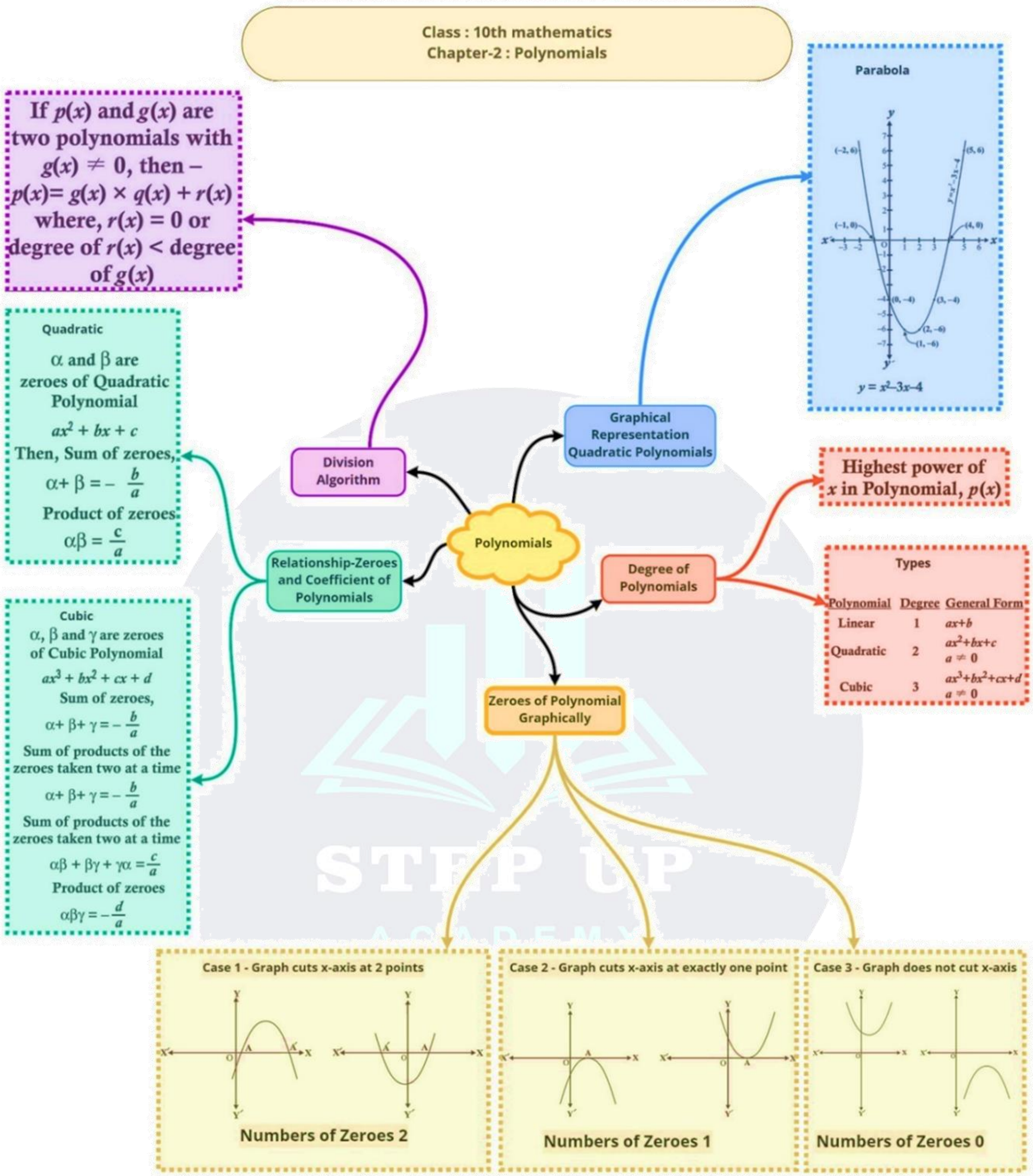
$$\text{Product of zeroes} = -\frac{d}{a} = -\frac{\text{Constant term}}{\text{Coefficient of x}^3}$$

12. The quadratic **polynomial** whose sum of the zeroes $= (\alpha + \beta)$ and product of zeroes $= (\alpha\beta)$ is given by: k $[x^2 - (\alpha + \beta)x + (\alpha\beta)]$, where k is real.

If a, b and g are the zeroes of a cubic polynomial f (x)= $ax^3 + bx^2 + cx + d$, a^1 0, then

f(x) = k (x - a)(x–b)(x–g)

f (x)= k {x^3 - (a + b + g)x^2 + (ab + bg + ga)x - abg}, where k is any non-zero real number.


Class : 10th mathematics
Chapter-2 : Polynomials
Polynomials
Division Algorithm
If p(x) and g(x) are two polynomials with g(x) ≠ 0, then – p(x)= g(x) × q(x) + r(x) where, r(x) = 0 or degree of r(x) < degree of g(x)
Graphical Representation Quadratic Polynomials
Parabola
y = x²–3x–4
Degree of Polynomials
Highest power of x in Polynomial, p(x)
Types
Polynomial Degree General Form
Linear 1 ax+b
Quadratic 2 ax²+bx+c a ≠ 0
Cubic 3 ax³+bx²+cx+d a ≠ 0
Relationship-Zeroes and Coefficient of Polynomials
Quadratic
α and β are zeroes of Quadratic Polynomial
ax² + bx + c
Then, Sum of zeroes,
α+ β = – b/a
Product of zeroes
αβ = c/a
Cubic
α, β and γ are zeroes of Cubic Polynomial
ax³ + bx² + cx + d
Sum of zeroes,
α+ β+ γ = – b/a
Sum of products of the zeroes taken two at a time
α+ β+ γ = – b/a
Sum of products of the zeroes taken two at a time
αβ + βγ + γα = c/a
Product of zeroes
αβγ = – d/a
Zeroes of Polynomial Graphically
Case 1 - Graph cuts x-axis at 2 points
Numbers of Zeroes 2
Case 2 - Graph cuts x-axis at exactly one point
Numbers of Zeroes 1
Case 3 - Graph does not cut x-axis
Numbers of Zeroes 0

Important Questions

Multiple Choice Questions

1. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is
   (a) 10
   (b) -10
   (c) 5
   (d) -5

2. If the zeroes of the quadratic polynomial $x^2 + (a + 1) x + b$ are 2 and -3, then
   (a) a = -7, b = -1
   (b) a = 5, b = -1
   (c) a = 2, b = -6
   (d) a – 0, b = -6

3. The number of polynomials having zeroes as -2 and 5 is
   (a) 1
   (b) 2
   (c) 3
   (d) more than 3

4. If one of the zeroes of the cubic polynomial x3 + $ax^2 + bx + c$ is -1, then the product of the other two zeroes is
   (a) b – a + 1
   (b) b – a – 1
   (c) a – b + 1
   (d) a – b – 1

5. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are
   (a) both positive
   (b) both negative
   (c) one positive and one negative
   (d) both equal

5. The zeroes of the quadratic polynomial $x^2 + kx + k$, k? 0,
   (a) cannot both be positive
   (b) cannot both be negative
   (c) are always unequal
   (d) are always equal

6. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, c # 0 are equal, then
   (a) c and a have opposite signs
   (b) c and b have opposite signs
   (c) c and a have the same sign
   (d) c and b have the same sign

7. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it
   (a) has no linear term and the constant term is negative.
   (b) has no linear term and the constant term is positive.
   (c) can have a linear term but the constant term is negative.
   (d) can have a linear term but the constant term is positive.

8. The number of polynomials having zeroes as 4 and 7 is
   (a) 2
   (b) 3
   (c) 4
   (d) more than 4

9. A quadratic polynomial, whose zeores are -4 and -5, is
   (a) x^2-9x + 20
   (b) $x^2 + 9x + 20$
   (c) x^2-9x- 20
   (d) $x^2 + 9x$- 20

10. The zeroes of the quadratic polynomial $x^2 + 1750x + 175000$ are
   (a) both negative
   (b) one positive and one negative
   (c) both positive
   (d) both equal

Very Short Questions:

1. Find the quadratic polynomial whose zeros are -3 and 4.

2. If one zero of the quadratic polynomial $x^2 – 5x – 6$ is 6 then find the other zero.

3. If both the zeros of the quadratic polynomial $ax^2 + bx + c$ are equal and opposite in sign, then find the value of b.
4. What number should be added to the polynomial $x^2 - 5x + 4$, so that 3 is the zero of the polynomial?
5. Can a quadratic polynomial $x^2 + kx + k$ have equal zeros for some odd integer k > 1?
6. If the zeros of a quadratic polynomial $ax^2 + bx + c$ are both negative, then can we say a, b and c all have the same sign? Justify your answer.
7. If the graph of a polynomial intersects the x-axis at only one point, can it be a quadratic polynomial?
8. If the graph of a polynomial intersects the x-axis at exactly two points, is it necessarily a quadratic polynomial?

Short Questions :

1. If one of the zeros of the quadratic polynomial $f(x) = 4x^2 - 8kx - 9$ is equal in magnitude but opposite in sign of the other, find the value of k.
2. If one of the zeros of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is -3 then find the value of k.
3. If 1 is a zero of the polynomial $p(x) = ax^2 - 3(a - 1)x - 1$, then find the value of a.
4. If α and β are zeros of polynomial $p(x) = x^2 - 5x + 6$, then find the value of α + B – 3aß.
5. Find the zeros of the polynomial $p(x) = 4x^2 - 12x + 9$.
6. Obtain the zeros of quadratic polynomial $3x^2 - 8x + 4\sqrt{3}$ and verify the relation between its zeros and coefficients.
7. If α and β are the zeros of the polynomial $6y^2 - 7y + 2$, find a quadratic polynomial whose zeros are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.
8. If one zero of the polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

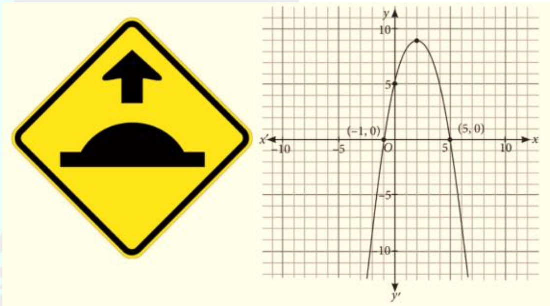
Long Questions :

1. Verify that the numbers given alongside the cubic polynomial below are their zeros. Also verify the relationship between the zeros and the coefficients.

   $x^3 - 4x^2 + 5x - 2$; 2,1,1
2. Find a cubic polynomial with the sum of the zeros, sum of the products of its zeros taken two at a time, and the product of its zeros as 2, -7, -14 respectively.
3. Find the zeros of the polynomial $f(x) = x^3 - 5x^2 - 2x + 24$, if it is given that the product of its two zeros is 12.
4. If α, β, γ bezerosofpo1ynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of α-1 + β-1 + γ-1.
5. Find the zeros of the polynomial $f(x) = -12x^2 + 39x - 28$, if it is given that the zeros are in AP.

Case Study Questions:

1. ABC construction company got the contract of making speed humps on roads. Speed humps are parabolic in shape and prevents overspeeding, minimise accidents and gives a chance for pedestrians to cross the road. The mathematical representation of a speed hump is shown in the given graph.



Based on the above information, answer the following questions.

i. The polynomial represented by the graph can be ___ polynomial.
   a. Linear
   b. Quadratic
   c. Cubic
   d. Zero

ii. The zeroes of the polynomial represented by the graph are:
   a. 1, 5
   b. 1, -5
   c. -1, 5
   d. -1, -5

iii. Sum of zeroes of the polynomial represented by the graph are:
   a. 4
   b. 5
   c. 6
   d. 7

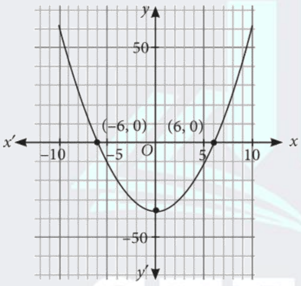
iv. If α and β are the zeroes of the polynomial represented by the graph such that β > α, β > α, then |8 α + β| = |8 α + β| =

a. 1
b. 2
c. 3
d. 4

v. The expression of the polynomial represented by the graph is:

a. $x^2 - 4x - 5$
b. $x^2 + 4x + 5$
c. $x^2 + 4x - 5$
d. $-x^2 + 4x + 5$

2. While playing in garden, Sahiba saw a honeycomb and asked her mother what is that. She replied that it's a honeycomb made by honey bees to store honey. Also, she told her that the shape of the honeycomb formed is parabolic. The mathematical representation of the honeycomb structure is shown in the graph.



Based on the above information, answer the following questions.

i. Graph of a quadratic polynomial is ____ in shape.

a. Straight line.
b. Parabolic.
c. Circular.
d. None of these.

ii. The expression of the polynomial represented by the graph is:

a. $x^2 - 49$
b. $x^2 - 64$
c. $x^2 - 36$
d. $x^2 - 81$

iii. Find the value of the polynomial represented by the graph when x = 6.

a. -2
b. -1
c. 2
d. 1

iv. The sum of zeroes of the polynomial $x^2 + 2x - 3$ is:

a. -1
b. -2
c. 2
d. 1

v. If the sum of zeroes of polynomial $at^2 + 5t + 3a$ is equal to their product, then find the value of a.

a. -5
b. -3
c. 5353
d. −53−53

Assertion Reason Questions:

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Assertion: $x^2 + 7x + 12$ has no real zeroes.

Reason: A quadratic polynomial can have at the most two zeroes.

2. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Assertion: If the sum of the zeroes of the quadratic polynomial $x^2-2kx+8$ is 2 then value of k is 1.

Reason: Sum of zeroes of a quadratic polynomial ax^2+bx+c is $-b/a$

Answer Key

Multiple Choice Questions-

1. (b) -10
2. (d) a – 0, b = -6
3. (d) more than 3
4. (a) b – a + 1
5. (b) both negative
6. (a) cannot both be positive
7. (c) c and a have the same sign
8. (a) has no linear term and the constant term is negative.
9. (d) more than 4
10. (b) $x^2 + 9x + 20$
11. (a) both negative

Very Short Answer :

1. Sum of zeros = -3 + 4 = 1,

   Product of zeros = – 3 x 4 = -12

   ∴ Required polynomial = $x^2 - x - 12$
2. Let α,6 be the zeros of given polynomial.

   Then α + 6 = 5 3 ⇒ α = -1
3. Let α and -α be the roots of given polynomial.

   Then $\alpha + (-\alpha) = 0 \Rightarrow -\frac{b}{a} = 0 \Rightarrow b = 0.$
4. Let $f(x) = x^2 - 5x + 4$

   Then $f(3) = 3^2 - 5 \times 3 + 4 = -2$

   For f(3) = 0, 2 must be added to f(x).
5. No, for equal zeros, k = 0, 4 ⇒ k should be even.
6. Yes, because $-\frac{b}{a}$ = sum of zeros < 0, so that $\frac{b}{a} = 0 >$ 0. Also the product of the zeros $= \frac{c}{a} = 0 > 0.$
7. Yes, because every quadratic polynomial has at the most two zeros.
8. No, $x^4 - 1$ is a polynomial intersecting the x-axis at exactly two points.

Short Answer :

1. Let one root of the given polynomial be α.

   Then the other root = -α

   Sum of the roots = (-α) + α = 0

   $\Rightarrow -\frac{b}{a} = 0 \text{ or } -\frac{8k}{4} = 0 \text{ or } k = 0$
2. Since – 3 is a zero of the given polynomial

   ∴ $(k - 1)(-3)^2 + k(-3) + 1 = 0$:

   ⇒ 9k – 9 – 3k + 1 = 0 ⇒ k = 4/3.
3. Put x = 1 in p(x)

   ∴ $p(1) = a(1)^2 - 3(a - 1) \times 1 - 1 = 0$

   ⇒ a – 3a + 3 – 1 = 0 ⇒ 2a = -2 ⇒ a = 1
4. Here, α + β = 5, αβ = 6

   = α + β – 3αβ = 5 – 3 x 6 = -13
5. $p(x) = 4x^2 - 12x + 9 = (2x - 3)+$

   For zeros, p(x) = 0

   $\Rightarrow (2x - 3)(2x - 3) = 0 \Rightarrow x = \frac{3}{2}$
6. We have,

   $$\alpha+\beta=-\left(\frac{-7}{6}\right)=\frac{7}{6};\ \alpha\beta=\frac{2}{6}=\frac{1}{3}$$
7. Let $p(y) = 6y^2 - 7y + 2$

   Now, $$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{7}{6\times\frac{1}{3}}=\frac{7}{2}$$

   $$\frac{1}{\alpha}\times\frac{1}{\beta}=\frac{1}{\alpha\beta}=\frac{1}{\frac{1}{3}}=3$$

   The required polynomial =

   $$y^2-\frac{7}{2}y+3=\frac{1}{2}(2y^2-7y+6)$$
8. Let α and β be the zeros of the polynomial. Then as per question β = 7α

   Now sum of zeros $=\alpha+\beta=\alpha+7\alpha=-\left(\frac{-8}{3}\right)$

   $$\Rightarrow \quad 8\alpha=\frac{8}{3} \quad \text{or} \quad \alpha=\frac{1}{3}$$

   and $$\alpha\times\beta=\alpha\times7\alpha=\frac{2k+1}{3}$$

   $$\Rightarrow \quad 7\alpha^2=\frac{2k+1}{3} \quad \Rightarrow \quad 7\left(\frac{1}{3}\right)^2=\frac{2k+1}{3}$$

   $$\left(\because\ \alpha=\frac{1}{3}\right)$$

   $$\Rightarrow \quad \frac{7}{9}=\frac{2k+1}{3} \quad \Rightarrow \quad \frac{7}{3}=2k+1$$

   $$\Rightarrow \quad \frac{7}{3}-1=2k \quad \Rightarrow \quad k=\frac{2}{3}$$

Long Answer :

1. Let p(x) = x³ – 4x² + 5x – 2

On comparing with general polynomial px) ax³ + bx² + cx + d, we get a = 1, b = -4, c = 5 and d = -2

Given zeros 2, 1, 1.

∴ p(2) = (2)³ – 4(2)² + 5(2) – 2 = 8 – 16 + 10 – 2 = 0

and p(1) = (1)³ – 4(1)² + 5(1) – 2 = 1 – 4 + 5 – 2 = 0

Hence, 2, 1 and I are the zeros of the given cubic polynomial.

Again, consider α = 2, β = 1, γ = 1

∴ α + 13 + y = 2 + 1 + 1 = 4

and $\alpha+\beta+\gamma=\frac{\text{-(Coefficient of } x^2)}{\text{Coefficient of } x^3}=\frac{-b}{a}=\frac{-(-4)}{1}=4$

$\alpha\beta+\beta\gamma+\gamma\alpha=(2)(1)+(1)(1)+(1)(2)=2+1+2=5$

and $\alpha\beta+\beta\gamma+\gamma\alpha=\frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}=\frac{c}{a}=\frac{5}{1}=5$

$\alpha\beta\gamma=(2)(1)(1)=2$

and $\alpha\beta\gamma=\frac{\text{-(Constant term)}}{\text{Coefficient of } x^3}=\frac{-d}{a}=\frac{-(-2)}{1}=2$

2. Let the cubic polynomial be p(x) = ax³ + bx² + cx + d. Then

Sum of zeros $=\frac{-b}{a}=2$

Sum of the products of zeros taken two at a time $=\frac{c}{a}=-7$

and product of the zeros $=\frac{-d}{a}=-14$

$\Rightarrow \quad \frac{b}{a}=-2,\ \frac{c}{a}=-7,\ -\frac{d}{a}=-14 \ \text{ or } \ \frac{d}{a}=14$

$\therefore \quad p(x)=ax^3+bx^2+cx+d$

$\Rightarrow \quad p(x)=a\left[x^3+\frac{b}{a}x^2+\frac{c}{a}x+\frac{d}{a}\right]$

p(x) = a[x³ + (-2)x² + (-7)x + 14] ⇒ p(x) = a[x³ – 2x² – 7x + 14]

For real value of a = 1, p(x) = x³ – 2x² – 7x + 14

3. Let α, β and γ be the zeros of polynomial (fx) such that αβ = 12.

We have, $\alpha+\beta+\gamma=\frac{-b}{a}=\frac{-(-5)}{1}=5$

$\alpha\beta+\beta\gamma+\gamma=\frac{-b}{a}=\frac{-(-5)}{1}=5$

and $\alpha\beta\gamma=\frac{-d}{a}=\frac{-24}{1}=-24$

Putting αβ = 12 in αβγ = –24, we get

$12\gamma=-24 \quad \Rightarrow \quad \gamma=-\frac{24}{12}=-2$

Now, α + β + γ = 5 α + β – 2 = 5

= α + β = 7 a = 7 – β

= (7 – β) β =12 ⇒ 7β – β² – 12

= β² + 7β + 12 = 0 ⇒ β² – 3β – 4β + 12 = O

= β = 4 or β = 3

β = 4 or β = 3

∴ α = 3 or α = 4

4. ∵ p(x) = 6x³ + 3x² – 5x + 1 so a = 6, b = 3, c = -5, d = 1

∴ α, β and γ are zeros of the polynomial p(x).

$\therefore \quad \alpha+\beta+\gamma=\frac{-b}{a}=\frac{-3}{6}=\frac{-1}{2}$

$\alpha\beta+\alpha\gamma+\beta\gamma=\frac{c}{a}=\frac{-5}{6}$ and $\alpha\beta\gamma=\frac{-d}{a}=\frac{-1}{6}$

Now, $\alpha^{-1}+\beta^{-1}+\gamma^{-1}=\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

$=\frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}=\frac{-5/6}{-1/6}=5$

5. If α, β, γ are in AP., then,

β – α = γ – β ⇒ 2β = α + γ

$\alpha+\beta+\gamma=-\frac{b}{a}=\frac{-(-12)}{1}=12$

⇒ α + γ = 12 – β .... (i)

From (i) and (ii)

2β = 12 – β or 3β = 12 or β = 4

Putting the value of β in (i), we have

8 = α + γ

$\alpha\beta\gamma=-\frac{d}{a}=\frac{-(-28)}{1}=28$ .... (iii)

(αγ) 4 = 28 or αγ = 7 or γ = 7α ..... (iv)

Putting the value of γ = 7α in (iii), we get

$\Rightarrow 8=\alpha+\frac{7}{\alpha} \Rightarrow 8\alpha=\alpha^2+7$

⇒ α² – 8α + 7 = 0 ⇒ α² – 7α – 1α + 7 = 0

⇒ α(α – 7)-1 (α – 7) = 0 ⇒ (α – 1)(α – 7) = 0

⇒ α = 1 or α = 7

Putting $\alpha = 1$ in (iv), we get

$$\gamma = \frac{7}{1}$$

or $\gamma = 7$

and $\beta = 4$

$\therefore$ zeros are 1, 7, 4.

Putting $\alpha = 1$ in (iv), we get

$$\gamma = \frac{7}{7}$$

or $\gamma = 1$

and $\beta = 4$

$\therefore$ zeros are 7, 4, 1.

Case Study Answers:

1. **Answer :**

i. (b) Quadratic

Solution:

Since, the given graph is parabolic is shape, therefore it will represent a quadratic polynomial.

[∵ Graph of quadratic polynomial is parabolic in shape)

ii. (c) -1, 5

Solution:

Since, the graph cuts the x-axis at -1, 5. So the polynomial has 2 zeroes i.e., -1 and 5.

iii. (a) 4

Solution:

Sum of zeroes = -1 + 5 = 4

iv. (c) 3

Solution:

Since α and β are zeroes of the given polynomial and $\beta > \alpha$, $\beta > \alpha$,

$\therefore \alpha = -1 \therefore \alpha = -1$ and $\beta = 5\beta = 5$

$\therefore |8\alpha + \beta| = |8(-1) + 5| = |-8 + 5| = |-3| = 3.$

$\therefore |8\alpha + \beta| = |8(-1)+5| = |-8+5| = |-3| = 3.$

v. (d) $-x^2 + 4x + 5$

Solution:

Since the zeroes of the given polynomial are -1 and 5.

$\therefore$ Required polynomial p(x)

$= k^2 \{x^2 - (-1 + 5)x + (-1)(5)\} = k(x^2 - 4x - 5)$

For k = -1, we get,

$p(x) = -x^2 + 4x + 5$, which is the required polynomial.

2. **Answer :**

i. (b) Parabolic.

Solution:

Graph of a quadratic polynomial is a parabolic in shape.

ii. (c) $x^2 - 36$

Solution:

Since the graph of the polynomial cuts the x-axis at (-6, 0) and (6, 0). So, the zeroes of polynomial are -6 and 6.

$\therefore$ Required polynomial is

$p(x) = x^2 - (-6 + 6)x + (-6)(6) = x^2 - 36$

iii. (c) 2

Solution:

We have, $p(x) = x^2 - 36$

Now, $p(6) = 6^2 - 36 = 36 - 36 = 0$

iv. (b) -2

Solution:

Let $f(x) = x^2 + 2x - 3$. Then,

$$\text{Sum of zeroes} = \frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$= -\frac{(2)}{1} = -2$$

v. (d) −53−53

Solution:

The given polynomial is $at^2 + 5t + 3a$

Given, sum of zeroes = product of zeroes

$$\Rightarrow \frac{-5}{a} = \frac{3a}{a}$$

$$\Rightarrow a = \frac{-5}{3}$$

Assertion Reason Answers:

1. (d) Assertion (A) is false but reason (R) is true.
2. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Pair of Linear Equations in Two Variables 3

1. A pair of Linear Equations in two variables:

- An equation of the form ax + by + c = 0, where a, b and c are real numbers, such that a and bare not both zero, is called a **linear equation in two variables**.
- Two linear equations in same two variables x and y are called **pair of linear equations in two variables.**

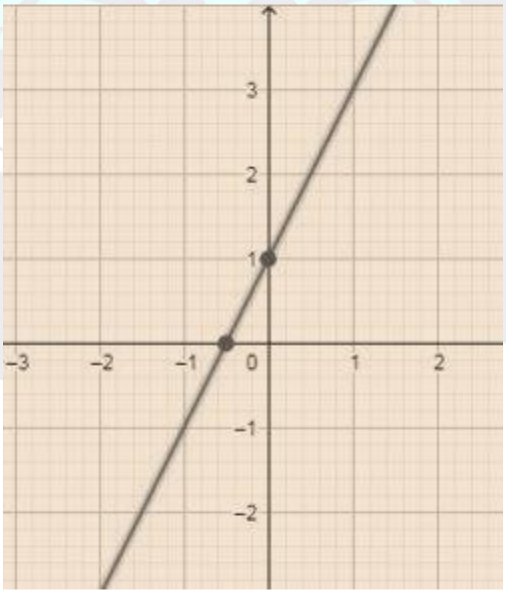
Geometrical Representation of a Linear Equation

Geometrically, a linear equation in two variables can be represented as a straight line.

2x – y + 1 = 0

⇒ y = 2x + 1

Graph of y = 2x + 1



Plotting a Straight Line

The graph of a linear equation in two variables is a straight line. We plot the straight line as follows:

- Take any value for one of the variables (x_1=0) and substitute it in the equation to get the corresponding value of the other variable (y_1).
- Repeat this again (put y_2 = 0, get x_2) to get two pairs of values for the variables which represent two points on the Cartesian plane. Draw a line through the two points.

2. Types of Polynomials based on Degree

Linear Polynomial

A polynomial whose degree is one is called a linear polynomial.

For example, 2x+1 is a linear polynomial.

Quadratic Polynomial

A polynomial of degree two is called a quadratic polynomial.

For example, $3x^2 + 8x + 5$ is a quadratic polynomial.

Cubic Polynomial

A polynomial of degree three is called a cubic polynomial.

For example, $2x^3 + 5x^2 + 9x + 15$ is a cubic polynomial.

3. Graph of the polynomial x^n

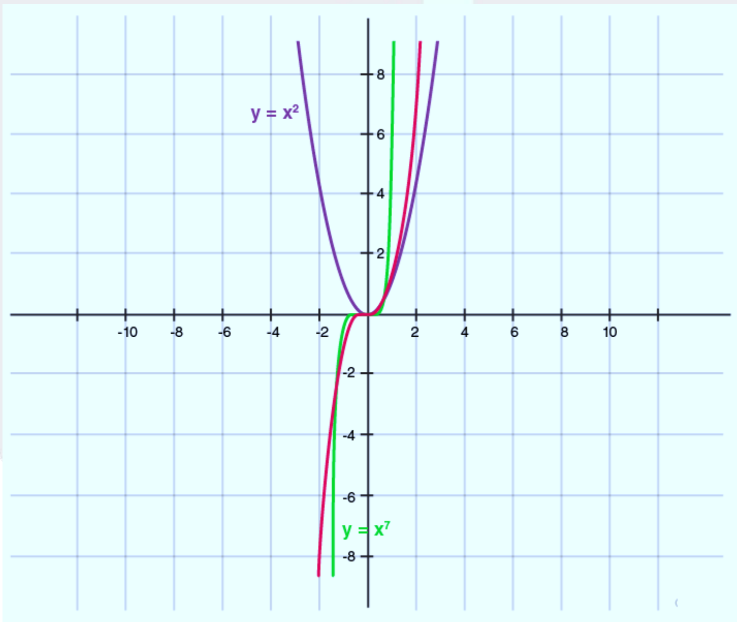
For a polynomial of the form $y = x^n$ where n is a whole number:

as n increases, the graph becomes steeper or draws closer to the Y-axis

If n is odd, the graph lies in the first and third quadrants

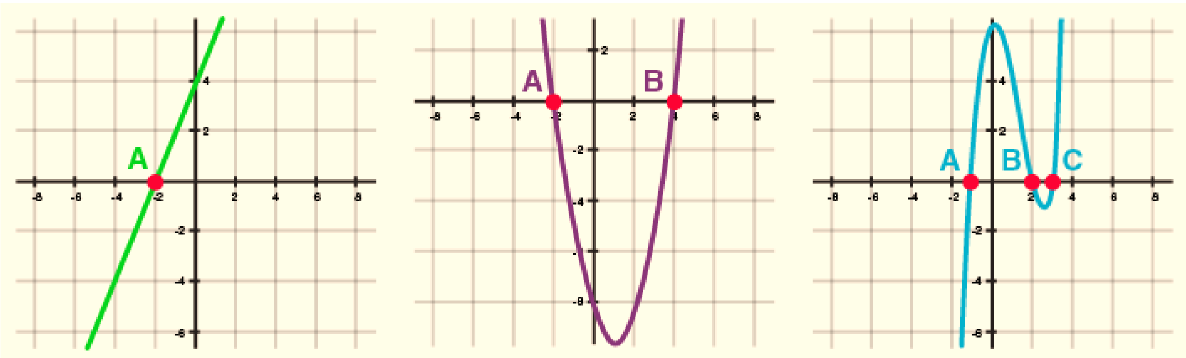
If n is even, the graph lies in the first and second quadrants

The graph of $y = -x^n$ is the reflection of the graph of $y = x^n$ on the x-axis



4. Geometrical Meaning of Zeros of a Polynomial

Geometrically, zeros of a polynomial are the points where its graph cuts the x-axis.



(i) One zero **(ii) Two zeros** **(iii) Three zeros**

Here A, B and C correspond to the zeros of the polynomial represented by the graphs.

Number of Zeros

In general, a polynomial of degree n has at most n zeros.

- A linear polynomial has one zero,
- A quadratic polynomial has at most two zeros.
- A cubic polynomial has at most 3 zeros.

5. The **general form** of a pair of linear equations in two variables is

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

where a_1, a_2, b_1, b_2, c_1 and c_2 are real numbers, such that

6. A system of linear equations in two variables represents two lines in a plane. For two given lines in a plane there could be three possible cases:

i. The two lines are intersecting, i. e., they **intersect at one point**.

ii. The two lines are **parallel**, i.e., they do not intersect at any real point.

iii. The two lines are **coincident** lines, i.e., one line overlaps the other line.

7. A system of simultaneous linear equations is said to be

➢ **Consistent**, if it has **at least one solution.**

➢ **In-consistent**, if it has **no solution.**

8. If the lines

i. Intersect at a point, then that point gives the **unique solution** of the system of equations. In this case system of equations is said to be **consistent.**

ii. Coincide (overlap), then the pair of equations will have **infinitely many solutions**. System of equations is said to be **consistent.**

iii. are parallel, then the pair of equations has **no solution**. In this case pair of equations is said to be **inconsistent.**

9. Solution of a pair of Linear Equations in two variable:

System of equations can be solved using **Algebraic** and **Graphical Methods.**

10. **Graphical Method:**

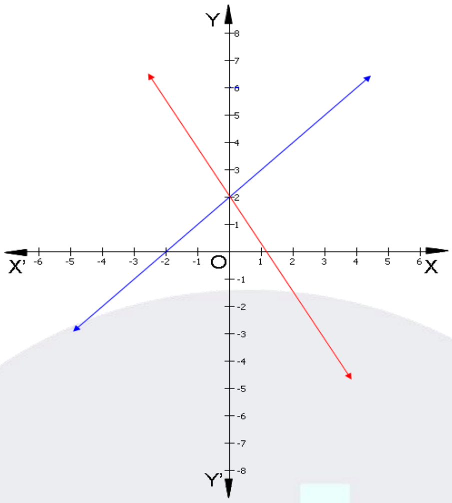
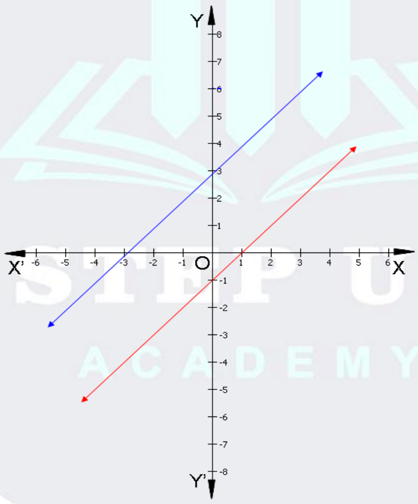
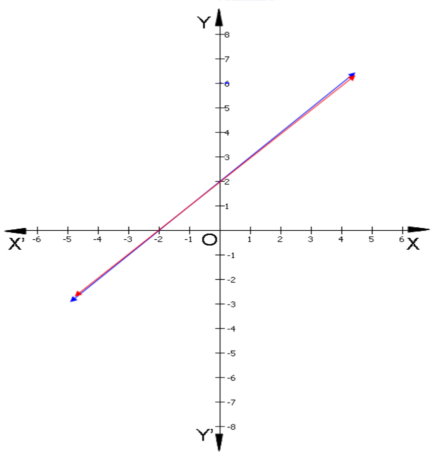
➢ A linear equation in two variables is represented geometrically by a **straight line.**

➢ The graph of a pair of linear equations in two variables is represented by two lines. Steps:

i. Draw the graphs of both the equations by finding two solutions for each.

ii. Plot the points and draw the lines passing through them to represent the equations.

iii. The behaviour of lines representing a pair of linear equations in two variables and the existence of solutions can be summarised as follows:

Ratio of Coefficients	Graphical Representation	Nature of Solution	Defined as
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Lines are intersecting 	Unique solution	Consistent pair of equations
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Lines are parallel 	No solution	Inconsistent pair of equations
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Lines are coincident 	Infinitely many solutions	Dependent (consistent) pair of equations

11. Algebraic Method:

The most commonly used **Algebraic Methods** to solve a pair of linear equations in two variables are:

i. Substitution method

ii. Elimination method

iii. Cross-multiplication method

12. Substitution Method:

Steps followed for solving linear equations in two variables,using **substitution method**:

Step 1: Express the value of one variable, say y in terms of other variable x from either equation, whichever is convenient.

Step 2: Substitute the value of y in other equation and reduce it to an equation in one variable, i.e. in terms of x. There will be three possibilities:

a. If reduced equation is linear in x, then solve it for x to get **a unique solution.**

b. If reduced equation is a true statement without x, then system has **infinite solutions.**

c. If reduced equation is a false statement without x, then system has **no solution.**

Step 3: Substitute the value of x obtained in step 2, in the equation used in step 1, to obtain the value of y.

Step 4: The values of x and y so obtained is the coordinates of the solution of system of equations.

13. Elimination Method:

Steps followed for solving linear equations in two variables, by **elimination Method**:

Step 1: Multiply both the equations by some suitable non-zero constants to make the coefficients of variable x (or y) equal.

Step 2: Add or subtract both the equations to eliminate the variable whose coefficients are equal.

a. If an equation in one variable y (or x) is obtained, solve it for variable y (or x).

b. If a true statement involving no variable is obtained then the system has **infinite solutions**.

c. If a false statement involving no variable is obtained then the system has **no solution**.

Step 3: Substitute the value of variable y (or x) in either of the equation to get the value of other variable.

14. Equations which are not linear but can be reduced to linear form by some suitable substitutions are called equations reducible to linear form.

Reduced equation can be solved by any of the algebraic method (substitution, elimination or cross multiplication) of solving linear equation.

15. While solving problems based on time, distance and speed; following knowledge may be useful:

If speed of a boat in still water = u km/hr,

Speed of the current = v km/hr Then,

Speed upstream = (u - v) km/hr

Speed downstream = (u + v) km/hr

16. Factorization of Polynomials

Quadratic polynomials can be factorized by splitting the middle term.

For example, consider the polynomial $2x^2 - 5x + 3$

Splitting the middle term:

The middle term in the polynomial $2x^2 - 5x + 3$ is -5x. This must be expressed as a sum of two terms such that the product of their coefficients is equal to the product of 2 and 3 (coefficient of x^2 and the constant term)

-5 can be expressed as $(-2) + (-3)$, as $-2 \times -3 = 6 = 2 \times 3$

Thus, $2x^2 - 5x + 3 = 2x^2 - 2x - 3x + 3$

Now, identify the common factors in individual groups

$2x^2 - 2x - 3x + 3 = 2x\,(x-1) - 3\,(x-1)$

Taking $(x-1)$ as the common factor, this can be expressed as:

$2x\,(x-1) - 3\,(x-1) = (x-1)\,(2x-3)$

17. Relationship between Zeroes and Coefficients of a Polynomial

For Quadratic Polynomial:

If α and β are the roots of a quadratic polynomial ax^2+bx+c, then,

$\alpha + \beta = -b/a$

Sum of zeroes = -coefficient of x /coefficient of x^2

$\alpha\beta = c/a$

Product of zeroes = constant term / coefficient of x^2

For Cubic Polynomial

If α, β and γ are the roots of a cubic polynomial ax^3+bx^2+cx+d, then

$\alpha + \beta + \gamma = -b/a$

$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$

$\alpha\beta\gamma = -d/a$

18. Division Algorithm

To divide one polynomial by another, follow the steps given below.

Step 1: Arrange the terms of the dividend and the divisor in the decreasing order of their degrees.

Step 2: To obtain the first term of the quotient, divide the highest degree term of the dividend by the highest degree term of the divisor Then carry out the division process.

Step 3: The remainder from the previous division becomes the dividend for the next step. Repeat this process until the degree of the remainder is less than the degree of the divisor.

$$
\begin{array}{r|l}
 & x-2 \\
\hline
-x^2+x-1 & -x^3+3x^2-3x+5 \\
 & \mp x^3 \pm x^2 \mp x \\
\hline
 & 2x^2-2x+5 \\
 & \_2x^2-2x+2 \\
\hline
 & 3
\end{array}
$$

Class : 10th mathematics
Chapter- 3: Pair of Linear Equations in Two Variables

Pair of Linear Equations in Two Variables

Algebraic Methods

Substitution

Solve: $7x-15y = 2$ –(i)
$x+2y = 3$ –(ii)

Solution: From equation (ii), $x = 3-2y$
substitute value of x in eq. (i)
$7(3-2y)-15y = 2$
$-29y = -19 \Leftrightarrow y = \frac{19}{29}$
Now, from $x = 3-2y$
$x = 3-2\left(\frac{19}{29}\right) = \left(\frac{49}{29}\right)$

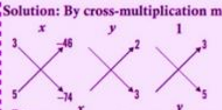
By Elimination

Solve: $x+3y = 6$ – (i)
$2x+3y = 12$ – (ii)
Now, Adding equation (i) and (ii)
$3x = 18$ or $x = 6$
Again, from (i) ×2– (ii)
$3y = 0$ or, $y = 0$
Hence, $x = 6, y = 0$

By Cross Multiplication

Solve: $2x+3y-46 = 0$ –(i)
$3x+5y-74 = 0$ –(ii)

Solution: By cross-multiplication method



Then, $\frac{x}{3(-74)-5(-46)} = \frac{y}{(-46)(3)-(-74)(2)} = \frac{1}{2(5)-3(3)}$

$\frac{x}{-222+230} = \frac{y}{-138+148} = \frac{1}{10-9}$

$\frac{x}{8} = \frac{y}{10} = \frac{1}{1} \Leftrightarrow \frac{x}{8} = \frac{1}{1}$ and $\frac{y}{10} = \frac{1}{1}$

i.e. $x = 8$ and $y = 10$

General Form

$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$
$a_1, b_1, c_1, a_2, b_2, c_2$, – Real numbers

Solution Graphically

Each solution (x, y), corresponds to a point on the line representing the equation and vice-versa

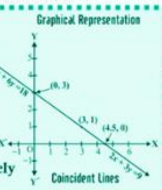
Graphically Presentation

Pair of Lines = $2x+3y-9 = 0$
$4x+6y-18 = 0$

$\frac{a_1}{a_2} = \frac{2}{4}, \frac{b_1}{b_2} = \frac{3}{6}, \frac{c_1}{c_2} = \frac{-3}{-18}$

Compare the Ratios = $\frac{a}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Algebraic Interpretation = Infinitely many solutions – Dependent

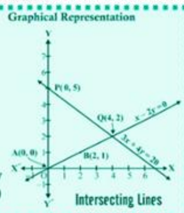


Pair of Lines = $x-2y = 0$
$3x+4y-20 = 0$

$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-2}{4}, \frac{c_1}{c_2} = \frac{0}{-20}$

Compare the Ratios = $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Algebraic Interpretation : Exactly one solution – consistent (unique)

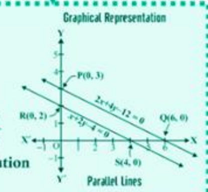


Pair of Lines = $x+2y-4 = 0$
$2x+4y-12 = 0$

$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4}, \frac{c_1}{c_2} = \frac{-4}{-12}$

Compare the Ratios = $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Algebraic Interpretation : No solution – Inconsistent



Important Questions

Multiple Choice Questions-

1. Graphically, the pair of equations $7x - y = 5$; $21x - 3y = 10$ represents two lines which are
   (a) intersecting at one point
   (b) parallel
   (c) intersecting at two points
   (d) coincident
2. The pair of equations $3x - 5y = 7$ and $-6x + 10y = 7$ have
   (a) a unique solution
   (b) infinitely many solutions
   (c) no solution
   (d) two solutions
3. If a pair of linear equations is consistent, then the lines will be
   (a) always coincident
   (b) parallel
   (c) always intersecting
   (d) intersecting or coincident
4. The pair of equations $x = 0$ and $x = 5$ has
   (a) no solution
   (b) unique/one solution
   (c) two solutions
   (d) infinitely many solutions
5. The pair of equation $x = -4$ and $y = -5$ graphically represents lines which are
   (a) intersecting at (- 5, – 4)
   (b) intersecting at (- 4, – 5)
   (c) intersecting at (5, 4)
   (d) intersecting at (4, 5)
6. One equation of a pair of dependent linear equations is $2x + 5y = 3$. The second equation will be
   (a) $2x + 5y = 6$
   (b) $3x + 5y = 3$
   (c) $-10x - 25y + 15 = 0$
   (d) $10x + 25y = 15$
7. If $x = a$, $y = b$ is the solution of the equations $x + y = 5$ and $2x - 3y = 4$, then the values of a and b are respectively
   (a) 6, -1
   (b) 2, 3
   (c) 1, 4
   (d) 19/5, 6/5
8. The graph of $x = -2$ is a line parallel to the
   (a) x-axis
   (b) y-axis
   (c) both x- and y-axis
   (d) none of these
9. The graph of $y = 4x$ is a line
   (a) parallel to x-axis
   (b) parallel to y-axis
   (c) perpendicular to y-axis
   (d) passing through the origin
10. The graph of $y = 5$ is a line parallel to the
   (a) x-axis
   (b) y-axis
   (c) both axis
   (d) none of these

Very Short Questions:

1. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then find value of k.
2. Find the value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions.
3. Do the equations $4x + 3y - 1 = 5$ and $12x + 9y = 15$ represent a pair of coincident lines?
4. Find the co-ordinate where the line $x - y = 8$ will intersect y-axis.
5. Write the number of solutions of the following pair of linear equations:
   $x + 2y - 8 = 0$, $2x + 4y = 16$
6. Is the following pair of linear equations consistent? Justify your answer.
   $2ax + by = a$, $4ax + 2by - 2a = 0$; $a, b \neq 0$
7. For all real values of c, the pair of equations
   $x - 2y = 8$, $5x + 10y = c$
   have a unique solution. Justify whether it is true or false.
8. Does the following pair of equations represent a pair of coincident lines? Justify your answer.
   $\frac{x}{2} + y + \frac{2}{5} = 0, 4x + 8y + \frac{5}{16} = 0.$
9. If $x = a$, $y = b$ is the solution of the pair of equation $x - y = 2$ and $x + y = 4$, then find the value of a and b.
10. $\frac{3}{2}x + \frac{5}{3}y = 7$
    $9x - 10y = 14$

Short Questions :

1. Solve: ax + by = a – b and bx – ay = a + b
2. Solve the following linear equations:

   152x – 378y = -74 and -378x + 152y = -604
3. Solve for x and y

   $\frac{b}{a}x+\frac{a}{b}y=a^2+b^2; \quad x+y=2ab$
4. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

   2x + 3y = 7

   (a – b)x + (a + b)y = 3a + b – 2

   (ii) for which value of k will the following pair of linear equations have no solution?

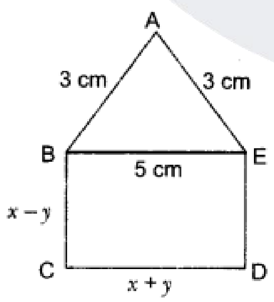
   3x + y = 1

   (2k – 1)x + (k – 1)y = 2k + 1
5. Find whether the following pair of linear equations has a unique solution. If yes, find the

   7x – 4y = 49 and 5x – y = 57
6. Solve for x and y.

   $\frac{6}{x-1}-\frac{3}{y-2}=1; \frac{5}{x-1}+\frac{1}{y-2}$

   where $x \neq 1, y \neq 2$
7. In ΔABC, LA = x, ∠B = 3x, and ∠C = y if 3y – 5x = 30°, show that triangle is right angled.
8. In Fig. 3.1, ABCDE is a pentagon with BE||CD and BC||DE. BC is perpendicular to CD. If the perimeter of ABCDE is 21 cm. Find the value of x and y.



9. Five years ago, A was thrice as old as B and ten years later, A shall be twice as old as B. What are the present ages of A and B?

Long Questions :

1. Form the pair of linear equations in this problem and find its solution graphically: 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.
2. Show graphically the given system of equations

   2x + 4y = 10 and 3x + 6y = 12 has no solution.
3. Solve the following pairs of linear equations by the elimination method and the substitution method:

   (i) 3x – 5y – 4 = 0 and 9x = 2y + 7

   (ii) $\frac{x}{2}+\frac{2y}{3} = -1$ and $x-\frac{y}{3} = 3$
4. Draw the graph of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.
5. A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days, she has to pay Rs. 1000/- as hostel charges whereas a student B, who takes food for 26 days, pays 1180 as hostel charges. Find the fixed charges and the cost of food per day.
6. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?
7. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by one man alone and that by one boy alone to finish the work.
8. A boat covers 25 km upstream and 44 km downstream in 9 hours. Also, it covers 15 km upstream and 22 km downstream in 5 hours. Find the speed of the boat in still water and that of the stream.

Case Study Questions:

1. A part of monthly hostel charges in a college is fixed and the remaining depends on the number of days one has taken food in the mess. When a student Anu takes food for 25 days, she has to pay ₹ 4500 as hostel charges, whereas another student Bindu who takes food for 30 days, has to pay ₹ 5200 as hostel charges.

Considering the fixed charges per month by ₹ x and the cost of food per day by ₹ y, then answer the following questions.

i. Represent algebraically the situation faced by both Anu and Bindu.
   a. x + 25y = 4500, x + 30y = 5200
   b. 25x + y = 4500, 30x + y = 5200
   c. x - 25y = 4500, x - 30y = 5200
   d. 25x - y = 4500, 30x - y = 5200

ii. The system of linear equations, represented by above situations has.
   a. No solution.
   b. Unique solution.
   c. Infinitely many solutions.
   d. None of these.

iii. The cost of food per day is:
   a. ₹ 120
   b. ₹ 130
   c. ₹ 140
   d. ₹ 1300

iv. The fixed charges per month for the hostel is:
   a. ₹ 1500
   b. ₹ 1200
   c. ₹ 1000
   d. ₹ 1300

v. If Bindu takes food for 20 days, then what amount she has to pay?
   a. ₹ 4000
   b. ₹ 3500
   c. ₹ 3600
   d. ₹ 3800

2. From Bengaluru bus stand, if Riddhima buys 2 tickets to Malleswaram and 3 tickets to Yeswanthpur, then total cost is ₹ 46; but if she buys 3 tickets to Malleswaram and 5 tickets to Yeswanthpur, then total cost is ₹ 74.

Consider the fares from Bengaluru to Malleswaram and that to Yeswanthpur as ₹ x and ₹ y respectively and answer the following questions.

i. 1st situation can be represented algebraically as:
   a. 3x - 5y = 74
   b. 2x + 5y = 74
   c. 2x - 3y = 46
   d. 2x + 3y = 46

ii. 2nd situation can be represented algebraically as:
   a. 5x + 3y = 74
   b. 5x - 3y = 74
   c. 3x + 5y = 74
   d. 3x - 5y = 74

iii. Fare from Bengaluru to Malleswaram is:
   a. ₹ 6
   b. ₹ 8
   c. ₹ 10
   d. ₹ 2

iv. Fare from Bengaluru to Yeswanthpur is:
   a. ₹ 10
   b. ₹ 12
   c. ₹ 14
   d. ₹ 16

v. The system of linear equations represented by both situations has:
   a. Infinitely many solutions.
   b. No solution.
   c. Unique solution.
   d. None of these.

Assertion reason questions-

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
   a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. (C) Assertion (A) is true but reason (R) is false.

d. (d) Assertion (A) is false but reason (R) is true.

Assertion: The graph of the linear equations 3x+2y=12 and 5x-2y=4 gives a pair of intersecting lines.

Reason: The graph of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ gives a pair of intersecting lines if $a_1/a_2 \neq b_1/b_2$

2. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Assertion: If the pair of lines are coincident, then we say that pair of lines is consistent and it has a unique solution.

Reason: If the pair of lines are parallel, then the pairs has no solution and is called inconsistent pair of equations.

Answer Key

Multiple Choice Questions-

1. (b) -10
2. (d) a – 0, b = -6
3. (d) more than 3
4. (a) b – a + 1
5. (b) both negative
6. (a) cannot both be positive
7. (c) c and a have the same sign
8. (a) has no linear term and the constant term is negative.
9. (d) more than 4
10. (b) $x^2 + 9x + 20$
11. (a) both negative

Very Short Answer :

1. Since the given lines are parallel

$$\therefore \frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1} \quad i.e., k = \frac{15}{4}.$$

2. The given system of equations will have infinitely many solutions if $\frac{c}{6} = \frac{-1}{-2} = \frac{2}{3}$ which is not possible

∴ For no value of c, the given system of equations have infinitely many solutions.

3. Here, $\frac{4}{12} = \frac{3}{9} \neq \frac{6}{15} \quad i.e., \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Given equations do not represent a pair of coincident lines.

4. The given line will intersect y-axis when x = 0.

∴ 0 – y = 8 ⇒ y = -8

Required coordinate is (0, -8).

5. Here, $\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ The given pair of linear equations has infinitely many solutions.

6. Yes,

Here, $\frac{a_1}{a_2} = \frac{2a}{4a} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{b}{2b} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-a}{-2a} = \frac{1}{2}$

$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ The given system of equations is consistent.

7. Here, $\frac{a_1}{a_2} = \frac{1}{5}, \frac{b_1}{b_2} = \frac{-2}{+10} = \frac{-1}{5}, \frac{c_1}{c_2} = \frac{8}{c}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

8. Here, $a_1 = \frac{1}{2}, \quad b_1 = 1, \quad c_1 = \frac{2}{5}$

and $a_2 = 4, \ b_2 = 8, \ c_2 = \frac{5}{16}$

$$\frac{a_1}{a_2}=\frac{\frac{1}{2}}{4}=\frac{1}{8},\quad \frac{b_1}{b_2}=\frac{1}{8},\quad \frac{c_1}{c_2}=\frac{\frac{2}{5}}{\frac{5}{16}}=\frac{32}{25}$$

$\because \quad \frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

∴ The given system does not represent a pair of coincident lines.

x – y = 2 ... (i)

x + y = 4 ... (ii)

9. On adding (i) and (ii), we get 2x = 6 or x = 3

From (i), 3 – y ⇒ 2 = y = 1

a = 3, b = 1.

On comparing the ratios $\frac{a_1}{a_2},\frac{b_1}{b_2}$, and, $\frac{c_1}{c_2}$ find out whether the following pair of linear equations consistent or inconsistent. is consistent or inconsistent.

10. We have, $\frac{3}{2}x+\frac{5}{3}y=7$ ....(i)

$9x - 10y = 14$ ....(ii)

Here, $a_1=\frac{3}{2},\ b_1=\frac{5}{3},\ c_1=7$

$a_2 = 9,\ b_2 = -10,\ c_2 = 14$

Thus, $\frac{a_1}{a_2}=\frac{3}{2\times9}=\frac{1}{6},\ \frac{b_1}{b_2}=\frac{5}{3\times(-10)}=-\frac{1}{6}$

Hence, $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$. So, it has unique solution and it is consistent.

Short Answer :

1. The given system of equations may be written as

ax + by – (a – b) = 0

bx – ay – (a + b) = 0

By cross-multiplication, we have

$$\frac{x}{\begin{matrix} b & -(a-b) \\ -a & -(a+b)\end{matrix}}=\frac{-y}{\begin{matrix} a & -(a-b) \\ b & -(a+b)\end{matrix}}=\frac{1}{\begin{matrix} a & b \\ b & -a\end{matrix}}$$

$$\Rightarrow \frac{x}{b\times-(a-b)-(-a)\times-(a-b)}$$

$$=\frac{-y}{a\times-(a+b)-b\times-(a-b)}=\frac{1}{-a^2-b^2}$$

$$\Rightarrow \frac{x}{-b(a+b)-(-a)\times-(a-b)}$$

$$\frac{-y}{-a(a+b)+b(a-b)}=\frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \frac{x}{-b^2-a^2}=\frac{-y}{-a^2-b^2}=\frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \frac{x}{-(a^2-b^2)}=\frac{y}{(a^2+b^2)}=\frac{1}{-(a^2+b^2)}$$

$$\Rightarrow x=-\frac{(a^2+b^2)}{-(a^2+b^2)}=1 \ \ and \ \ y=\frac{(a^2+b^2)}{-(a^2+b^2)}=-1$$

Hence, the solution of the given system of equations is x = 1, y = –1

2. We have, 152x – 378y = -74 ...(i)

-378x + 152y = -604 ......(ii)

Adding equation (i) and (ii), we get

$$\begin{array}{r} 152x-378y=-74 \\ -378x+152y=-604 \\ \hline -226x-226y=-678 \end{array}$$

$\Rightarrow \quad -226(x+y) = -678$

$\Rightarrow \quad x+y=\frac{-678}{-226}$

$\Rightarrow \quad x + y = 3$ ....(iii)

Subtracting equation (ii) from (i), we get

$$\begin{array}{r} 152x-378y=-74 \\ \mp378x\pm152y=\mp604 \\ \hline 530x-530y=530 \end{array}$$

$\Rightarrow \quad x - y = 1$ ....(iv)

Adding equations (iii) and (iv), we get

$$\begin{array}{r} x+y=3 \\ x-y=1 \\ \hline 2x=4 \end{array} \quad \Rightarrow \quad x = 2$$

Putting the value of x in (iii), we get

2 + y = 3 ⇒ y = 1

Hence, the solution of given system of equations is x = 2, y = 1.

3. We have, $\frac{b}{a}x+\frac{a}{b}y=a^2+b^2$ ....(i)

$x + y = 2ab$ ....(ii)

Multiplying (ii) by b/a, we get

$\frac{b}{a}x+\frac{b}{a}y=2b^2$ ....(iii)

Subtracting (iii) from (i), we get

$$\left(\frac{a}{b}-\frac{b}{a}\right)y=a^2+b^2-2b^2$$

$$\Rightarrow \left(\frac{a^2-b^2}{ab}\right)y=(a^2-b^2)$$

$$\Rightarrow y=(a^2-b^2)\times\frac{ab}{(a^2-b^2)}$$

$$\Rightarrow y = ab$$

4. (i) We have, 2x + 3y = 7

(a – b) x + (a + b) y = 3a + b – 2 ... (ii)

Here, a_1 = 2, b_1 = 3, c_1 = 7 and

a_2 = a – b, b_2 = a + b, c_2 = 3a + b – 2

For infinite number of solutions, we have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2} \Rightarrow \frac{2}{a-b}=\frac{3}{a+b}=\frac{7}{3a+b-2}$$

Now, $\frac{2}{a-b}=\frac{3}{a+b}$

⇒ $2a + 2b = 3a - 3b$

⇒ $2a - 3a = 3b - 2b$

⇒ $-a = -5b$ ....(iii)

Again, we have

$$\frac{3}{a+b}=\frac{7}{3a+b-2}$$

⇒ $9a + 3b - 6 = 7a + 7b$

⇒ 9a – 7a + 3b – 75 -6 = 0 ⇒ 2a – 45 – 6 = 0 => 2a – 4b = 6

⇒ a – 2b = 3 ...(iv)

Putting a = 5b in equation (iv), we get

56 – 2b = 3 or 3b = 3 i.e., b = $\frac{3}{3}$ =1

Putting the value of b in equation (ii), we get a = 5(1) = 5

Hence, the given system of equations will have an infinite number of solutions for a = 5 and b = 1.

(ii) We have, 3x + y = 1, 3x + y – 1 = 0 ...(i)

(2k – 1) x + (k – 1) y = 2k + 1

⇒ (2k – 1) x + (k – 1) y – (2k + 1) = 0 ......(ii)

Here, a_1 = 3, b_1 = 1, C_1 = -1

a_2 = 2k – 1, b_2 = k – 1, C_2 = -(2k + 1)

For no solution, we must have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2} \Rightarrow \frac{3}{2k-1}=\frac{1}{k-1}\neq\frac{1}{2k+1}$$

Now, $\frac{3}{2k-1}=\frac{1}{k-1}$

⇒ $3k - 3 = 2k - 1$

⇒ 3k – 2k = 3 – 1 ⇒ k = 2

5. Hence, the given system of equations will have no solutions for k = 2.

We have, 7x – 4y = 49 ....(i)

and 5x – 6y = 57 ....(ii)

Here, $a_1 = 7, b_1 = -4, c_1 = 49$

$a_2 = 5, b_2 = -6, c_2 = 57$

So, $\frac{a_1}{a_2}=\frac{7}{5}, \frac{b_1}{b_2}=\frac{-4}{-6}=\frac{2}{3}$

Since, $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

So, system has a unique solution.

Multiply equation (i) by 5 and equation (ii) by 7 and subtract

$$\begin{aligned} 35x-20y &= 245 \\ \underline{{}_-35x\mp42y} &= \underline{{}_-399} \\ 22y &= -154 \end{aligned} \Rightarrow y = -7$$

Put y = -7 in equation (ii)

5x – 6(-7)57 ⇒ 5x = 57 – 42 ⇒ x = 3

hence, x = 3 and y = -7.

6. Let $\frac{1}{x-1}=p$ and $\frac{1}{y-2}=q$

The given equations become

$6p - 3q = 1$ ....(i)

$5p + q = 2$ ....(ii)

Multiply equation (ii) by 3 and add in equation (i)

$$\begin{aligned} 15p+3q &= 6 \\ 6p-3q &= 1 \\ 21p \quad &= 7 \end{aligned} \Rightarrow p=\frac{7}{21}=\frac{1}{3}$$

Putting this value in equation (i) we get

$6\times\frac{1}{3}-3q=1$ ⇒ $2 - 3q = 1$

⇒ $3q = 1$, ⇒ $q=\frac{1}{3}$

Now, $\frac{1}{x-1}=p=\frac{1}{3}$ ⇒ $x - 1 = 3$

⇒ $x = 4$

$\frac{1}{y-2}=q=\frac{1}{3}$ ⇒ $y - 2 = 3$

⇒ $y = 5$

Hence, x = 4 and y= 5.

7. ∠A + 2B + ∠C = 180°

(Sum of interior angles of A ABC) $x + 3x + y = 180°$

$4x + y = 180°$ ...(i)

$3y - 5x = 30°$ (Given) ...(ii) Multiply equation (i) by 3 and subtracting from eq. (ii), we get

-17x = – 510 = x = 910 = 30°

17 then _A = x = 30° and 2B = 3x = 3 X 30o = 90°

∠C = y = 180° – (∠A + ∠B) = 180° – 120° = 60°

∠A = 30°, ∠B = 90°, ∠C = 60° Hence ΔABC is right triangle right angled at B.

8. Since BC||DE and BE||CD with BC||CD.

BCDE is a rectangle.

Opposite sides are equal BE = CD

∴ $x + y = 5$ …… (i)

DE = BC = $x - y$

Since perimeter of ABCDE is 21 cm.

AB + BC + CD + DE + EA = 21

$3 + x - y + x + y + x - y + 3 = 21 \Rightarrow 6 + 3x - y = 21$

$3x - y = 15$ ….. (iii)

Adding (i) and (ii), we get

$4x = 20 \Rightarrow x = 5$

On putting the value of x in (i), we get y = 0

Hence, x = 5 and y = 0.

9. Let the present ages of B and A be x years and y years respectively. Then

B's age 5 years ago = (x – 5) years

and A's age 5 years ago = (- 5) years

(-5) = 3 (x – 5) = 3x – y = 10 …….(i)

B's age 10 years hence = (x + 10) years

A's age 10 years hence = (y + 10) years

y + 10 = 2 (x + 10) = 2x – y = -10 …….. (ii)

On subtracting (ii) from (i) we get x = 20

Putting x = 20 in (i) we get

(3 × 20) – y = 10 ⇒ y = 50

∴ x = 20 and y = 50

Hence, B's present age = 20 years and A's present age = 50 years.

Long Answer :

1. Let x be the number of girls and y be the number of boys.

According to question, we have

x = y + 4

⇒ x – y = 4 ……(i)

Again, total number of students = 10

Therefore, x + y = 10 …(ii)

Hence, we have following system of equations

x – y = 4

and x + y = 10

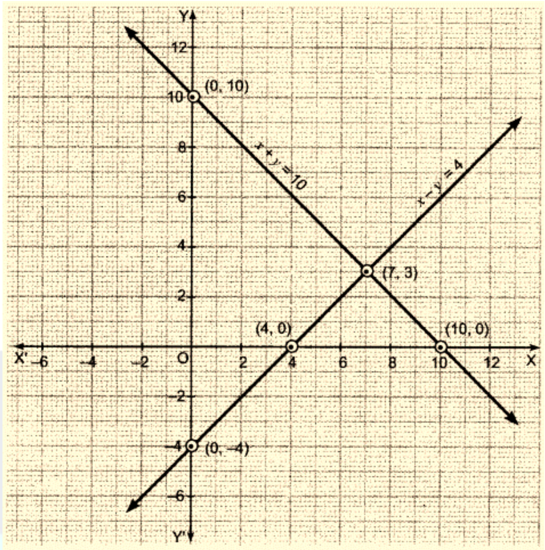
From equation (i), we have the following table:

x	0	4	7
y	–4	0	3

From equation (ii), we have the following table:

x	0	10	7
y	10	0	3

Plotting this, we have



Here, the two lines intersect at point (7,3) i.e., x = 7, y = 3.

So, the number of girls = 7

and number of boys = 3.

2. We have, 2x + 4y = 10

⇒ 4y = 10 – 2x ⇒ $y = \frac{5-x}{2}$

Thus, we have the following table:

x	1	3	5
y	2	1	0

Plot the points A (1, 2), B (3, 1) and C (5,0) on the graph paper. Join A, B and C and extend it on both sides to obtain the graph of the equation 2x + 4y = 10.

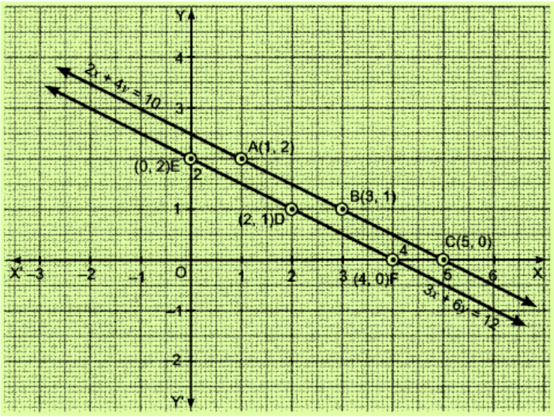
We have, 3x + 6y = 12

⇒ 6y = 12 – 3x ⇒ $y = \frac{4-x}{2}$

Thus, we have the following table :

x	2	0	4
y	1	2	0

Plot the points D (2, 1), E (0, 2) and F (4,0) on the same graph paper. Join D, E and F and extend it on both sides to obtain the graph of the equation 3x + 6y = 12.



We find that the lines represented by equations 2x + 4y = 10 and 3x + y = 12 are parallel. So, the two lines have no common point. Hence, the given system of equations has no solution.

3. (i) We have, 3x – 5y – 4 = 0

⇒ 3x – 5y = 4 …….(i)

Again, 9x = 2y + 7

9x – 2y = 7 …(ii)

By Elimination Method:

Multiplying equation (i) by 3, we get

9x – 15y = 12 … (iii)

Subtracting (ii) from (iii), we get

$$\begin{array}{r} 9x-15y=12 \\ \underline{{}_{-}9x\mp2y={}_{-}7} \\ -13y=\ 5 \end{array}$$

$$\Rightarrow\ y=\frac{5}{13}$$

Putting the value of y in equation (ii), we have

$$9x-2\left(-\frac{5}{13}\right)=7\Rightarrow 9x+\frac{10}{13}=7\Rightarrow 9x=7-\frac{10}{13}$$

$$\Rightarrow\quad 9x=\frac{91-10}{13}\Rightarrow 9x=\frac{81}{13}\Rightarrow x=\frac{9}{13}$$

Hence, the required solution is $x=\frac{9}{13},\quad y=-\frac{5}{13}$

By Substitution Method:

Expressing x in terms of *y* from equation (i),

we have $x=\frac{4+5y}{3}$

Substituting the value of *x* in equation (ii),

we have $9\times\left(\frac{4+5y}{3}\right)-2y=7$

$$\Rightarrow\quad 3\times(4+5y)-2y=7$$

$$\Rightarrow\quad 12=15y-2y=7$$

$$\Rightarrow\quad 13y=7-12$$

$$\therefore\quad y=-\frac{5}{13}$$

Putting the value of *y* in equation (i), we have

$$3x-5\times\left(-\frac{5}{13}\right)=4$$

$$\Rightarrow\quad 3x+\frac{25}{13}=4$$

$$\Rightarrow\quad 3x=4-\frac{25}{13}$$

$$\Rightarrow\quad 3x=\frac{27}{13}$$

Hence, the required solution is

$x=\frac{9}{13},\ y=-\frac{5}{13}.$

(ii) We have, $\frac{x}{2}+\frac{2y}{3}=-1\Rightarrow\frac{3x+4y}{6}=-1$

∴ 3x + 4*y* = -6 ....(i)

and $x-\frac{y}{3}=3\Rightarrow\frac{3x-y}{3}=3$

∴ 3x – *y* = 9 ....(ii)

By Elimination Method:

Subtracting (ii) from (i), we have

5y = – 15 or y = – 55 = -3

Putting the value of y in equation (i), we have

3x + 4 × (-3) = -6 ⇒ 3x = – 6 + 12

∴ 3x – 12 = -6 ⇒ 3x = 6

∴ x = 63 = 2

Hence, solution is x = 2, y = -3.

By Substitution Method:

Expressing x in terms of y from equation (i), we have

$$3\times\left(\frac{-6-4y}{3}\right)-y=9\ \Rightarrow\ -6-4y-y=9\ \Rightarrow\ -6-5y=9$$

Substituting the value of x in equation (ii), we have

∴ -5y = 9 + 6 = 15

$$y\ =-\frac{15}{5}\ =-\ 3$$

Putting the value of y in equation (i), we have

3x + 4 × (-3) = -6 ⇒ 3x – 12 = -6

∴ 3x = 12 – 6 = 6

∴ x = $\frac{6}{3}$ = 2

Hence, the required solution is x = 2, y = – 3.

4. We have,' x – y + 1 = 0 and 3x + 2y – 12 = 0

Thus, x – y = -1 => x = y – 1 ...(i)

3x + 2y = 12 => $x = \frac{12-2y}{3}$ ... (ii)

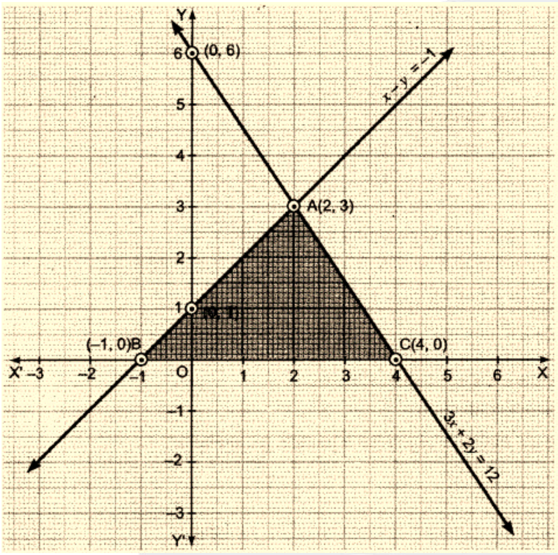
From equation (i), we have

x	–1	0	2
y	0	1	3

From equation (ii), we have

x	0	4	2
y	6	0	3

Plotting this, we have



ABC is the required (shaded) region and point of intersection is (2, 3).

∴ The vertices of the triangle are (-1, 0), (4, 0), (2, 3).

5. Let the fixed charge be *x and the cost of food per day be by.

Therefore, according to question,

x + 20y = 1000 ...(i)

x + 26y = 1180 ...(ii)

Now, subtracting equation (ii) from (i), we have

$$\begin{array}{r} x+20y = 1000 \\ \underline{-x \pm 26y = -1000} \\ -6y = -180 \end{array}$$

$$\therefore \quad y = \frac{-180}{-6} = 30$$

Putting the value of y in equation (i), we have

x + 20 x 30 = 1000 ⇒ x + 600 = 1000 ⇒ x = 1000 – 600 = 400

Hence, fixed charge is ₹400 and cost of food per day is ₹30.

6. Let x be the number of questions of right answer and y be the number of questions of wrong answer.

According to question,

3x – y = 40 ... (i)

and 4x – 2y = 50

or 2x – y = 25 ...(ii)

Subtracting (ii) from (i), we have

$$\begin{array}{r} 3x - y = 40 \\ \underline{-2x \mp y = -25} \\ x = 15 \end{array}$$

Putting the value of x in equation (i), we have

3 x 15 – y = 40 ⇒45 – y = 40

∴ y = 45 – 40 = 5

Hence, total number of questions is x + i.e.., 5 + 15 = 20.

7. Let one man alone can finish the work in x days and one boy alone can finish the work in y days

Then, One day work of one man $= \frac{1}{x}$,

One day work of one boy $\frac{1}{y}$

∴ One day work of 8 men $= \frac{8}{x}$,

One day work of 12 boys $= \frac{12}{y}$

Since 8 men and 12 boys can finish the work in 10 days

$$10\left(\frac{8}{x}+\frac{12}{y}\right)=1 \Rightarrow \frac{80x}{x}+\frac{120}{y}=1 \quad ...(i)$$

Again, 6 men and 8 boys can finish the work in 14 days

$$\therefore 14\left(\frac{6}{x}+\frac{8}{y}\right)=1 \Rightarrow \frac{84}{x}+\frac{112}{y}=1 \quad ..(ii)$$

Put $\frac{1}{x}=u$ and $\frac{1}{y}=v$ in equations (i) and (ii), we get

80*u* + 120*v* – 1 = 0 and 84*u* + 112*v* – 1 = 0

By using cross-multiplication, we have

$$\frac{u}{120\times-1-112\times-1}=\frac{-v}{80\times-1-84-1}=\frac{1}{80\times112-84\times120}$$

$$\Rightarrow \frac{u}{-120+112}=\frac{-v}{-80+84}=\frac{1}{8960-10080}$$

$$\Rightarrow \frac{u}{-8}=\frac{-v}{4}=\frac{1}{-1120}$$

Hence, one man alone can finish the work in 140 days and one boy alone can finish the work in 280 days.

8. Let the speed of the boat in still water be x km/h and that of the stream be y km/h. Then,

Speed upstream (x – y) km/h

Speed downstream (x + y) km/h

Now, time taken to cover 25 km upstream

$= \frac{25}{x-y}$ hours

Time taken to cover 44 km downstream

$= \frac{44}{x+y}$ hours

The total time of journey is 9 hours

$\frac{25}{x-y} + \frac{44}{x+y} = 9$ ...(i)

Time taken to cover 15 km upstream = $\frac{15}{x-y}$

Time taken to cover 22 km downstream = $\frac{22}{x+y}$

In this case, total time of journey is 5 hours.

$\therefore \frac{15}{x-y} + \frac{22}{x+y} = 5$ ...(ii)

Put $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$ in equations (i) and (ii), we get

25u + 44v = 9 ⇒ 25u + 44v – 9 = 0 ...(iii)

15u + 22v = 5 ⇒ 15u + 22v – 5 = 0 ...(iv)

By cross-multiplication, we have

$\Rightarrow u = \frac{22}{110} = \frac{1}{5}$ and $v = \frac{1}{11}$

We have, $u = \frac{1}{5} \Rightarrow \frac{1}{x+y} = \frac{1}{5} \Rightarrow x - y = 5$ (v)

and $v = \frac{1}{11} \Rightarrow \frac{1}{x+y} = \frac{1}{11} \Rightarrow x + y = 11$ (vi)

Solving equations (v) and (vi), we get x = 8 and y = 3.

Hence, speed of the boat in still water is 8 km/h and speed of the stream is 3 km/h.

Case Study Answers:

1. **Answer :**

i. (a) x + 25y = 4500, x + 30y = 5200

Solution:

For student Anu:

Fixed charge + cost of food for 25 days = ₹ 4500

i.e., x + 25y = 4500

For student Bindu:

Fixed charges + cost of food for 30 days = ₹ 5200

i.e., x + 30y = 5200

ii. (b) Unique solution.

Solution:

From above, we have $a_1 = 1$, $b_1 = 25$

$c_1 = -4500$ and $a_2 = 1$, $b_2 = 30$,

$c_2 = -5200$

$\therefore \frac{a_1}{a_2} = 1, \frac{b_1}{b_2} = \frac{25}{30} = \frac{5}{6}, \frac{c_1}{c_2} = \frac{-4500}{-5200} = \frac{45}{52}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Thus, system of linear equations has unique solution.

iii. (c) ₹ 140

Solution:

We have, x + 25y = 4500

and x + 30y = 5200

Subtracting (i) from (ii), we get

5y = 700 ⇒ y = 140

∴ Cost of food per day is ₹ 140

iv. (c) ₹ 1000

Solution:

We have, x + 25y = 4500

⇒ x = 4500 - 25 × 140

⇒ x = 4500 - 3500 = 1000

∴ Fixed charges per month for the hostel is ₹ 100

v. (d) ₹ 3800

Solution:

We have, x = 1000, y = 140 and Bindu takes food for 20 days.

∴ Amount that Bindu has to pay = ₹ (1000 + 20 × 140) = ₹ 3800

2. **Answer :**

i. (d) 2x + 3y = 46

Solution:

1st situation can be represented algebraically as.

2x + 3y = 46

ii. (c) 3x + 5y = 74

Solution:

2^{nd} situation can be represented algebraically as:

$3x + 5y = 74$

iii. (b) ₹ 8

Solution:

We have, $2x + 3y = 46$.........(i)

$3x + 5y = 74$.........(ii)

Multiplying (i) by 5 and (ii) by 3 and then subtracting, we get

$10x - 9x = 230 - 222 \Rightarrow x = 8$

∴ Fare from Bengaluru to Malleswaram is ₹ 8.

iv. (a) ₹ 10

Solution:

Putting the value of x in equation (i), we g

$3y = 46 - 2 \times 8 = 30 \Rightarrow y = 10$

∴ Fare from Bengaluru to Yeswanthpur is ₹ 10

v. (c) Unique solution.

Solution:

We have, $a_1 = 2, b_1 = 3, c_1 = -46$ and

$a_2 = 3, b_2 = 5, C_2 = -74$

$$\therefore \frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{3}{5} = \frac{5}{6}, \frac{c_1}{c_2} = \frac{-46}{-74} = \frac{23}{37}$$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus system of linear equations has unique solution.

Assertion reason Answer-

1. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
2. (d) Assertion (A) is true but reason (R) is false.

Quadratic Equations | 4

1. Introduction to Quadratic equation

If p(x) is a quadratic polynomial, then p(x) = 0 is called a **quadratic equation.**

The general or standard form of a quadratic equation, in the variable x, is given by $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

2. Roots of the quadratic equation

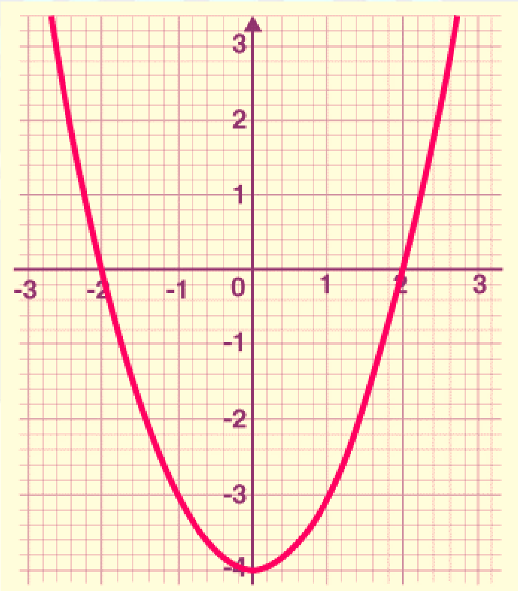
The value of x that satisfies an equation is called the **zeroes** or **roots** of the equation.

A real number α is said to be a solution/root of the quadratic equation $ax^2 + bx + c = 0$ if $a\alpha^2 + b\alpha + c = 0$.

A quadratic equation has **at most two roots.**

Graphically, the roots of a quadratic equation are the points where the graph of the quadratic polynomial cuts the x-axis.

Consider the graph of a quadratic equation $x^2 - 4 = 0$:



Graph of a Quadratic Equation

In the above figure, -2 and 2 are the roots of the quadratic equation $x^2-4=0$

Note:

- If the graph of the quadratic polynomial cuts the x-axis at two distinct points, then it has real and distinct roots.
- If the graph of the quadratic polynomial touches the x-axis, then it has real and equal roots.
- If the graph of the quadratic polynomial does not cut or touch the x-axis then it does not have any real roots.

3. The standard form of a Quadratic Equation

The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$.

'a' is the coefficient of x^2. It is called the quadratic coefficient. 'b' is the coefficient of x. It is called the linear coefficient. 'c' is the constant term.

4. A quadratic equation can be solved by following algebraic methods:

i. Splitting the middle term (factorization)

ii. Completing squares

iii. Quadratic formula

5. Splitting the middle term (or factorization) method

- If $ax^2 + bx + c$, $a \neq 0$, can be reduced to the product of two linear factors, then the roots ofthe quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
- Steps involved in solving quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ by **splitting the middle term** (or factorization) method:

Step 1: Find the product ac.

Step 2: Find the factors of 'ac' that add to up to b, using the following criteria:

i. If ac > 0 and b > 0, then both the factors are positive.

ii. If ac > 0 and b < 0, then both the factors are negative.

iii. If ac < 0 and b > 0, then larger factor is positive and smaller factor is negative.

iv. If ac < 0 and b < 0, then larger factor is negative and smaller factor is positive.

Step 3: Split the middle term into two parts using the factors obtained in the above step.

Step 4: Factorize the quadratic equation obtained in the above step by grouping method. Two factors will be obtained.

Step 5: Equate each of the linear factors to zero to get the value of x.

6. Completing the square method

- Any quadratic equation can be converted to the form $(x + a)^2 – b^2 = 0$ or $(x – a)^2 + b^2 = 0$ by adding and subtracting the constant term. This method of finding the roots of quadratic equation is called the method of completing the square.
- The steps involved in solving a quadratic equation by **completing the square**, are as follows:

Step 1: Make the coefficient of x^2 unity.

Step 2: Express the coefficient of x in the form $2 \times x \times p$.

Step 3: Add and subtract the square of p.

Step 4: Use the square identity $(a + b)^2$ or $(a – b)^2$ to obtain the quadratic equation in the required form $(x + a)^2 - b^2 = 0$ or $(x – a)^2 + b^2 = 0$.

Step 5: Take the constant term to the other side of the equation.

Step 6: Take the square root on both the sides of the obtained equation to get the roots of the given quadratic equation.

7. Quadratic formula

The roots of a quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ can be calculated by using the **quadratic formula:**

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a} \text{ where } b^2 – 4ac \geq 0$$

If $b^2 – 4ac < 0$, then equation does not have real roots.

The quadratic formula is used to find the roots of a quadratic equation. This formula helps to evaluate the solution of quadratic equations replacing the factorization method. If a quadratic equation does not contain real roots, then the quadratic formula helps to find the imaginary roots of that equation. The quadratic formula is also known as Shreedhara Acharya's formula. In this article, you will learn the quadratic formula, derivation and proof of the quadratic formula, along with a video lesson and solved examples.

An algebraic expression of degree 2 is called the **quadratic equation**. The general form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are real numbers, also called "numeric coefficients" and $a \neq 0$. Here, x is an unknown variable for which we need to find the solution. We know that the **quadratic formula** used to find the solutions (or roots) of the quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here,

a, b, c = Constants (real numbers)

$a \neq 0$

x = Unknown, i.e. variable

The above formula can also be written as:

$$x = \frac{-b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$or$$

$$x = \frac{-b}{2a} \pm \sqrt{(\frac{b}{2a})^2 - \frac{c}{a}}$$

What is the Quadratic Formula used for?

The quadratic formula is used to find the roots of a quadratic equation and these roots are called the solutions of the quadratic equation. However, there are several methods of solving quadratic equations such as factoring, completing the square, graphing, etc.

Roots of Quadratic Equation by Quadratic Formula

We know that a second-degree polynomial will have at most two zeros, and therefore a quadratic equation will have at most two roots.

In general, if α is a root of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$; then, $a\alpha^2 + b\alpha + c = 0$. We can also say that $x = \alpha$ is a solution of the quadratic equation or α satisfies the equation, $ax^2 + bx + c = 0$.

Note: Roots of the quadratic equation $ax^2 + bx + c = 0$ are the same as zeros of the polynomial $ax^2 + bx + c$.

One of the easiest ways to find the roots of a quadratic equation is to apply the quadratic formula.

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $b^2 - 4ac$ is called the discriminant and is denoted by D.

The sign of plus (+) and minus (-) in the quadratic formula represents that there are two solutions for quadratic equations and are called the roots of the quadratic equation.

Root 1:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

And

Root 2:

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

8. Derivation of Quadratic Formula

We can derive the quadratic formula in different ways using various techniques.

Derivation Using Completing the Square Technique

Let us write the standard form of a quadratic equation.

$$ax^2 + bx + c = 0$$

Divide the equation by the coefficient of x^2, i.e., a.

$$x^2 + (b/a) x + (c/a) = 0$$

Subtract c/a from both sides of this equation.

$$x^2 + (b/a) x = -c/a$$

Now, apply the method of completing the square.

Add a constant to both sides of the equation to make the LHS of the equation as complete square.

Adding $(b/2a)^2$ on both sides,

$$x^2 + (b/a) x + (b/2a)^2 = (-c/a) + (b/2a)^2$$

Using the identity $a^2 + 2ab + b^2 = (a + b)^2$,

$$[x + (b/2a)]^2 = (-c/a) + (b^2/4a^2)$$

$$[x + (b/2a)]2 = (b^2 – 4ac)/4a2$$

Take the square root on both sides,

Shortcut Method of Derivation

Write the standard form of a quadratic equation.

$$ax^2 + bx + c = 0$$

Multiply both sides of the equation by 4a.

$$4a (ax^2 + bx + c) = 4a(0)$$

$$4a^2x^2 + 4abx + 4ac = 0$$

$$4a^2x^2 +4abx = -4ac$$

Add a constant on sides such that LHS will become a complete square.

Adding b^2 on both sides,

$$4a^2x^2 + 4abx + b^2 = b^2 – 4ac$$

$$(2ax)^2 + 2(2ax)(b) + b^2 = b^2 – 4ac$$

Using algebraic identity $a^2 + 2ab + b^2 = (a + b)^2$,

$$(2ax + b)^2 = b^2 – 4ac$$

Taking square root on both sides,

$$2ax + b = \pm \sqrt{(b^2 – 4ac)}$$

$$2ax = -b \pm \sqrt{(b^2 – 4ac)}$$

$$x = [-b \pm\sqrt{(b^2 – 4ac)}]/2a$$

9. Nature of Roots

Based on the value of the discriminant, $D = b^2 – 4ac$, the roots of a quadratic equation can be of three types.

Case 1: If D>0, the equation has two distinct real roots.

Case 2: If D=0, the equation has two equal real roots.

Case 3: If D<0, the equation has no real roots.

The number of roots of a polynomial equation is equal to its degree. So, a quadratic equation has two roots. Some methods for finding the roots are:

Factorization method

Quadratic Formula

Completing the square method

All the quadratic equations with real roots can be factorized. The physical significance of the roots is that at the roots of an equation, the graph of the equation intersects x-axis. The x-axis represents the real line in the Cartesian plane. This means that if the equation has unreal roots, it won't intersect x-axis and hence it cannot be written in factorized form. Let us now go ahead and learn how to determine whether a quadratic equation will have real roots or not.

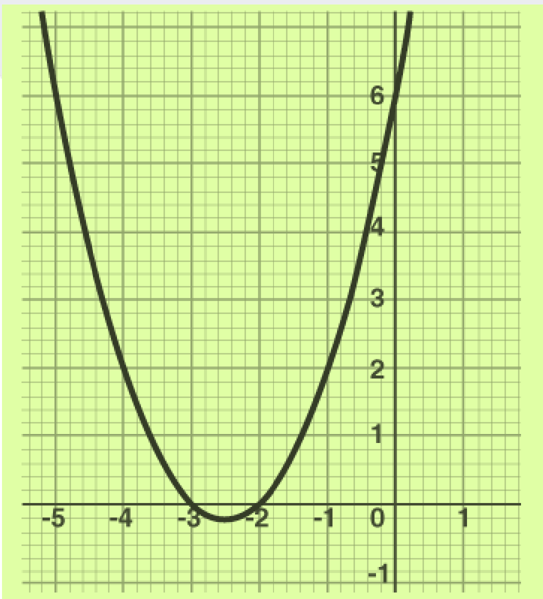
Nature Of Roots Of Quadratic Equation

Value of Discriminant	Nature of Roots	
D > 0	Real, Distinct	
	D is a perfect square	Rational roots
	D is not a perfect square	Irrational roots
D = 0	Real, Equal	
D < 0	Complex, Distinct (A pair of complex conjugates)	

10. Graphical Representation of a Quadratic Equation

The graph of a quadratic polynomial is a parabola. The roots of a quadratic equation are the points where the parabola cuts the x-axis i.e. the points where the value of the quadratic polynomial is zero.

Now, the graph of $x^2 + 5x + 6 = 0$ is:



In the above figure, -2 and -3 are the roots of the quadratic equation

$x^2 + 5x + 6 = 0$.

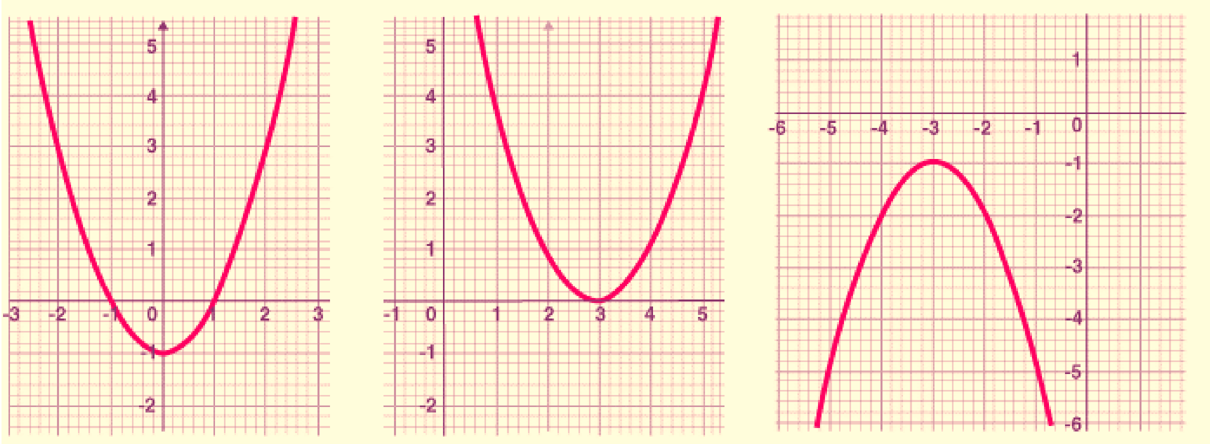
For a quadratic polynomial $ax^2 + bx + c$,

If $a > 0$, the parabola opens upwards.

If $a < 0$, the parabola opens downwards.

If $a = 0$, the polynomial will become a first-degree polynomial and its graph is linear.

The discriminant, $D = b^2 - 4ac$



(a) (b) (c)

Nature of graph for different values of D.

If $D > 0$, the parabola cuts the x-axis at exactly two distinct points. The roots are distinct. This case is shown in the above figure in a, where the quadratic polynomial cuts the x-axis at two distinct points.

If $D = 0$, the parabola just touches the x-axis at one point and the rest of the parabola lies above or below the x-axis. In this case, the roots are equal.

This case is shown in the above figure in b, where the quadratic polynomial touches the x-axis at only one point.

If $D < 0$, the parabola lies entirely above or below the x-axis and there is no point of contact with the x-axis. In this case, there are no real roots.

This case is shown in the above figure in c, where the quadratic polynomial neither cuts nor touch the x-axis.

11. Discriminant of a quadratic equation

For the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, the expression $b^2 - 4ac$ is known as **discriminant.**

12. **Nature of the roots** of a quadratic equation:

i. If $b^2 - 4ac > 0$, the quadratic equation has **two distinct real roots.**

ii. If $b^2 - 4ac = 0$, the quadratic equation has **two equal real roots.**

iii. If $b^2 - 4ac < 0$, the quadratic equation has **no real roots**.

13. There are many equations which are not in the quadratic form but can be reduced to the quadratic form by simplifications.

14. **Application of quadratic equations**

- The applications of quadratic equation can be utilized in solving real life problems.
- Following points can be helpful in solving word problems:

i. Every two digit number ‘xy’ where x is a ten’s place and y is a unit’s place can be expressed as xy = 10x + y.

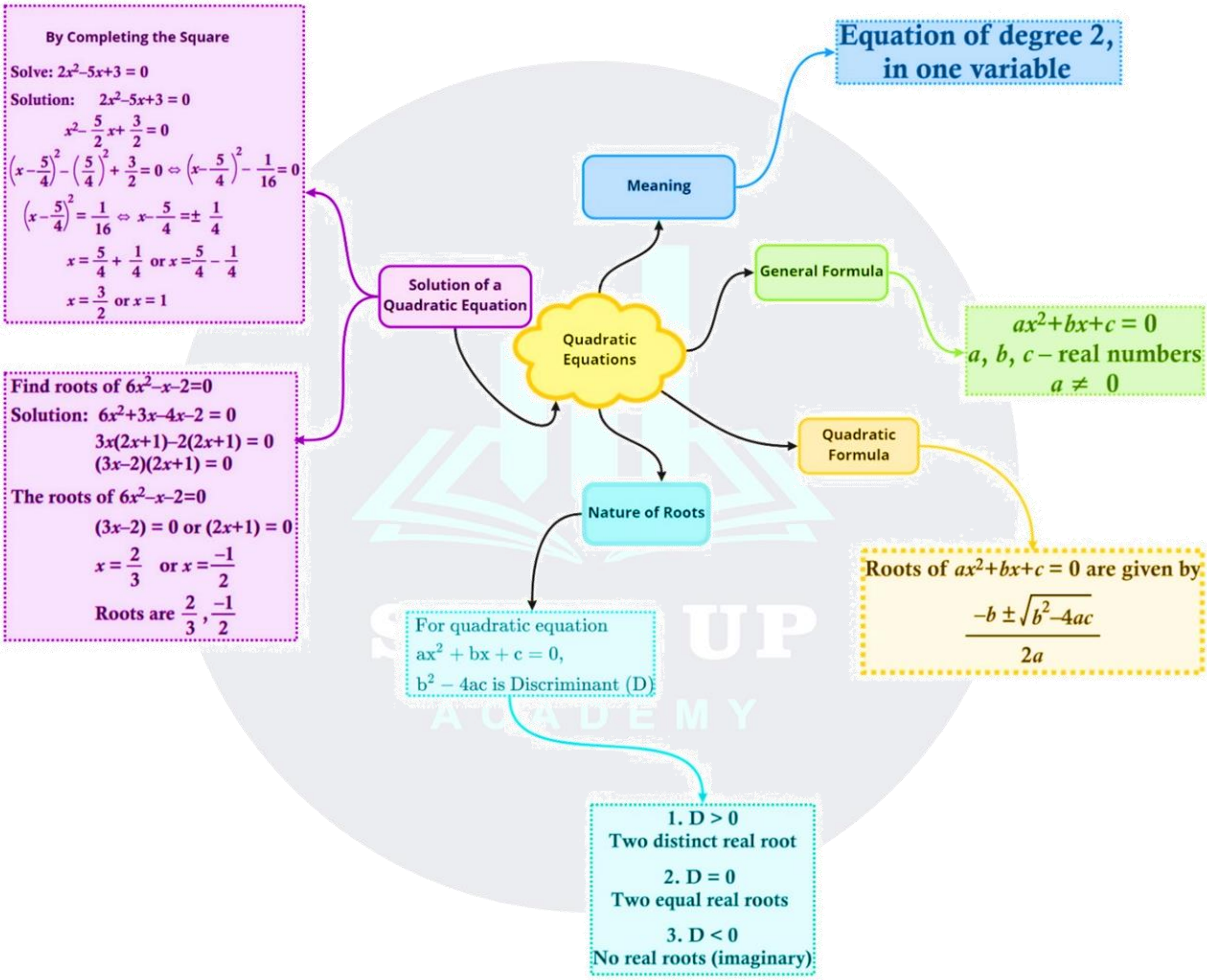
ii. Downstream: It means that the boat is running in the direction of the stream Upstream: It means that the boat is running in the opposite direction of the stream Thus, if Speed of boat in still water is x km/h.

And the speed of stream is y km/h.

Then the speed of boat downstream will be (x + y) km/h and in upstream it will be (x − y) km/h.

iii. If a person takes x days to finish a work, then his one day's work = $\frac{1}{x}$.

Class : 10th mathematics
Chapter-4 : Quadratic Equations



Important Questions

Multiple Choice Questions

1. Which of the following is not a quadratic equation
   (a) $x^2 + 3x - 5 = 0$
   (b) $x^2 + x^3 + 2 = 0$
   (c) $3 + x + x^2 = 0$
   (d) $x^2 - 9 = 0$
2. The quadratic equation has degree
   (a) 0
   (b) 1
   (c) 2
   (d) 3
3. The cubic equation has degree
   (a) 1
   (b) 2
   (c) 3
   (d) 4
4. A bi-quadratic equation has degree
   (a) 1
   (b) 2
   (c) 3
   (d) 4
5. The polynomial equation $x(x + 1) + 8 = (x + 2)\{x - 2)$ is
   (a) linear equation
   (b) quadratic equation
   (c) cubic equation
   (d) bi-quadratic equation
6. The equation $(x - 2)^2 + 1 = 2x - 3$ is a
   (a) linear equation
   (b) quadratic equation
   (c) cubic equation
   (d) bi-quadratic equation
7. The quadratic equation whose roots are 1 and $-1/2$
   (a) $2x^2 + x - 1 = 0$
   (b) $2x^2 - x - 1 = 0$
   (c) $2x^2 + x + 1 = 0$
   (d) $2x^2 - x + 1 = 0$
8. The quadratic equation whose one rational root is $3 + \sqrt{2}$ is
   (a) $x^2 - 7x + 5 = 0$
   (b) $x^2 + 7x + 6 = 0$
   (c) $x^2 - 7x + 6 = 0$
   (d) $x^2 - 6x + 7 = 0$
9. The equation $2x^2 + kx + 3 = 0$ has two equal roots, then the value of k is
   (a) $\pm\sqrt{6}$
   (b) ± 4
   (c) $\pm 3\sqrt{2}$
   (d) $\pm 2\sqrt{6}$
10. The sum of the roots of the quadratic equation $x^2 - 4x + 4 = 0$ is
    (a) 3
    (b) 6
    (c) -3
    (d) 4

Very Short Questions:

1. What will be the nature of roots of quadratic equation $2x^2 + 4x - 7 = 0$?
2. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - 54 = 0$, then find the value of k.
3. If $ax^2 + bx + c = 0$ has equal roots, find the value of c.
4. If a and b are the roots of the equation $x^2 + ax - b = 0$, then find a and b.
5. Show that $x = -2$ is a solution of $3x^2 + 13x + 14 = 0$.
6. Find the discriminant of the quadratic equation $4\sqrt{2}x^2 + 8x + 2\sqrt{2} = 0$).
7. State whether the equation $(x + 1)(x - 2) + x = 0$ has two distinct real roots or not. Justify your answer.
8. Is 0.3 a root of the equation $x^2 - 0.9 = 0$? Justify.
9. For what value of k, is 3 a root of the equation $2x^2 + x + k = 0$?
10. Find the values of k for which the quadratic equation $9x^2 - 3kx + k = 0$ has equal roots.

Short Questions :

1. Find the roots of the following quadratic equations by factorisation:

   (i) √2x2 + 7x + 5√2 = 0 (ii) 2x2 – x + $\frac{1}{8}$ = 0

2. Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

   (i) $2x^2 + x - 4 = 0$

   (ii) $4x^2 + 4\sqrt{3}x + 3 = 0$

3. Find the roots of the following quadratic equations by applying the quadratic formula.

   (i) $2x^2 - 7x + 3 = 0$

   (ii) $4x^2 + 4\sqrt{3}x + 3 = 0$

4. Using quadratic formula solve the following quadratic equation:

   $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

5. Find the roots of the following equation:

   $$\frac{1}{x+3} - \frac{1}{x-6} = \frac{9}{20};\ x \neq -3, 6$$

6. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

   (i) $3x^2 - 4\sqrt{3}x + 4 = 0$) (ii) $2x^2 - 6x + 3 = 0$

7. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

   (i) $2x^2 + kx + 3 = 0$

   (ii) $kx(x - 2) + 6 = 0$

8. If the roots of the quadratic equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal, prove that $2a = b + c$.

9. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, show that $c^2 = a^2(1 + m^2)$.

10. If sin θ and cos θ are roots of the equation $ax^2 + bx + c = 0$, prove that $a^2 - b^2 + 2ac = 0$.

Long Questions :

1. Using quadratic formula, solve the following equation for x:

   $abx^2 + (b^2 - ac)x - bc = 0$

2. Find the value of p for which the quadratic equation

   $(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$ has equal roots. Also find these roots.

3. Solve for

   $$x: \frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3};\ x \neq 5, 7$$

4. The sum of the reciprocals of Rehman's age (in years) 3 years ago and 5 years from now is Find his present age.

5. The difference of two natural numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

6. The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

7. The sum of two numbers is 15 and the sum of their reciprocals is 3. Find the numbers.

8. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210. Find her marks in the two subjects.

9. A train travels 360 km at a uniform speed. If the speed has been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

10. The sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Case Study Question:

1. If p(x) is a quadratic polynomial i.e., $p(x) = ax^2 + bx + c$, a≠0a≠0 then p(x) = 0 is called a quadratic equation. Now, answer the following questions.

   i. Which of the following is correct about the quadratic equation $ax^2 + bx + c = 0$?
      a. a, b and c are real numbers c ≠ 0
      b. a, b and c are rational numbers, a ≠ 0
      c. a, b and c are integers, a, band c ≠ 0
      d. a, b and c are real numbers a ≠ 0

   ii. The degree of a quadratic equation is:
      a. 1
      b. 2
      c. 3
      d. Other than 1

   iii. Which of the following is a quadratic equation?
      a. $x(x + 3) + 7 = 5x - 11$
      b. $(x - 1)^2 - 9 = (x - 4)(x + 3)$
      c. $x^2(2x + 1) - 4 = 5x^2 - 10$
      d. $x(x - 1)(x + 7) = x(6x - 9)$

iv. Which of the following is incorrect about the quadratic equation $ax^2 + bx + c = 0$?

a. If $a\alpha^2 + b\alpha + c = 0$ then $x = -\alpha$ is the solution of the given quadratic equation.

b. The additive inverse of zeroes of the polynomial $ax^2 + bx + c$ is the roots of the given equation.

c. If α is a root of the given quadratic equation, then its other root is $-\alpha$

d. All of these.

v. Which of the following is not a method of finding solutions of the given quadratic equation:

a. Factorisation method

b. Completing the square method

c. Formula method

d. None of these

2. Quadratic equations started around 3000 B.C. with the Babylonians. They were one of the world's first civilisation, and came up with some great ideas like agriculture, irrigation and writing. There were many reasons why Babylonians needed to solve quadratic equations. for example to know what amount of crop you can grow on the square field. Based on the above information, represent the following questions in the form of quadratic equation.

i. The sum of squares of two consecutive integers is 650.

a. $x^2 + 2x - 650 = 0$

b. $2x^2 + 2x - 649 = 0$

c. $x^2 - 2x - 650 = 0$

d. $2x^2 + 6x - 550 = 0$

ii. The sum of two numbers is 15 and the sum of their reciprocals is 310310.

a. $x^2 + 10x - 150 = 0$

b. $15x^2 - x + 150 = 0$

c. $x^2 - 15x + 50 = 0$

d. $3x^2 - 10x + 15 = 0$

iii. Two numbers differ by 3 and their product is 504.

a. $3x^2 - 504 = 0$

b. $x^2 - 504x + 3 = 0$

c. $504x^2 + 3 = x$

d. $x^2 + 3x - 504 = 0$

iv. A natural number whose square diminished by 84 is thrice of 8 more of given number.

a. $x^2 + 8x - 84 = 0$

b. $3x^2 - 84x + 3 = 0$

c. $x^2 - 3x - 108 = 0$

d. $x^2 - 11x + 60 = 0$

v. A natural number when increased by 12, equals 160 times its reciprocal.

a. $x^2 - 12x + 160 = 0$

b. $x^2 - 160x + 12 = 0$

c. $12x^2 - x - 160 = 0$

d. $x^2 + 12x - 160 = 0$

Assertion Reason Questions-

1. **Directions:** In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

a. Both A and R are true and R is the correct explanation for A.

b. Both A and R are true and R is the correct explanation for A.

c. A is true but R is false.

d. A is false but R is true.

Assertion: The product of two successive positive integral multiples of 5 is 300, then the two numbers are 15 and 20.

Reason: The product of two consecutive integrals is a multiple of 2.

2. **Directions:** In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

a. Both A and R are true and R is the correct explanation for A.

b. Both A and R are true and R is the correct explanation for A.

c. A is true but R is false.

d. A is false but R is true.

Assertion: The roots of the quadratic equation x2 + 2x + 2 = 0 are imaginary.

Reason: If discriminant D = b2 – 4ac < 0 then the roots of the quadratic equation ax2 + bx + c = 0 are imaginary.

Answer Key

Multiple Choice questions-

1. (b) -10
2. (b) $x^2 + x^3 + 2 = 0$
3. (c) 2
4. (d) 4
5. (a) linear equation
6. (b) quadratic equation
7. (b) $2x^2 - x - 1 = 0$
8. (d) $x^2 - 6x + 7 = 0$
9. (d) $\pm 2\sqrt{6}$
10. (d) 4

Very Short Answer :

1. $D = b^2 - 4ac$

   $\Rightarrow 4^2 - 4 \times 2\,(-7)$

   $\Rightarrow 16 + 56 = 72 > 0$

   Hence, roots of quadratic equation are real and unequal.

2. $\therefore \frac{1}{2}$ is a root of quadratic equation.

   $\therefore$ It must satisfy the quadratic equation.

   $$x^2 + kx - \frac{5}{4} = 0$$

   $$\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0 \Rightarrow \frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$$

   $$\frac{1 + 2k - 5}{4} = 0 \Rightarrow 2k - 4 = 0 \Rightarrow k = 2$$

3. For equal roots D = 0

   i.e., b2 – 4ac = 0

   ⇒ b2 = 4 ac

   $\Rightarrow c = \frac{b^2}{4a}$

4. Sum of the roots = a + b = $-\frac{B}{A} = -a$

   Product of the roots = ab = $\frac{B}{A} = -b$

   = a + b = – a and ab = -b

   ⇒ 2a = -b and a = -1

   ⇒ b = 2 and a = -1

5. Put the value of x in the quadratic equation,

   $\Rightarrow$ LHS = $3x^2 + 13x + 14$

   $\Rightarrow 3(-2)^2 + 13(-2) + 14$

   $\Rightarrow 12 - 26 + 14 = 0$

   ⇒ RHS Hence, x = -2 is a solution.

6. $D = b^2 - 4ac = (8)^2 - 4(4\sqrt{2})(2\sqrt{2})$

   $\Rightarrow 64 - 64 = 0$

7. $(x + 1)(x - 2) + x = 0$

   $\Rightarrow x^2 - x - 2 + x = 0$

   $\Rightarrow x^2 - 2 = 0$

   $D = b^2 - 4ac$

   $\Rightarrow (-4(1)(-2) = 8 > 0$

   $\therefore$ Given equation has two distinct real roots.

8. $\because$ 0.3 is a root of the equation $x^2 - 0.9 = 0$

   $\therefore x^2 - 0.9 = (0.3)^2 - 0.9 = 0.09 - 0.9 \neq 0$

   Hence, 0.3 is not a root of given equation.

9. 3 is a root of $2x^2 + x + k = 0$, when

   $\Rightarrow 2(3)^2 + 3 + k = 0$

   $\Rightarrow 18 + 3 + k = 0$

   $\Rightarrow k = -21$

10. For equal roots:

    D = 0

    $\Rightarrow b^2 - 4ac = 0$

    $\Rightarrow (-3k)^2 - 4 \times 9 \times k = 0$

    $\Rightarrow 9k^2 = 36k$

    $\Rightarrow k = 4$

Short Answer :

1. (i) We have, $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

   $= \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

   $x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

   $= (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

   $\therefore$ Either $\sqrt{2}x + 5 = 0$ or $x + \sqrt{2} = 0$

   $\therefore x = -\frac{5}{\sqrt{2}}$ or $x = -\sqrt{2}$

   Hence, the roots are $-\frac{5}{\sqrt{2}}$ and $-\sqrt{2}$.

(ii) We have, $2x^2 - x + 18 = 0$

$$\Rightarrow \frac{16x^2-8x+1}{8}=0$$

$$\Rightarrow 16x^2-8x+1=0$$

$$\Rightarrow 16x^2-4x-4x+1=0$$

$$\Rightarrow 4x(4x-1)-1(4x-1)=0$$

$$\Rightarrow (4x-1)(4x-1)=0$$

So, either $4x-1=0$ or $4x-1=0$

$$x=\frac{1}{4} \text{ or } x=\frac{1}{4}$$

hence, the roots of the given equation are $\frac{1}{4}$ and $\frac{1}{4}$.

2. (i) We have, $2x^2 + x - 4 = 0$

On dividing both sides by 2, we have

$$x^2+\frac{x}{2}-2=0$$

$$\Rightarrow x^2+\frac{1}{2}x+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-2=0$$

$$\left[b=\frac{1}{2}(\text{coefficient of x})=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}\right]$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2-\frac{1}{16}-2=0$$

$$\Rightarrow \left(x+\frac{1}{4}\right)^2=\frac{1}{16}+2=\frac{1+32}{16}=\frac{33}{16}>0$$

$\Rightarrow$ Roots exist.

$$\therefore \quad x=-\frac{1}{4}\pm\frac{\sqrt{33}}{4} \text{ or } x=-\frac{1}{4}\pm\frac{\sqrt{33}}{4}$$

$$\Rightarrow x+\frac{1}{4}=\frac{\sqrt{33}}{4} \text{ or } x+\frac{1}{4}=-\frac{\sqrt{33}}{4}$$

$$\therefore \quad x=-\frac{1}{4}+\frac{\sqrt{33}}{4} \text{ or } x=-\frac{1}{4}-\frac{\sqrt{33}}{4}$$

$$\Rightarrow x=\frac{\sqrt{33}-1}{4} \text{ or } x=\frac{-\sqrt{33}+1}{4}$$

Hence, roots of given equation are

$$\frac{\sqrt{33}-1}{4} \text{ and } \frac{-\left(\sqrt{33}+1\right)}{4}$$

(ii) We have, $4x^2 + 4\sqrt{3}x + 3 = 0$

On dividing both sides by 4, we have

$$x^2+\sqrt{3}x+\frac{3}{4}=0$$

$$\Rightarrow x^2+\sqrt{3}x+\left(\frac{\sqrt{3}}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2+\frac{3}{4}=0$$

$$\Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)^2-\frac{3}{4}+\frac{3}{4}=0$$

$$\Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)^2=0 \quad \text{...(i)}$$

$\Rightarrow$ Roots exist.

$$\therefore \text{(i)} \Rightarrow x=-\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2}$$

Hence, roots of given equation are $-\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{2}$.

3. (i) We have, $2x^2 - 7x + 3 = 0$

Here, a = 2, b = -7 and c = 3

Therefore, $D = b^2 - 4ac$

$\Rightarrow D = (-7)^2 - 4 \times 2 \times 3 = 49 - 24 = 25$

$\because D > 0, \therefore$ roots exist.

$$\text{Thus, } x=\frac{-b\pm\sqrt{D}}{2a}=\frac{-(-7)\pm\sqrt{25}}{2\times2}=\frac{7\pm5}{4}$$

$$x=\frac{7+5}{4} \text{ or } \frac{7-5}{4}=3 \text{ or } \frac{1}{2}$$

So, the roots of given equation are 3 and $\frac{1}{2}$.

(ii) We have, $4x^2 + 4\sqrt{3}x + 3 = 0$

Here, a = 4, b = $4\sqrt{3}$ and c = 3

Therefore, $D = b^2 - 4ac = (4\sqrt{3})^2 - 4 \times 4 \times 3 = 48 - 48 = 0$

$\therefore D = 0$, roots exist and are equal.

$$\text{Thus, } x=\frac{-b\pm\sqrt{D}}{2a}=\frac{-4\sqrt{3}\pm0}{2\times4}=\frac{-\sqrt{3}}{2}$$

Hence, roots of given equation are $-\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{2}$.

4. We have, $p^2x^2 + (p^2 - q^2) x - q^2 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we have

$a = p^2$, $b = p^2 - q^2$ and $c = -q^2$

$\therefore D = b^2 - 4ac$

$\Rightarrow (p^2 - q)^2 - 4 \times p^2 \times (-q^2)$

$\Rightarrow (p^2 - q^2)^2 + 4p^2q^2$

$\Rightarrow (p^2 + q^3)^2 > 0$

So, the given equation has real roots given by

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(p^2-q^2)+(p^2+q^2)}{2p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$$

and

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(p^2-q^2)-(p^2+q^2)}{2p^2} = \frac{-2p^2}{2p^2} = -1$$

Hence, roots are $\frac{q^2}{p^2}$ and -1.

5. Given, $\frac{1}{x+3} - \frac{1}{x-6} = \frac{9}{20}; x \neq -3, 6$

$$\Rightarrow \frac{(x-6)-(x+3)}{(x+3)(x-6)} = \frac{9}{20}$$

$$\Rightarrow \frac{-9}{(x+3)(x-6)} = \frac{9}{20}$$

$\Rightarrow (x + 3)(x - 6) = -20$

$\Rightarrow -20$ or $x^2 - 3x + 2 = 0$

$\Rightarrow x^2 - 2x - x + 2 = 0$

$\Rightarrow x(x - 2) - 1(x - 2) = 0)$

$\Rightarrow (x - 1)(x - 2) = 0$

$\Rightarrow x = 1$ or $x = 2$

Both x = 1 and x = 2 are satisfying the given equation. Hence, x = 1, 2 are the solutions of the equation.

6. (i) We have, $3x^2 - 4\sqrt{3}x + 4 = 0$

Here, $a = 3$, $b = -4\sqrt{3}$ and $c = 4$

Therefore,

$D = b^2 - 4ac$

$\Rightarrow (-4\sqrt{3})^2 - 4 \times 3 \times 4$

$\Rightarrow 48 - 48 = 0$

Hence, the given quadratic equation has real and equal roots.

Thus,

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4\sqrt{3}) \pm \sqrt{0}}{2 \times 3} = \frac{2\sqrt{3}}{3}$$

Hence, equal roots of given equation are $\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$

(ii) We have, $2x^2 - 6x + 3 = 0$

Here, $a = 2$, $b = -6$, $c = 3$

Therefore, $D = b^2 - 4ac$

$= (-6)^2 4 \times 2 \times 3 = 36 - 24 = 12 > 0$

Hence, given quadratic equation has real and distinct roots.

Thus,

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-6) \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

Hence, equal roots of given equation are $\frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$

7. (i) We have, $2x^2 + kx + 3 = 0$

Here, $a = 2$, $b = k$, $c = 3$

$D = b^2 - 4ac = k^2 - 4 \times 2 \times 3 = k^2 - 24$

For equal roots

$D = 0$

i.e., $k^2 - 24 = 0$

$\Rightarrow k^2 = 24$

$\Rightarrow k = \pm \sqrt{24}$

$\Rightarrow k = + 2\sqrt{6}$

(ii) We have, $kx(x - 2) + 6 = 0$

$\Rightarrow kx^2 - 2kx + 6 = 0$

Here, $a = k$, $b = -2k$, $c = 6$

For equal roots, we have

$D = 0$

i.e., $b^2 - 4ac = 0$

$\Rightarrow (-2k)^2 - 4 \times k \times 6 = 0$

$\Rightarrow 4k^2 - 24k = 0$

$\Rightarrow 4k(k - 6) = 0$

Either $4k = 0$ or $k - 6 = 0$

$\Rightarrow k = 0$ or $k = 6$

But k = 0 6) because if k = 0 then given equation will not be a quadratic equation).

So, k = 6.

8. Since the equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ has equal roots, therefore discriminant

D = (b – c)² – 4(a – b) (c – a) = 0

⇒ $b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab)$

⇒ $b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$

⇒ $4a^2 + b^2 + c^2 - 4ab + 2bc - 4ac = 0$

⇒ $(2a)^2 + (-b)^2 + (-c)^2 + 2(2a)(-b) + 2(-b)(-c) + 2(-c)\,2a = 0$

⇒ $(2a - b - c)^2 = 0$

⇒ 2a – b – c = 0

⇒ 2a = b + c. Hence Proved.

9. The given equation is $(1 + m^2)x^2 + (2mc)x + (c^2 - a^2) = 0$

Here, $A = 1 + m^2$, $B = 2mc$ and $C = c^2 - a^2$

Since the given equation has equal roots, therefore $D = 0 = B^2 - 4AC = 0$.

⇒ $(2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

⇒ $4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$

⇒ $m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$. [Dividing throughout by 4]

⇒ $-c^2 + a^2(1 + m^2) = 0$

⇒ $c^2 = a(1 + m^2)$ Hence Proved

10. Sum of roots = $\frac{-B}{A} \Rightarrow \sin\theta + \cos\theta = \frac{-b}{c}$ ...(i)

Product of the roots = $\frac{C}{A} \Rightarrow \sin\theta . \cos\theta = \frac{c}{a}$ ..(ii)

Now, we have, $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow (\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta = 1$

$\Rightarrow \left(\frac{-b}{a}\right)^2 - 2.\frac{c}{a} = 1$

$\Rightarrow \frac{b^2}{a^2} - \frac{2c}{a} = 1$ or $b^2 - 2ac = a^2$

$\Rightarrow a^2 - b^2 + 2ac = 0$

Long Answer :

1. We have, $abx^2 + (b^2 - ac)x - bc = 0$

Here, $A = ab$, $B = b^2 - ac$, $C = -bc$

$$\therefore \quad x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\Rightarrow \quad x = \frac{-(b^2 - ac) \pm \sqrt{(b^2 - ac)^2 - 4(ab)(-bc)}}{2ab}$$

$$\Rightarrow \quad x = \frac{-(b^2 - ac) \pm \sqrt{(b^2 - ac)^2 - 4ab^2c}}{2ab}$$

$$\Rightarrow \quad x = \frac{-(b^2 - ac) \pm \sqrt{(b^2 - 2ab^2c + a^2c^2 + 4ab^2c)}}{2ab}$$

$$\Rightarrow \quad x = \frac{-(b^2 - ac) \pm \sqrt{(b^2 + ac)^2}}{2ab}$$

$$\Rightarrow \quad x = \frac{-(b^2 - ac) \pm (b^2 + ac)}{2ab}$$

$$\Rightarrow \quad x = \frac{-(b^2 - ac) + (b^2 + ac)}{2ab}$$

$$\text{or} \quad x = \frac{-(b^2 - ac) - (b^2 + ac)}{2ab}$$

$$x = \frac{2ac}{2ab} \quad \text{or} \quad x = \frac{-2b^2}{2ab}$$

$$\Rightarrow \quad x = \frac{c}{b} \quad \text{or} \quad x = \frac{-b}{a}$$

2. Since the quadratic equation has equal roots, D = 0

i.e., $b^2 - 4ac = 0$

In $(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$

Here, a = (2p + 1), b = -(7p + 2), c = (7p – 3)

For $p = -\frac{4}{7}$

$$\left(2\times\left(\frac{-4}{7}\right)+1\right)x^2 - \left(7\times\left(\frac{-4}{7}\right)+2\right)x + \left(7\times\left(\frac{-4}{7}\right)-3\right) = 0$$

$$\Rightarrow \quad \frac{-1}{7}x^2 + 2x - 7 = 0$$

$$\Rightarrow \quad x^2 - 14x + 49 = 0$$

$$\Rightarrow \quad (x - 7)^2 = 0$$

$$\Rightarrow \quad x = 7, 7$$

For $p = 4$,

$$(2\times 4 + 1)x^2 - (7\times 4 + 2)x + (7\times 4 - 3) = 0$$

$$\Rightarrow \quad 9x^2 - 30x + 25 = 0$$

$$\Rightarrow \quad 9x^2 - 15x - 15x + 25 = 0$$

$$\Rightarrow \quad 3x(3x - 5) - 5(3x - 5) = 0$$

$$\Rightarrow \quad (3x - 5)(3x - 5) = 0$$

$$\Rightarrow \quad x = \frac{5}{3}, \frac{5}{3}$$

3. $$\frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{(x-4)(x-7) + (x-6)(x-5)}{(x-5)(x-7)} = \frac{10}{3}$$

$$\Rightarrow \frac{x^2-7x-4x+28+x^2-5x-6x+30}{x^2-7x-5x+35}=\frac{10}{3}$$

$$\Rightarrow \frac{2x^2-22x+58}{x^2-12x+35}=\frac{10}{3}$$

$$\Rightarrow \frac{x^2-11x+29}{x^2-12x+35}=\frac{5}{3}$$

$$\Rightarrow 3x^2-33x+87=5x^2-60x+175$$

$$\Rightarrow 2x^2-27x+88=0$$

$$\Rightarrow 2x^2-16x-11x+88=0$$

$$\Rightarrow 2x(x-8)-11(x-8)=0$$

$$\Rightarrow (2x-11)(x-8)=0$$

$$\Rightarrow 2x-11=0 \text{ or } x-8=0$$

$$\Rightarrow x=\frac{11}{2} \text{ or } x=8$$

4. Let the present age of Rehman be x years.

So, 3 years ago, Rehman's age = (x – 3) years

And 5 years from now, Rehman's age = (x + 5) years

Now, according to question, we have

$$\frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

$$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$$

$$\Rightarrow \frac{2x+2}{(x-3)(x+5)}=\frac{1}{3}$$

$$\Rightarrow 6x+6=(x-3)(x+5)$$

$$\Rightarrow 6x+6=x^2+5x-3x-15$$

$$\Rightarrow x^2+2x-15-6x-6=0$$

$$\Rightarrow x^2-4x-21=0$$

$$\Rightarrow x^2-7x+3x-21=0$$

$$\Rightarrow x(x-7)+3(x-7)=0$$

$$\Rightarrow (x-7)(x+3)=0$$

$$\Rightarrow x=7 \text{ or } x=-3$$

But x ≠ -3 (age cannot be negative)

Therefore, present age of Rehman = 7 years.

5. Let the two natural numbers be x and y such that x > y.

According to the question,

Difference of numbers,

$$x - y = 5 \Rightarrow x = 5 + y \quad .....(i)$$

Difference of the reciprocals,

$$\frac{1}{y}-\frac{1}{x}=\frac{1}{10} \quad ....(ii)$$

Putting the value of (i) in (ii)

$$\frac{1}{y}-\frac{1}{5+y}=\frac{1}{10}$$

$$\Rightarrow \frac{5+y-y}{y(5+y)}=\frac{1}{10}$$

$$\Rightarrow 50=5y+y^2$$

$$\Rightarrow y^2+5y-50=0$$

$$\Rightarrow y^2+10y-5y-50=0$$

$$\Rightarrow y(y+10)-5(y+10)=0$$

$$\Rightarrow (y-5)(y+10)=0$$

∴ y is a natural number.

∵ y = 5

Putting the value of y in (i), we have

⇒ x = 5 + 5

⇒ x = 10

The required numbers are 10 and 5.

6. Let the two consecutive odd numbers be x and x + 2.

$$\Rightarrow x^2+(x+2)^2=394$$

$$\Rightarrow x^2+x^2+4+4x=394$$

$$\Rightarrow 2x^2+4x+4=394$$

$$\Rightarrow 2x^2+4x-390=0$$

$$\Rightarrow x^2+2x-195=0$$

$$\Rightarrow x^2+15x-13x-195=0$$

$$\Rightarrow x(x+15)-13(x+15)=0$$

$$\Rightarrow (x-13)(x+15)=0$$

$$\Rightarrow x-13=0 \text{ or } x+15=0$$

$$\Rightarrow x=13 \text{ or } x=-15$$

Hence, the numbers are 13 and 15 or -15 and -13.

7. Let the numbers be x and 15 – x.

According to given condition,

$$\frac{1}{x}+\frac{1}{15-x}=\frac{3}{10} \Rightarrow \frac{15-x+x}{x(15-x)}=\frac{3}{10}$$

⇒ 150 = 3x(15 – x)

⇒ 50 = 15x – x^2

⇒ x^2 – 15x + 50 = 0

$\Rightarrow x^2 - 5x - 10x + 50 = 0$

$\Rightarrow x(x - 5) - 10(x - 5) = 0$

$\Rightarrow (x - 5)(x - 10) = 0$

$\Rightarrow x = 5$ or 10.

When x = 5, then 15 – x = 15 – 5 = 10

When x = 10, then 15 – x = 15 – 10 = 5

Hence, the two numbers are 5 and 10.

8. Let Shefali's marks in Mathematics be x.

Therefore, Shefali's marks in English is (30 – x).

Now, according to question,

$\Rightarrow (x + 2)(30 - x - 3) = 210$

$\Rightarrow (x + 2)(27 - x) = 210$

$\Rightarrow 27x - x^2 + 54 - 2x = 210$

$\Rightarrow 25x - x^2 + 54 - 210 = 0$

$\Rightarrow 25x - x^2 - 156 = 0$

$\Rightarrow -(x^2 - 25x + 156) = 0$

$\Rightarrow x^2 - 25x + 156 = 0$

$= x^2 - 13x - 12x + 156 = 0$

$\Rightarrow x(x - 13) - 12(x - 13) = 0$

$\Rightarrow (x - 13)(x - 12) = 0$

Either x – 13 or x – 12 = 0

$\therefore x = 13$ or $x = 12$

Therefore, Shefali's marks in Mathematics = 13

Marks in English = 30 – 13 = 17

or Shefali's marks in Mathematics = 12

marks in English = 30 – 12 = 18.

9. Let the uniform speed of the train be x km/h.

Then, time taken to cover 360 km = $\frac{360}{x}$ h

Now, new increased speed = (x + 5) km/h

So, time taken to cover 360 km = $\frac{360}{x+5}$ h

According to question, $\frac{360}{x} - \frac{360}{x+5} = 1$

$$\Rightarrow 360\left(\frac{1}{x} - \frac{1}{x+5}\right) = 1$$

$$\Rightarrow \frac{360(x+5-x)}{x(x+5)} = 1$$

$$\Rightarrow \frac{360 \times 5}{x(x+5)} = 1$$

$$\Rightarrow 1800 = x^2 + 5x$$

$$\therefore x^2 + 5x - 1800 = 0$$

$$\Rightarrow x^2 + 45x - 40x - 1800 = 0$$

$$\Rightarrow x(x+45) - 40(x+45) = 0$$

$$\Rightarrow (x+45)(x-40) = 0$$

Either $x + 45 = 0$ or $x - 40 = 0$

$\therefore x = -45$ or $x = 40$

But x cannot be negative, so $x \neq -45$

therefore, $x = 40$

Hence, the uniform speed of train is 40 km/h

10. Let x be the length of the side of first square and y be the length of side of the second square.

Then, $x^2 + y^2 = 468$ ...(i)

Let x be the length of the side of the bigger square.

$4x - 4y = 24$

$\Rightarrow x - y = 6$ or $x = y + 6$ ...(ii)

Putting the value of x in terms of y from equation (ii), in equation (i), we get

$(y + 6)^2 + y^2 = 468$

$\Rightarrow y^2 + 12y + 36 + y^2 = 468$ or $232 + 12y - 432 = 0$

$\Rightarrow y^2 + 6y - 216 = 0$

$\Rightarrow y^2 + 18y - 12y - 216 = 0$

$\Rightarrow y(y + 18) - 12(y + 18) = 0$

$\Rightarrow (y + 18)(y - 12) = 0$

Either y + 18 = 0 or y – 12 = 0

$\Rightarrow y = -18$ or $y = 12$

But, sides cannot be negative, so y = 12

Therefore, x = 12 + 6 = 18

Hence, sides of two squares are 18 m and 12 m.

Case Study Answers:

1. **Answer :**

i. (d) a, b and c are real numbers a ≠ 0

ii. (b) 2

iii. (a) x(x + 3) + 7 = 5x - 11

Solution:

a. $x(x + 3) + 7 = 5x - 11$

$\Rightarrow x^2 + 3x + 7 = 5x - 11$

$\Rightarrow x^2 - 2x + 18 = 0$ is a quadratic equation.

b. $(x - 1)^2 - 9 = (x - 4)(x + 3)$

$\Rightarrow x^2 - 2x - 8 = x^2 - x - 12$

$\Rightarrow x - 4 = 0$ is not a quadratic equation.

c. $x^2(2x + 1) - 4 = 5x^2 - 10$

$\Rightarrow 2x^3 + x^2 - 4 = 5x^2 - 10$

$\Rightarrow 2x^3 - 4x^2 + 6 = 0$ is not a quadratic equation.

d. $x(x - 1)(x + 7) = x(6x - 9)$

$x^3 + 6x^2 - 7x = 6x^2 - 9x$

$x^3 + 2x = 0$ is not a quadratic equation.

iv. (d) All of these

v. (d) None of these

2. **Answer :**

i. (b) $2x^2 + 2x - 649 = 0$

Solution:

Let two consecutive integers be x, x + 1. Given, $x^2 + (x + 1)^2 = 650$.

$\Rightarrow 2x^2 + 2x + 1 - 650 = 0$

$\Rightarrow 2x^2 + 2x - 649 = 0$

ii. (c) $x^2 - 15x + 50 = 0$

Solution:

Let the two numbers be x and 15 - x. Given 1x+115−x=310

$\Rightarrow 10(15 - x + x) = 3x(15 - x)$

$\Rightarrow 50 = 15x - x^2$

$\Rightarrow x^2 - 15x + 50 = 0$

iii. (d) $x^2 + 3x - 504 = 0$

Solution:

Let the numbers be x and x + 3. Given, $x(x + 3) = 504$

$\Rightarrow x^2 + 3x - 504 = 0$

iv. (c) $x^2 - 3x - 108 = 0$

Solution:

Let the number be x. According to question, $x^2 - 84 = 3(x + 8)$

$\Rightarrow x^2 - 84 = 3x + 24$

$\Rightarrow x^2 - 3x - 108 = 0$

v. (d) $x^2 + 12x - 160 = 0$

Solution:

Let the number be x. According to question x+12=160xx+12=160x

$\Rightarrow x^2 + 12x - 160 = 0$

Assertion Reason Answer-

1. (b) Both A and R are true and R is the correct explanation for A.
2. (c) A is true but R is false.

Arithmetic Progressions 5

1. What is a Sequence?

- A **sequence** is an arrangement of numbers in a definite order according to some rule.
- The various numbers occurring in a sequence are called its **terms**.
- We denote the terms of a sequence by a_1, a_2, a_3...etc. Here, the subscripts denote the positionsof the terms in the sequence.
- In general, the number at the n^{th} place is called the n^{th} term of the sequence and is denoted by a_n. The n^{th} term is also called the **general term** of the sequence.
- A sequence having a finite number of terms is called a **finite sequence**.
- A sequence which do not have a last term and which extends indefinitely is known as an **infinite sequence**.

Sequences, Series and Progressions

A sequence is a finite or infinite list of numbers following a specific pattern. For example, 1, 2, 3, 4, 5,... is the sequence, an infinite sequence of natural numbers.

A series is the sum of the elements in the corresponding sequence. For example, 1 + 2 + 3 + 4 + 5....is the series of natural numbers. Each number in a sequence or a series is called a term.

A progression is a sequence in which the general term can be can be expressed using a mathematical formula.

2. Arithmetic Progression:

- An **arithmetic progression** is a list of numbers in which each term is obtained by adding a fixed number to the preceding term, except the first term.
- Each of the numbers of the sequence is called a **term** of an Arithmetic Progression. The fixed number is called the **common difference**. This common difference could be a positive number, a negative number or even zero.

3. General form and general term (n^{th} term) of an A.P:

- The **general form of an A.P.** is a, a + d, a + 2d, a + 3d....., where 'a' is the first term and 'd' is the common difference.
- The **general term (n^{th} term)** of an A.P is given by $a_n = a + (n - 1)\,d$, where 'a' is the first term and 'd' is the common difference.
- If the A.P a, a + d, a + 2d, …….., l is reversed to l, l – d, l – 2d, ….., a then the common difference changes to negative of the common difference of the original sequence.
- To find the **n^{th} term from the end**, we consider this AP backward such that the last term becomes the first term.

  l, (l - d), (l - 2d) ……

  The general term of this AP is given by $a_n = l + (n - 1)(-d)$

4. Algorithm to determine whether a sequence is an AP or not:

When we are given an algebraic formula for the general term of the sequence:

Step 1: Obtain a_n.

Step 2: Replace n by (n + 1) in a_n to get a_{n+1}

Step 3: Calculate $a_{n+1} - a_n$

Step 4: Check the value of $a_{n+1} - a_n$.

If $a_{n+1} - a_n$ is independent of n, then the given sequence is an A.P. Otherwise, it is not an A.P.

OR

A list of numbers a_1, a_2, a_3...... is an A.P, if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3$ ... give the same value, i.e., $a_{k+1} - a_k$ is same for all different values of k.

5. Sometimes we require certain number of terms in A.P. The following ways of selecting terms are generally very convenient.

Number of terms	Terms	Common difference
3	a – d, a, a + d	d
4	a – 3d, a – d, a + d, a + 3d	2d
5	a – 2d, a – d, a, a + d, a + 2d	d
6	a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d	2d

It should be noted that in case of an odd number of terms, the middle term is 'a' and the common difference is 'd' while in case of an even number of terms the middle terms are a – d, a + d and the common differences is 2d.

6. **Arithmetic mean:**

If three number a, b, c (in order) are in A.P. Then,

b – a = c – b = common difference

⇒ 2b = a + c

Thus a, b and c are in A.P., if and only if 2b = a + c. In this case, b is called the **Arithmetic mean** of a and c.

7. **Sum of n terms of an A.P:**

➢ **Sum of n terms of an A.P.** is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where 'a' is the first term, 'd' is the common difference and 'n' is the total number of terms.

➢ **Sum of n terms of an A.P.** is also given by:

$$S_n = \frac{n}{2}[a + l]$$

where 'a' is the first term and 'l' is the last term.

➢ **Sum of first n natural numbers** is given by $\frac{n(n+1)}{2}$

8. The n^{th} term of an A.P is the difference of the sum to first n terms and the sum to first (n - 1) terms of it. That is, $a_n = S_n - S_{n-1}$

9. **Common Difference**

The difference between two consecutive terms in an AP, (which is constant) is the "common difference"(d) of an A.P. In the progression: 2, 5, 8, 11, 14 ...the common difference is 3.

As it is the difference between any two consecutive terms, for any A.P, if the common difference is:

- positive, the AP is increasing.
- zero, the AP is constant.
- negative, the A.P is decreasing.

10. Finite and Infinite AP

- A finite AP is an A.P in which the number of terms is finite. For example, the A.P: 2, 5, 8……32, 35, 38
- An infinite A.P is an A.P in which the number of terms is infinite. For example: 2, 5, 8, 11…..

A finite A.P will have the last term, whereas an infinite A.P won't.

Finite sets are the sets having a finite/countable number of members. Finite sets are also known as countable sets as they can be counted. The process will run out of elements to list if the elements of this set have a finite number of members.

Examples of finite sets:

$$P = \{0, 3, 6, 9, \ldots, 99\}$$

$$Q = \{a : a \text{ is an integer}, 1 < a < 10\}$$

A set of all English Alphabets (because it is countable).

Another example of a Finite set:

A set of months in a year.

M = {January, February, March, April, May, June, July, August, September, October, November, December}

$$n(M) = 12$$

It is a finite set because the number of elements is countable.

Cardinality of Finite Set

If 'a' represents the number of elements of set A, then the cardinality of a finite set is n(A) = a.

So, the Cardinality of the set A of all English Alphabets is 26, because the number of elements (alphabets) is 26.

Hence, n (A) = 26.

Similarly, for a set containing the months in a year will have a cardinality of 12.

So, this way we can list all the elements of any finite set and list them in the curly braces or in Roster form.

Properties of Finite sets

The following finite set conditions are always finite.

- A subset of Finite set
- The union of two finite sets
- The power set of a finite set

Few Examples:

$$P = \{1, 2, 3, 4\}$$

$$Q = \{2, 4, 6, 8\}$$

$$R = \{2, 3)$$

Here, all the P, Q, R are the finite sets because the elements are finite and countable.

R

⊂

P, i.e R is a Subset of P because all the elements of set R are present in P. So, the subset of a finite set is always finite.

P U Q is {1, 2, 3, 4, 6, 8}, so the union of two sets is also finite.

The number of elements of a power set = 2^n.

The number of elements of the power set of set P is 24 = 16, as the number of elements of set P is 4. So it shows that the power set of a finite set is finite.

Non- Empty Finite set

It is a set where either the number of elements are big or only starting or ending is given. So, we denote it with the number of elements with n(A) and if n(A)is a natural number then it's a finite set.

Example:

S = {a set of the number of people living in India}

It is difficult to calculate the number of people living in India but it's somewhere a natural number. So, we can call it a non-empty finite set.

If N is a set of natural numbers less than n. So the cardinality of set N is n.

$$N = \{1,2,3....n\}$$

$$X = x_1, x_2, ..., x_n$$

$$Y = \{x : x_1 \in N, 1 \le i \le n\}, \text{ where i is the integer between 1 and n.}$$

Can we say that an empty set is a finite set?

Let's learn what is an empty set first.

An empty set is a set which has no elements in it and can be represented as { } and shows that it has no element.

$$P = \{\ \} \text{ Or } \emptyset$$

As the finite set has a countable number of elements and the empty set has zero elements so, it is a definite number of elements.

So, with a cardinality of zero, an empty set is a finite set.

What is Infinite set?

If a set is not finite, it is called an infinite set because the number of elements in that set is not countable and also we cannot represent it in Roster form. Thus, infinite sets are also known as uncountable sets.

So, the elements of an Infinite set are represented by 3 dots (ellipse) thus, it represents the infinity of that set.

Examples of Infinite Sets

- A set of all whole numbers, W = {0, 1, 2, 3, 4,...}
- A set of all points on a line
- The set of all integers

Cardinality of Infinite Sets

The cardinality of a set is n (A) = x, where x is the number of elements of a set A. The cardinality of an infinite set is n (A) = ∞ as the number of elements is unlimited in it.

Properties of Infinite Sets

- The union of two infinite sets is infinite
- The power set of an infinite set is infinite
- The superset of an infinite set is also infinite

11. Comparison of Finite and Infinite Sets

Let's compare the differences between Finite and Infinite set:

The sets could be equal only if their elements are the same, so a set could be equal only if it is a finite set, whereas if the elements are not comparable, the set is infinite.

Factors	Finite sets	Infinite sets
Number of elements	Elements are countable	The number of elements is uncountable
Continuity	It has a start and end elements	It is endless from the start or end. Both the sides could have continuity
Cardinality	n(A) = n, n is the number of elements in the set	n(A) = ∞ as the number of elements are uncountable
union	Union of two finite sets is finite	Union of two infinite sets is infinite
Power set	The power set of a finite set is also finite	The power set of an infinite set is infinite
Roster form	Can be easily represented in roster form	As the set in infinite set can't be represented in Roster form, so we use three dots to represent the infinity

How to know if a Set is Finite or Infinite?

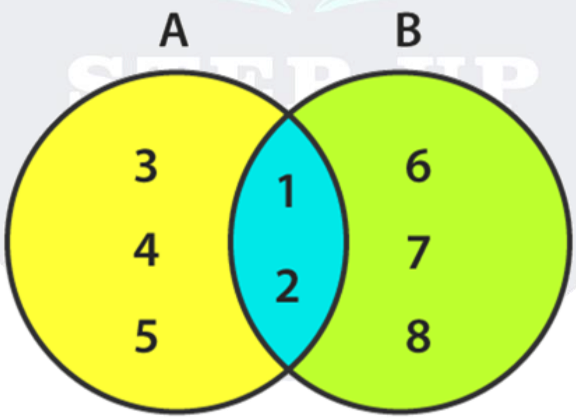
As we know that if a set has a starting point and an ending point both, it is a finite set, but it is infinite if it has no end from any side or both sides.

Points to identify a set is whether a finite or infinite are:

An infinite set is endless from the start or end, but both the side could have continuity unlike in Finite set where both start and end elements are there.

If a set has the unlimited number of elements, then it is infinite and if the elements are countable then it is finite.

12. Graphical Representation of Finite and Infinite Sets



Here in the above picture,

A = {1, 2, 3, 4, 5}

B = {1, 2, 6, 7, 8}

A U B = {1, 2, 3, 4, 5, 6, 7, 8}

A∩B = {1, 2}

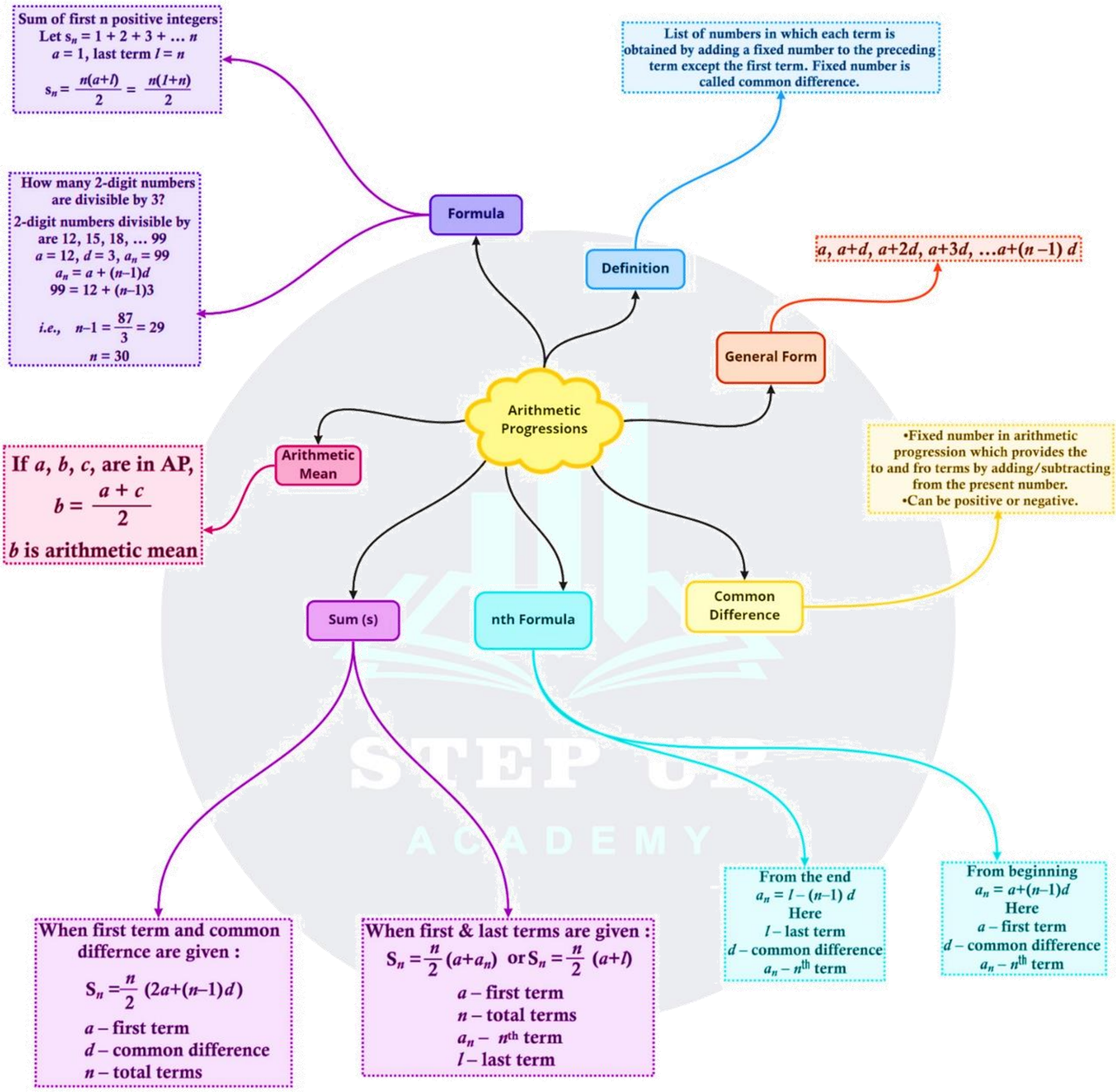
Both A and B are finite sets as they have a limited number of elements.

n(A) = 5 and n(B) = 5

AUB and A∩B are also finite.

So, a Venn diagram can represent the finite set but it is difficult to do the same for an infinite set as the number of elements can't be counted and bounced in a circle

Class : 10th mathematics
Chapter- 5 : Arithmetic Progressions



Important Questions

Multiple Choice Questions:

1. The n^{th} term of an A.P. is given by an = 3 + 4n. The common difference is
   (a) 7
   (b) 3
   (c) 4
   (d) 1
2. If p, q, r and s are in A.P. then r – q is
   (a) s – p
   (b) s – q
   (c) s – r
   (d) none of these
3. If the sum of three numbers in an A.P. is 9 and their product is 24, then numbers are
   (a) 2, 4, 6
   (b) 1, 5, 3
   (c) 2, 8, 4
   (d) 2, 3, 4
4. The $(n - 1)^{th}$ term of an A.P. is given by 7,12,17, 22,... is
   (a) 5n + 2
   (b) 5n + 3
   (c) 5n – 5
   (d) 5n – 3
5. The n^{th} term of an A.P. 5, 2, -1, -4, -7 ... is
   (a) 2n + 5
   (b) 2n – 5
   (c) 8 – 3n
   (d) 3n – 8
6. The 10th term from the end of the A.P. -5, -10, -15,..., -1000 is
   (a) -955
   (b) -945
   (c) -950
   (d) -965
7. Find the sum of 12 terms of an A.P. whose nth term is given by an = 3n + 4
   (a) 262
   (b) 272
   (c) 282
   (d) 292
8. The sum of all two-digit odd numbers is
   (a) 2575
   (b) 2475
   (c) 2524
   (d) 2425
9. The sum of first n odd natural numbers is
   (a) $2n^2$
   (b) 2n + 1
   (c) 2n – 1
   (d) n^2
10. The number of multiples lie between n and n^2 which are divisible by n is
   (a) n + 1
   (b) n
   (c) n – 1
   (d) n – 2

Very Short Questions:

1. Which of the following can be the n^{th} term of a_n AP?
   $4n + 3, 3n^2 + 5, n^2 + 1$ give reason.
2. Is 144 a term of the AP: 3, 7, 11, ...? Justify your answer.
3. The first term of a_n AP is p and its common difference is q. Find its 10^{th} term.
4. For what value of k: 2k, k + 10 and 3k + 2 are in AP?
5. If $a_n = 5 - 11n$, find the common difference.
6. If n^{th} term of an AP is $\frac{3+n}{4}$ find its 8^{th} term.
7. For what value of p are 2p + 1, 13, 5p – 3, three consecutive terms of AP?
8. In a_n AP, if d = -4, n = 7, a, = 4 then find a.
9. Find the 25^{th} term of the AP: $-5, \frac{-5}{2}, 0, \frac{-5}{2}$
10. Find the common difference of an AP in which $a_{18} - a_{14} = 32$.

Short Questions:

1. In which of the following situations, does the list of numbers involved to make an AP? If yes, give a reason.

(i) The cost of digging a well after every meter of digging, when it costs 150 for the first meter and rises by 50 for each subsequent meter.

(ii) The amount of money in the account every year, when 10,000 is deposited at simple interest at 8% per annum.

2. Find the 20^{th} term from the last term of the AP: 3, 8, 13, ..., 253.
3. If the sum of the first p terms of an AP is $ap^2 + bp$, find its common difference.
4. The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.
5. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.
6. Which term of the AP: 3, 8, 13, 18, ... is 78?
7. Find the 31^{st} term of an AP whose 11^{th} term is 38 and the 16^{th} term is 73.
8. An AP consists of 50 terms of which 3^{rd} term is 12 and the last term is 106. Find the 29^{th} term.
9. If the 8^{th} term of an AP is 31 and the 15^{th} term is 16 more than the 11^{th} term, find the AP.
10. Which term of the arithmetic progression 5, 15, 25, ... will be 130 more than its 31^{st} term?

Long Questions :

1. The sum of the 4^{th} and 8^{th} term of an AP is 24 and the sum of the 6^{th} and 10^{th} term is 44. Find the first three terms of the AP.
2. The sum of the first n terms of an AP is given by $s_n = 3n^2 - 4n$. Determine the AP and the 12^{th} term.
3. Divide 56 into four parts which are in AP such that the ratio of product of extremes to the product of means is 5 : 6.
4. In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the AP.
5. If s, denotes the sum of the first n terms of an AP, prove that $s_{30} = 3\,(s_{20} - s_{10})$.
6. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief?
7. The houses in a row are numbered consecutively from 1 to 49. show that there exists a value of X such that sum of numbers of houses preceeding the house numbered X is equal to sum of the numbers of houses following X. Find value of X.
8. If the ratio of the 11^{th} term of an AP to its 18^{th} term is 2:3, find the ratio of the sum of the first five terms to the sum of its first 10 terms.
9. Find the sum of the first 15 multiples of 8.
10. Find the sum of all two-digit natural numbers which when divided by 3 yield 1 as remainder.

Case Study Questions:

1. In a pathology lab, a culture test has been conducted. In the test, the number of bacteria taken into consideration in various samples is all 3-digit numbers that are divisible by 7, taken in order.

On the basis of above information, answer the following questions

i. How many bacteria are considered in the fifth sample?

a. 126

b. 140

c. 133

d. 149

ii. How many samples should be taken into consideration?

a. 129

b. 128

c. 130

d. 127

iii. Find the total number of bacteria in the first 10 samples.

a. 1365

b. 1335

c. 1302

d. 1540

iv. How many bacteria are there in the 7^{th} sample from the last?

a. 952

b. 945

c. 959

d. 966

v. The number of bacteria in 50^{th} sample is?

a. 546

b. 553

c. 448

d. 496

2. In a class the teacher asks every student to write an example of A.P. Two friends Geeta and Madhuri writes their progressions as -5, -2, 1, 4, ... and 187, 184, 181, ... respectively. Now, the teacher asks various students of the class the following questions on these two progressions. Help students to find the answers of the questions.

i. Find the 34^{th} term of the progression written by Madhuri.

a. 86

b. 88

c. -99

d. 190

ii. Find the sum of common difference of the two progressions.

a. 6

b. −6

c. 1

d. 0

iii. Find the 19^{th} term of the progression written by Geeta.

a. 49

b. 59

c. 52

d. 62

iv. Find the sum of first 10 terms of the progression written by Geeta.

a. 85

b. 95

c. 110

d. 200

v. Which term of the two progressions will have the same value?

a. 31

b. 33

c. 32

d. 30

Assertion Reason Questions:

1. **Directions:** In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

a. Both A and R are true and R is the correct explanation for A.

b. Both A and R are true and R is the correct explanation for A.

c. A is true but R is false.

d. A is false but R is true.

Assertion: 184 is the 50^{th} term of the sequence 3, 7, 11, ...

Reason: The nth term of A.P. is given by $a_n = a + (n - 1)d$

2. **Directions:** In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

a. Both A and R are true and R is the correct explanation for A.

b. Both A and R are true and R is the correct explanation for A.

c. A is true but R is false.

d. A is false but R is true.

Assertion: The nth term of A.P. is given by $a_n = a + (n - 1)d$

Reason: Common difference of the A.P. a, a + d, a + 2d, ……., is given by d = 2nd term – 1st term.

Answer Key

Multiple Choice Questions:

1. (c) 4
2. (c) s – r
3. (d) 2, 3, 4
4. (d) 5n – 3
5. (c) 8 – 3n
6. (a) -955
7. (a) 282
8. (b) 2475
9. (d) n^2
10. (d) n – 2

Very Short Answers:

1. 4n + 3 because nth term of an AP can only be a linear relation in n as $a_n = a + (n - 1)d$.
2. No, because here a = 3 an odd number and d = 4 which is even. so, sum of odd and even must be odd whereas 144 is an even number.
3. 210 = a + 9d = p + 99.
4. Given numbers are in AP
   ∴ (k + 10) – 2k = (3k + 2) – (k + 10)
   ⇒ -k + 10 = 2k – 8 or 3k = 18 or k = 6.
5. We have $a_n = 5 - 11n$
   Let d be the common difference
   $d = a_{n+1} - a_n$
   = 5 – 11(n + 1) – (5 – 11n)
   = 5 – 11n – 11 -5 + 11n = -11
6. $a_n = \frac{3+n}{4}$; So, $a_8 = \frac{3+8}{4} = \frac{11}{4}$
7. since 20 + 1, 13, 5p – 3 are in AP.
   ∴ second term – First term = Third term – second term
   ⇒ 13 – (2p + 1) = 5p – 3 – 13
   ⇒ 13 – 2p – 1 = 5p – 16
   ⇒ 12 – 2p = 5p – 16
   ⇒ -7p = – 28
   ⇒ p = 4
8. We know, $a_n = a + (n - 1)d$
   Putting the values given, we get
   ⇒ 4 = a + (7 – 1)(-4) or a = 4 + 24
   ⇒ a = 28
9. Here, a = -5, $b = \frac{-5}{2} - (-5) = \frac{5}{2}$
   We know,
   $a_{25} = a + (25 - 1)d$
   $= (-5) + 24\left(\frac{5}{2}\right) = -5 + 60 = 55$
10. Given, $a_{18} - a_{14} = 32$
   ⇒ (a + 17d) – (a + 13d) = 32
   ⇒ 17d – 13d = 32 or $d = \frac{32}{4}$

Short Answers:

1. (i) The numbers involved are 150, 200, 250, 300, ...
   Here 200 -150 = 250 – 200 = 300 – 250 and so on
   ∴ It forms an AP with a = 150, d = 50
   (ii) The numbers involved are 10,800, 11,600, 12,400, ...
   which forms an AP with a = 10,800 and d = 800.
2. We have, last term = l = 253
   And, common difference d = 2nd term – 1st term = 8 – 3 = 5
   Therefore, 20th term from end = l -(20 – 1) × d = 253 – 19 × 5 = 253 – 95 = 158.
3. $a_p = s_p - s_{p-1} = (ap^2 + bp) - [a(p-1)^2 + b(p-1)]$
   $= ap^2 + bp - (ap^2 + a - 2ap + bp - b)$
   $= ap^2 + bp - ap^2 - a + 2ap - bp + b = 2ap + b - a$.
   $= a_1 = 2a + b - a = a + b$ and $a_2 = 4a + b - a = 3a + b$
   ⇒ $d = a_2 - a_1 = (3a + b) - (a + b) = 2a$
4. Let the first term be ‘a’ and common difference be ‘d’.

Given, $a = 5$, $T_n = 45$, $s_n = 400$.

$Tn = a + (m - 1)d$

$\Rightarrow 45 = 5 + (m - 1)d$

$\Rightarrow (n - 1)\ d = 40$ ....(i)

$s_n = \frac{n}{2}(a + T_n)$

$\Rightarrow 400 = \frac{n}{2}(5 + 45)$

$\Rightarrow n = 2 \times 8 = 16$ substituting the value of n in (i)

$\Rightarrow (16 - 1)d = 40$

$\Rightarrow d = \frac{40}{15} = \frac{8}{3}$

5. Natural numbers between 101 and 999 divisible by both 2 and 5 are 110, 120, ... 990.

so, $a_1 = 110$, $d = 10$, $a_n = 990$

We know, $a_n = a_1 + (n - 1)d$

$990 = 110 + (n - 1)\ 10$

$(n - 1) = \frac{990-110}{10}$

$\Rightarrow n = 88 + 1 = 89$

6. Let a_n be the required term and we have given AP 3, 8, 13, 18, ...

Here, $a = 3$, $d = 8 - 3 = 5$ and $a_n = 78$

Now, $a_n = a + (n - 1)d$

$\Rightarrow 78 = 3 + (n - 1)\ 5$

$\Rightarrow 78 - 3 = (n - 1) \times 5$

$\Rightarrow 75 = (n - 1) \times 5$

$\Rightarrow \frac{75}{5} = n - 1$

$\Rightarrow 15 = n - 1$

$\Rightarrow n = 15 + 1 = 16$

Hence, 16^{th} term of given AP is 78.

7. Let the first term be a and common difference be d.

Now, we have

$a_{11} = 38 \Rightarrow a + (11 - 1)d = 38$

$\Rightarrow a + 10d = 38$ ...(i)

and $a_{16} = 73 \Rightarrow a + (16 - 1)d = 73$

$\Rightarrow a + 15d = 73$ ...(ii)

Now subtracting (ii) from (i), we have

$$\text{Now, } \begin{array}{r} a + 10d = 38 \\ \underline{-a \pm 15d = -73} \\ -5d = -35 \end{array} \quad \text{or} \quad 5d = 35$$

$\therefore\ d = \frac{35}{5} = 7$

Putting the value of d in equation (i), we have

$a + 10 \times 7 = 38$

$\Rightarrow a + 70 = 38$

$\Rightarrow a = 38 - 70$

$\Rightarrow a = -32$

We have, $a = -32$ and $d = 7$

Therefore, $a_{31} = a + (31 - 1)d$

$\Rightarrow a_{31} = a + 30d$

$\Rightarrow (-32) + 30 \times 7$

$\Rightarrow -32 + 210$

$= a_{31} = 178$

8. Let a be the first term and d be the common difference.

since, given AP consists of 50 terms, so $n = 50$

$a_3 = 12$

$\Rightarrow a + 2d = 12$ ...(i)

Also, $a_{50} = 106$

$\Rightarrow a + 49d = 106$ ... (ii)

subtracting (i) from (ii), we have

$47d = 94$

$\Rightarrow d = \frac{94}{47} = 2$

Putting the value of d in equation (i), we have

$a + 2 \times 2 = 12$

$\Rightarrow a = 12 - 4 = 8$

Here, $a = 8$, $d = 2$

$\therefore$ 29th term is given by

$a_{29} = a + (29 - 1)d = 8 + 28 \times 2$

$\Rightarrow a_{29} = 8 + 56$

$\Rightarrow a_{29} = 64$

9. Let a be the first term and d be the common difference of the AP.

We have, $a_8 = 31$ and $a_{15} = 16 + a_{11}$

$\Rightarrow a + 7d = 31$ and $a + 14d = 16 + a + 10d$

$\Rightarrow a + 7d = 31$ and $4d = 16$

$\Rightarrow a + 7d = 31$ and $d = 4$

$\Rightarrow a + 7 \times 4 = 31$

$\Rightarrow a + 28 = 31$

$\Rightarrow a = 3$

Hence, the AP is a, a + d, a + 2d, a + 3d.....

i.e., 3, 7, 11, 15, 19, ...

10. We have, $a = 5$ and $d = 10$

$\therefore a_{31} = a + 30d = 5 + 30 \times 10 = 305$

Let nth term of the given AP be 130 more than its 31^{st} term. Then,

$a_n = 130 + a_{31}$

∴ a + (n – 1)d = 130 + 305

⇒ 5 + 10(n – 1) = 435

⇒ 10(n – 1) = 430

⇒ n – 1 = 43

⇒ n = 44

Hence, 44^{th} term of the given AP is 130 more than its 31^{st} term.

Long Answers:

1. We have, $a_4 + a_8 = 24$

⇒ a + (4 – 1)d + a + (8 – 1) d = 24

⇒ 2a + 3d + 7d = 24

⇒ 2a + 10d = 24

⇒ 2(a + 5d) = 24

∴ a + 5d = 12

and, $a_6 + a_{10} = 44$

⇒ a + (6 – 1)d + a + (10 – 1) d = 44

⇒ 2a + 5d + 9d = 44

⇒ 2a + 14d = 44

⇒ a + 7d = 22

subtracting (i) from (ii), we have

2d = 10

$\therefore d = \frac{10}{2} = 5$

Putting the value of d in equation (i), we have

a + 5 × 5 = 12

⇒ a = 12 – 25 = -13

Here, a = -13, d = 5

Hence, first three terms are

-13, -13, + 5, -13 + 2 × 5 i.e., -13, -8, -3

2. We have, $s_n = 3n^2 - 4n$ ...(i)

Replacing n by (n – 1), we get

$s_{n-1} = 3(n-1)^2 - 4(n-1)$ ....(ii)

We know, .

$a_n = s_n - s_{n-1} = \{3n^2 - 4n\} - \{3(n-1)^2 - 4(n-1)\}$.

$= \{3n^2 - 4n\} - \{3n^2 + 3 - 6n - 4n + 4\}$

$= 3n^2 - 4n - 3n^2 - 3 + 6n + 4n - 4 = 6n - 7$

so, nth term an = 6n – 7

To get the AP, substituting n = 1, 2, 3... respectively in (iii), we get

$a_1 = 6 \times 1\text{-}7 = -1$,

$a_2 = 6 \times 2 - 7 = 5$

$a_3 = 6 \times 3 - 7 = 11, \ldots$

Hence, AP is – 1,5, 1:1, ...

Also, to get 12th term, substituting n = 12 in (iii), we get

$a_{12} = 6 \times 12 - 7 = 72 - 7 = 65$

3. Let the four parts be a – 3d, a-d, a + d, a + 3d.

Given, (a – 3d) + (a – d) + (a + d) + (a + 3d) = 56

⇒ 4a = 56 or a = 14

Also, $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{5}{6}$

$\Rightarrow \frac{a^2-9d^2}{a^2-d^2} = \frac{5}{6} \Rightarrow 6(196 - 9d^2) = 5(196 - d^2)$

$[\because a = 14]$

$\Rightarrow 6 \times 196 - 54d^2 = 5 \times 196 - 5d^2$

$\Rightarrow 49d^2 = 6 \times 196 - 5 \times 196 = 196$

$\Rightarrow d^2 = 4$ or ± 2

∴ Required parts are 14 – 3 × 2, 14 – 2, 14 + 2, 14 + 3 × 2

or 14 – 3(–2), 14 + 2, 14 – 2, 14 + 3(–2)

i.e., 8, 12, 16, 20

4. Let ‘a’ be the first term and ‘d be the common difference.

*n*th term of AP is $a_n = a + (n-1)d$

and sum of AP is $S_n = \frac{n}{2}[2a + (n-1)d]$

Sum of first 10 terms = $210 = \frac{10}{2}[2a + 9d]$

$\Rightarrow 42 = 2a + 9d \quad \Rightarrow \quad 2a + 9d = 42$ ...(i)

15^{th} term from the last = $(50 - 15 + 1)^{th} = 36^{th}$ term

$\Rightarrow a_{36} = a + 35d$

Sum of last 15 terms = $2565 = \frac{15}{2}[2a_{36} + (15-1)d]$

$\Rightarrow 2565 = \frac{15}{2}[2a_{36} + (15-1)d]$

$\Rightarrow 2565 = 15[a + 35d + 7d]$

$\Rightarrow a + 42d = 171$ ...(ii)

(i) –2 × (ii), we get

$9d - 84d = 42 - 342 \Rightarrow 75d = 300$

$\Rightarrow d = \frac{300}{75} = 4$

Putting the value of d in (ii)

$42 \times 4 + a = 171 \Rightarrow a = 171 - 168$

$\Rightarrow a = 3$

$\Rightarrow a_{50} = a + 49d = 3 + 49 \times 4 = 199$

So, the AP formed is 3, 7, 11, 15, ... and 199.

5. $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{30} = \frac{30}{2}[2a + 29d] \Rightarrow S_{30} = 30a + 435d$$

$$\Rightarrow S_{20} = \frac{20}{2}[2a + 19d] \Rightarrow S_{20} = 20a + 19d$$

$$S_{10} = \frac{10}{2}[2a + 9d] \Rightarrow S_{10} = 10a + 45d$$

$$3(S_{20} - S_{10}) = 3[20a + 190d - 10a - 45d]$$

$$= 3[10a + 145d] = 30a + 435d = S_{30}$$

[From (i)]

Hence, $S_{30} = 3(S_{20} - S10)$ Hence proved.

6. Let total time be n minutes

Total distance covered by thief = 100 n metres

Total distance covered by policeman = 100 + 110 + 120 + ... + $(n-1)$ terms

$\therefore 100m = \frac{n-1}{2}[100(2) + (n-2)10]$

$\Rightarrow 200n = (n-1)(180 + 10n)$

$\Rightarrow 102 - 30n - 180 = 0$

$\Rightarrow n^2 - 3n - 18 = 0$

$\Rightarrow (n-6)(n+3) = 0$

$\Rightarrow n = 6$

Policeman took $(n-1) = (6-1) = 5$ minutes to catch the thief.

7. The numbers of houses are 1, 2, 3, 4..........49.

The numbers of the houses are in AP, where $a = 1$ and $d = 1$

sum of n terms of an AP = $\frac{n}{2}[2a + (n-1)d]$

Let X^{th} number house be the required house.

sum of number of houses preceding X^{th} house is equal to sx-1 i.e.,

$$S_{x-1} = \frac{X-1}{2}[2a + (X-1-1)d]$$

$$\Rightarrow S_{x-1} = \frac{X-1}{2}[2 + (X-2)]$$

$$\Rightarrow S_{x-1} = \frac{X-1}{2}[2 + X - 2]$$

$$\Rightarrow S_{x-1} = \frac{X(X-1)}{2}$$

Sum of numbers of houses following X^{th} house is equal to $S_{49} - S_x$

$$= \frac{49}{2}[2a + (49-1)d] - \frac{X}{2}[2a + (X-1)d]$$

$$= \frac{49}{2}(2+48) - \frac{X}{2}(2+X-1) = \frac{49}{2}(50) - \frac{X}{2}(X+1)$$

$$= 2(49) - \frac{X}{2}(X+1)$$

Now, we are given that

Sum of number of houses before X is equal to sum of number of houses after X. i.e., $S_{x-1} = S_{49} - S_x$

$$\Rightarrow \frac{X(X-1)}{2} = 25(49) - X\frac{(X+1)}{2}$$

$$\Rightarrow \frac{X^2}{2} - \frac{X}{2} = 1225 - \frac{X^2}{2} - \frac{X}{2}$$

$\Rightarrow X^2 = 1225$ $\Rightarrow X = \sqrt{1225}$

$\Rightarrow X = \pm 35$

since number of houses is positive integer,

$\therefore X = 35$

8. Given,

$$\frac{a_{11}}{a_{18}} = \frac{a+10d}{a+17d} = \frac{2}{3}$$

[Using formula an = a + (n – 1)d]

$\Rightarrow 3a + 30d = 2a + 34d$

$\Rightarrow a = 4d$ ...(i)

$$\frac{S_5}{S_{10}} = \frac{\frac{5}{2}(2a+4d)}{5(2a+9d)}$$

[Using formula $S_n = \frac{n}{2}[2a + (n-1)d]$]

$$= \frac{8d + 4d}{2(8d + 9d)} \quad [\because a = 4d]$$

$$= \frac{12d}{34d} = \frac{6}{17}$$

Hence, $S_5 : S_{10} = 6 : 17$.

9. The first 15 multiples of 8 are 8, 16, 24, ... 120

Clearly, these numbers are in AP with first term a = 8 and common difference, d = 16 – 8 = 8

Thus, $S_{15} = \frac{15}{2}[2 \times 8 + (15-1) \times 8]$

$$= \frac{15}{2}[16 + 14 \times 8] = \frac{15}{2}[16 + 112]$$

$$= \frac{15}{2} \times 128 = 15 \times 64 = 960$$

10. Two-digit natural numbers which when divided by 3 yield 1 as remainder are:

10, 13, 16, 19, ..., 97, which forms an AP.

with a = 10, d = 3, a_n = 97

$a_n = 97 = a + (n - 1) d = 97$

or $10 + (n - 1)3 = 97$

⇒ $(n - 1) = \frac{87}{3} = 29$

⇒ n = 30

Now, s_{30} = [2 × 10 + 29 × 3) = 15(20 + 87)

= 15 × 107 = 1605

Case Study Answers:

1. Answer:

Here the smallest 3-digit number divisible by 7 is 105. So, the number of bacteria taken into consideration is 105, 112, 119, ... 994 So, first term (a) = 105, d = 7 and last term = 994.

i. (c) 133

Solution:

$t_5 = a + 4d = 105 + 28 = 133$

ii. (b) 128

Solution:

Let n samples be taken under consideration

∵ Last term = 994

⇒ a + (n – d)d = 994

⇒ 105 + (n – 1)7 = 994

⇒ n = 128

iii. (a) 1365

Solution:

Total number of bacteria in first 10 samples

$$= S_{10} = \frac{10}{2}[2(105) + 9(7)]1365$$

iv. (a) 952

Solution:

t_7 from end = (128 - 7 + 1) term from beginning = 122th term = 105 + 121(7) = 952

v. (c) 448

Solution:

t_{50} = 105 + 49 × 7 = 448

2. Answer:

Geeta's A.P. is -5, -2, 1, 4, ... Here, first term (a_1) = -5 and conunon difference (d_1) = -2 + 5 = 3 Similarly, Madhuri's A.P. is 187, 184, 181, ... Here first term (a_2) = 187 and common difference (d_2) = 184 - 187 = -3

i. (b) 88

Solution:

$t_{34} = a_2 + 33d_2 = 187 + 33(-3) = 88$

ii. (d) 0

Solution:

Required sum = 3 + (-3) = 0

iii. (a) 49

Solution:

$t_{19} = a_1 + 18d_1 = (-5) + 18(3) = 49$

iv. (a) 85

Solution:

$$S_{10} = \frac{n}{2}[2a_1 + (n-1)d_1] = \frac{10}{2}[2(-5) + 9(3)] = 85$$

v. (b) 33

Solution:

Let n^{th} terms of the two A.P s be equal.

∵ $-5 + (n - 1)3 = 187 + (n - 1)(-3)$

⇒ $6(n - 1) = 192$

⇒ n = 33

Assertion Reason Answers:

1. (a) Both A and R are true and R is the correct explanation for A.

2. (b) A is false but R is true.

Triangles 6

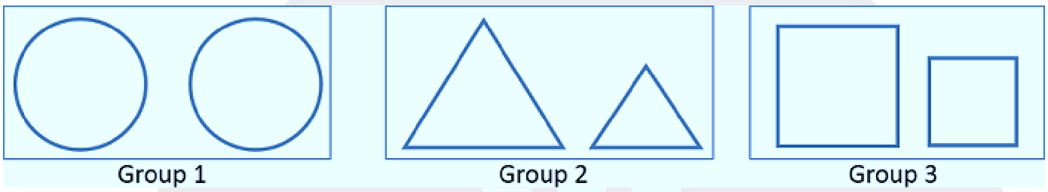
1. **Congruent Figures:**

Two geometrical figures are called **congruent** if they superpose exactly on each other, that is, they are of the same shape and size.

2. **Similar Figures:**

Two figures are **similar**, if they are of the same shape but not necessarily of the same size.

3. All congruent figures are similar but the similar figures need not to be congruent.



4. Two **polygons** having the same number of sides are **similar** if
   - i. their corresponding angles are equal and
   - ii. their corresponding sides are in the same ratio (or proportion).

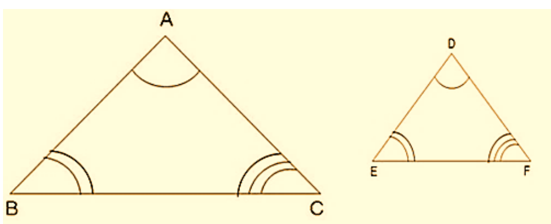
   Note: Same ratio of the corresponding sides means the **scale factor** for the polygons.

5. Important facts related to similar figures are:
   - i. All circles are similar.
   - ii. All squares are similar.
   - iii. All equilateral triangles are similar.
   - iv. The ratio of any two corresponding sides in two equiangular triangles is always same.

6. Two **triangles are similar** (~) if
   - i. Their corresponding angles are equal.
   - ii. Their corresponding sides are in same ratio.

7. If the angles in two triangles are:
   - i. Different, the triangles are neither similar nor congruent.
   - ii. Same, the triangles are similar.
   - iii. Same and the corresponding sides are of the same size, the triangles are congruent.

In the given figure, $A \leftrightarrow D$, $B \leftrightarrow E$ and $C \leftrightarrow F$, which means triangles ABC and DEF are similar which is represented by $\Delta\, ABC \sim \Delta\, DEF$



8. If $\Delta\ ABC \sim \Delta\ PQR$, then

i. $\angle A = \angle P$

ii. $\angle B = \angle Q$

iii. $\angle C = \angle R$

iv. $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

9. Equiangular Triangles:

Two triangles are **equiangular** if their corresponding angles are equal. The ratio of any two corresponding sides in such triangles is always the same.

10. Basic Proportionality Theorem (Thales Theorem):

If a line is drawn parallel to one side of a triangle to intersect other two sides in distinct points, the other two sides are divided in the same ratio.

11. Converse of BPT:

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

12. A line drawn through the mid-point of one side of a triangle which is parallel to another side bisects the third side. In other words, the line joining the mid-points of any two sides of a triangle is parallel to the third side.

13. AAA (Angle-Angle-Angle) similarity criterion:

If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.

14. AA (Angle-Angle) similarity criterion:

If two angles of a triangle are respectively equal to two angles of another triangle, then by the angle sum property of a triangle their third angles will also be equal.

Thus, **AAA similarity criterion** changes to **AA similarity criterion** which can be stated as follows:

If two angles of one triangle are respectively equal to two angles of other triangle, then the two triangles are similar.

15. Converse of *AAA* similarity criterion:

If two triangles are similar, then their corresponding angles are equal.

16. SSS (Side-Side-Side) similarity criterion:

If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

17. Converse of SSS similarity criterion:

If two triangles are similar, then their corresponding sides are in constant proportion.

18. SAS (Side-Angle-Side) similarity criterion:

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

19. Converse of SAS similarity criterion:

If two triangles are similar, then one of the angles of one triangle is equal to the corresponding angle of the other triangle and the sides including these angles are in constant proportion.

20. **RHS (Right angle-Hypotenuse-Side) criterion:**

If in two right triangles, hypotenuse and one side of one triangle are proportional to the hypotenuse and one side of another triangle, then the two triangles are similar. This criteria is referred as the RHS similarity criterion

21. **Pythagoras Theorem:**

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Thus, in triangle *ABC* right angled at *B*, $AB^2 + BC^2 = AC^2$

22. **Converse of Pythagoras Theorem:**

If in a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

23. The **ratio of the areas of two similar triangles** is equal to the square of the ratio of their corresponding sides.

$$\text{Thus, if } \Delta\, ABC \sim \Delta\, PQR, \text{ then} \frac{ar\ \Delta\, ABC}{ar\ \Delta\, PQR} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

Also, the ration of the areas of two similar triangles is equal to the ration of the squares of the corresponding medians.

24. Some important results of similarity are:

In an equilateral or an isosceles triangle, the altitude divides the base into two equal parts.

If a perpendicular is drawn from the vertex of the right triangle to the hypotenuse, then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

The area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

In an equilateral triangle, three times the square of one side is equal to four times the square of one of its altitudes.

25. **Triangle**

A triangle can be defined as a polygon which has three angles and three sides. The interior angles of a triangle sum up to 180 degrees and the exterior angles sum up to 360 degrees. Depending upon the angle and its length, a triangle can be categorized in the following types-

- **Scalene Triangle** – All the three sides of the triangle are of different measure
- **Isosceles Triangle** – Any two sides of the triangle are of equal length
- **Equilateral Triangle** – All the three sides of a triangle are equal and each angle measures 60 degrees
- **Acute angled Triangle** – All the angles are smaller than 90 degrees
- **Right angle Triangle** – Anyone of the three angles is equal to 90 degrees
- **Obtuse-angled Triangle** – One of the angles is greater than 90 degrees

26. **Similarity Criteria of Triangles**

To find whether the given two triangles are similar or not, it has four criteria. They are:

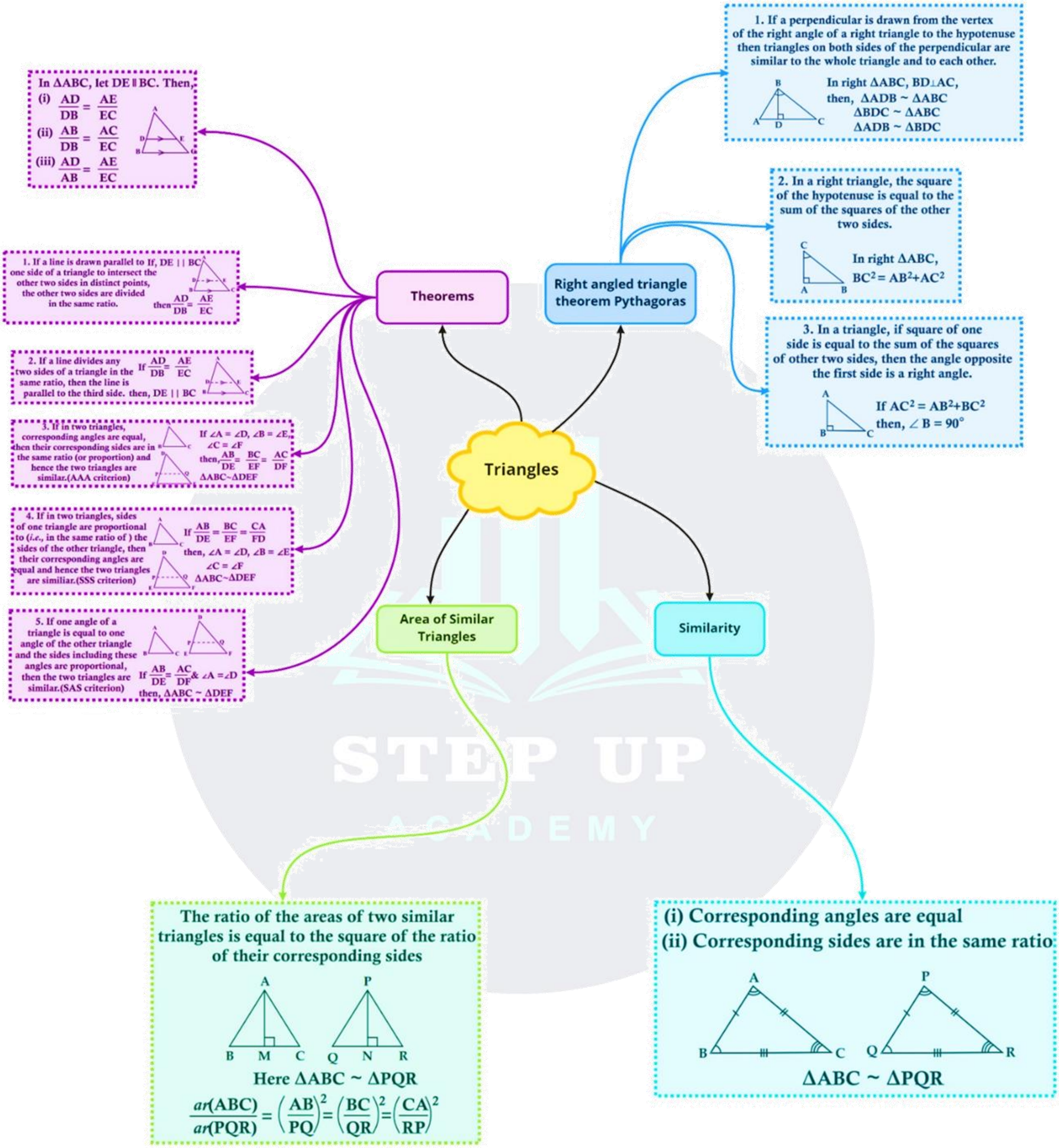
Side-Side- Side (SSS) Similarity Criterion – When the corresponding sides of any two triangles are in the same ratio, then their corresponding angles will be equal and the triangle will be considered as similar triangles.

Angle Angle Angle (AAA) Similarity Criterion – When the corresponding angles of any two triangles are equal, then their corresponding side will be in the same ratio and the triangles are considered to be similar.

Angle-Angle (AA) Similarity Criterion – When two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are considered as similar.

Side-Angle-Side (SAS) Similarity Criterion – When one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are said to be similar.

Class : 10th mathematics
Chapter- 6 : Triangles



Important Questions

Multiple Choice Questions:

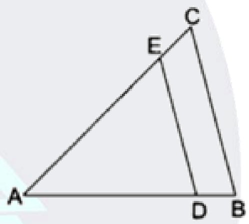
1. If in triangles ABC and DEF, $\frac{AB}{EF} = \frac{AC}{DE}$, then they will be similar when
   (a) ∠A = ∠D
   (b) ∠A = ∠E
   (c) ∠B = ∠E
   (d) ∠C = ∠F
2. A square and a rhombus are always
   (a) similar
   (b) congruent
   (c) similar but not congruent
   (d) neither similar nor congruent
3. Which geometric figures are always similar?
   (a) Circles
   (b) Circles and all regular polygons
   (c) Circles and triangles
   (d) Regular
4. ΔABC ~ ΔPQR, ∠B = 50° and ∠C = 70° then ∠P is equal to
   (a) 50°
   (b) 60°
   (c) 40°
   (d) 70°
5. In triangle DEF, GH is a line parallel to EF cutting DE in G and and DF in H. If DE = 16.5, DH = 5, HF = 6 then GE = ?
   (a) 9
   (b) 10
   (c) 7.5
   (d) 8
6. In a rectangle Length = 8 cm, Breadth = 6 cm. Then its diagonal = ...
   (a) 9 cm
   (b) 14 cm
   (c) 10 cm
   (d) 12 cm

Very Short Questions:

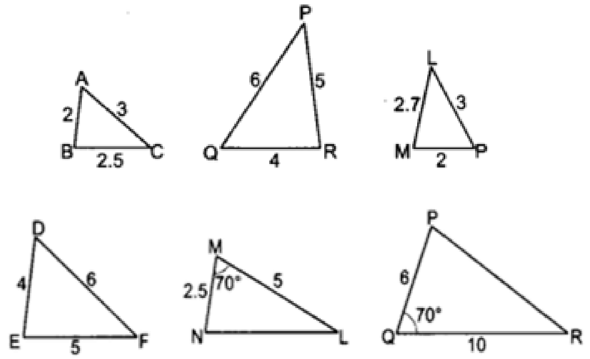
1. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why?
2. A and B are respectively the points on the sides PQ and PR of a ΔPQR such that PQ = 12.5 cm, PA = 5 cm, BR = 6 cm, and PB = 4 cm. Is AB || QR? Give reason.
3. Is the triangle with sides 12 cm, 16 cm and 18 cm a right triangle? Give reason.
4. In triangles PQR and TSM, ∠P = 55°, ∠Q = 25°, ∠M = 100°, and ∠S = 25°. Is ΔQPR ~ ΔTSM? Why?
5. If ABC and DEF are similar triangles such that ∠A = 47° and ∠E = 63°, then the measures of ∠C = 70°. Is it true? Give reason.

Short Questions:

1. In Fig. 7.10, DE || BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, find the value of x.



2. E and F are points on the sides PQ and PR respectively of a ΔPQR. Show that EF ||QR if PQ = 1.28 cm, PR= 2.56 cm, PE = 0.18 cm and PF = 0.36 cm.
3. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.
4. In Fig. 7.13, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$
5. In Fig. 7.14, DE || OQ and DF || OR Show that EF || QR.
6. Using converse of Basic Proportionality Theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side.
7. State which pairs of triangles in the following figures are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form.



8. In Fig. 7.17, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and AB = 5cm. Find the value of DC.
9. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB.
10. S and T are points on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

Long Questions:

1. Using Basic Proportionality Theorem, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.
2. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.
3. If AD and PM are medians of triangles ABC and PQR respectively, where ΔABC ~ ΔPQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$
4. In Fig. 7.37, ABCD is a trapezium with AB || DC. If ΔAED is similar to ΔBEC, prove that AD = BC.

Case Study Questions:

1. Rahul is studying in X Standard. He is making a kite to fly it on a Sunday. Few questions came to his mind while making the kite. Give answers to his questions by looking at the figure.

i. Rahul tied the sticks at what angles to each other?
   a. 30º
   b. 60º
   c. 90º
   d. 60º

ii. Which is the correct similarity criteria applicable for smaller triangles at the upper part of this kite?
   a. RHS
   b. SAS
   c. SSA
   d. AAS

iii. Sides of two similar triangles are in the ratio 4:9. Corresponding medians of these triangles are in the ratio:
   a. 2 : 3
   b. 4 : 9
   c. 81 : 16
   d. 16 : 81

iv. What is the area of the kite, formed by two perpendicular sticks of length 6cm and 8cm?
   a. 48 cm^2
   b. 14 cm^2
   c. 24 cm^2
   d. 96 cm^2

Assertion Reason Questions:

1. **Directions:** In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
c. Assertion (A) is true but reason (R) is false.
d. Assertion (A) is false but reason (R) is true.

Assertion: If ΔABC and ΔPQR are congruent triangles, then they are also similar triangles.

Reason: All congruent triangles are similar but the similar triangles need not be congruent.

Answer Key

Multiple Choice Questions:

1. (b) ∠A = ∠E
2. (d) neither similar nor congruent
3. (b) Circles and all regular polygons
4. (b) 60°
5. (a) 9
6. (c) 10cm

Very Short Answers:

1. Since the perimeters and two sides are proportional

∴ The third side is proportional to the corresponding third side.

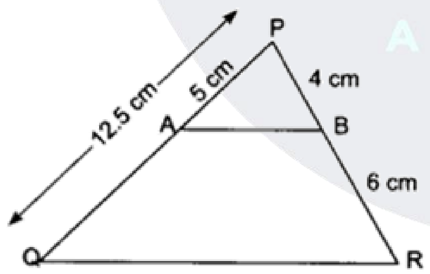
i.e., The two triangles will be similar by SSS criterion.

2. Yes, $\frac{PA}{AQ} = \frac{5}{12.5-5} = \frac{5}{7.5} = \frac{2}{3}$

$\frac{PB}{BR} = \frac{4}{6} = \frac{2}{3}$

Since $\frac{PA}{AQ} = \frac{PB}{BR} = \frac{2}{3}$

∴ AB || QR



3. Here, $12^2 + 16^2 = 144 + 256 = 400 \neq 18^2$

∴ The given triangle is not a right triangle.

4. Since, ∠R = 180° – (∠P + ∠Q)

= 180° – (55° + 25°) = 100° = ∠M

∠Q = ∠S = 25° (Given)

ΔQPR ~ ΔSTM

i.e., . ΔQPR is not similar to ΔTSM.

5. Since ΔABC ~ ΔDEF

∴ ∠A = ∠D = 47°

∠B = ∠E = 63°

∴ ∠C = 180° – (∠A + ∠B) = 180° – (47° + 63°) = 70°

∴ Given statement is true.

Short Answers:

1. In ΔABC, we have

DE || BC,

∴ $\frac{AD}{DB} = \frac{AE}{EC}$

[By Basic Proportionality Theorem]

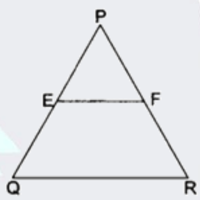
$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

⇒ x(x – 1) = (x – 2) (x + 2)

⇒ $x^2 - x = x^2 - 4$

⇒ x = 4

2.



We have, PQ = 1.28 cm, PR = 2.56 cm

PE = 0.18 cm, PF = 0.36 cm

Now, EQ = PQ – PE = 1.28 – 0.18 = 1.10 cm and

FR = PR – PF = 2.56 – 0.36 = 2.20 cm

Now, $\frac{PF}{EQ} = \frac{0.18}{1.18} = \frac{18}{110} = \frac{9}{55}$

And, $\frac{PF}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55}$ ∴ $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF || QR [By the converse of Basic Proportionality Theorem]

3. Let AB be a vertical pole of length 6m and BC be its shadow and DE be tower and EF be its shadow. Join AC and DF.

Now, in ΔABC and ΔDEF, we have

∠B = ∠E = 90°

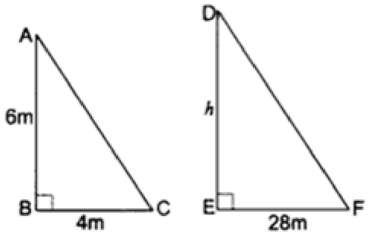
∠C = ∠F (Angle of elevation of the Sun)

∵ ΔABC ~ ΔDEF (By AA criterion of similarity)

Thus, $\frac{AB}{DE} = \frac{BC}{EF}$

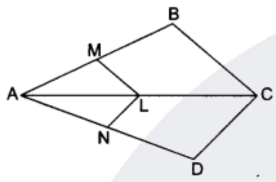
$\Rightarrow \quad \frac{6}{h} = \frac{4}{28}$ (Let DE = h)

$\Rightarrow \quad \frac{6}{h} = \frac{1}{7} \quad \Rightarrow h = 42$



h = 42 Hence, height of tower, DE = 42m

4.



Firstly, in ΔABC, we have

LM || CB (Given)

Therefore, by Basic Proportionality Theorem, we have

$$\frac{AM}{AB} = \frac{AL}{AC} \quad ...(i)$$

Again, in ΔACD, we have

LN || CD (Given)

∴ By Basic Proportionality Theorem, we have

$$\frac{AN}{AD} = \frac{AL}{AC} \quad ...(ii)$$

Now, from (i) and (ii), we have $\frac{AM}{AB} = \frac{AN}{AD}$.

5. In ΔPOQ, we have DE || OQ (Given)

∴ By Basic Proportionality Theorem,

we have $\frac{PE}{EQ} = \frac{PD}{DO}$ ...(i)

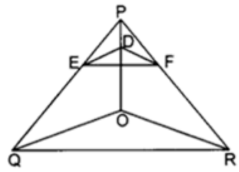
Similarly, in ΔPOR, we have

DF || OR (Given)

$$\frac{PD}{DO} = \frac{PF}{FR} \quad ...(ii)$$

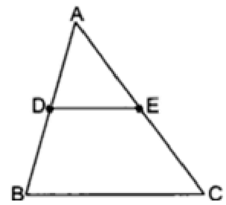
Now, from (i) and (ii), we have

$\frac{PE}{EQ} = \frac{PF}{FR} \quad \Rightarrow \quad$ EF || QR



[Applying the converse of Basic Proportionality Theorem in ΔPQR]

6.



Given: ΔABC in which D and E are the mid-points of sides AB and AC respectively.

To prove: DE || BC

Proof: Since D and E are the mid-points of AB and AC respectively

∴ AD = DB and AE = EC

$\Rightarrow \quad \frac{AD}{DB} = 1 \quad$ and $\quad \frac{AE}{EC} = 1$

$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC}$

DB EC Therefore, DE || BC (By the converse of Basic Proportionality Theorem)

7. (i) In ΔABC and ΔQRP, we have

$$\frac{BC}{RP} = \frac{2.5}{5} = \frac{25}{50} = \frac{1}{2}$$

Hence, $\frac{AB}{QR} = \frac{AC}{QP} = \frac{BC}{RP}$

∴ ΔABC ~ ΔQRP, by SSS criterion of similarity.

(ii) In ΔLMP and ΔFED, we have

$$\frac{LP}{FD} = \frac{3}{6} = \frac{1}{2}, \quad \frac{MP}{ED} = \frac{2}{4} = \frac{1}{2}, \quad \frac{LM}{FE} = \frac{2.7}{5}$$

Hence, $\frac{LP}{FD} = \frac{MP}{ED} \neq \frac{LM}{FE}$

∴ ΔLMP is not similar to ΔFED.

(iii) In ΔNML and ΔPQR, we have

$\angle M = \angle Q = 70^o$

Now, $\frac{MN}{QP} = \frac{2.5}{6} = \frac{5}{12}$ and $\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$

Hence, $\frac{MN}{QP} \neq \frac{ML}{QR}$

ΔNML is not similar to ΔPQR.

8. In ΔAOB and ΔCOD, we have

∠AOB = ∠COD [Vertically opposite angles]

$\Rightarrow \frac{AO}{OC} = \frac{BO}{OD}$ [Given]

So, by SAS criterion of similarity, we have

ΔAOB ~ ΔCOD

$\Rightarrow \frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC}$

$\Rightarrow \frac{1}{2} = \frac{5}{DC}$ [∵ AB = 5 cm]

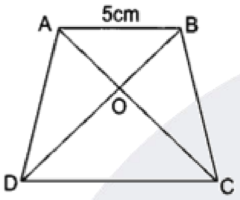
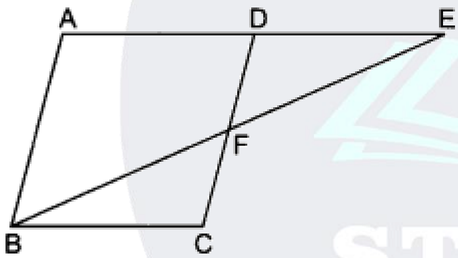


Fig. 7.17

⇒ DC = 10 cm.

9.



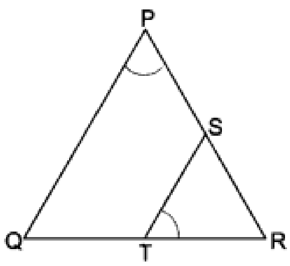
In ΔABE and ΔCFB, we have

∠AEB = ∠CBF (Alternate angles)

∠A = ∠C (Opposite angles of a parallelogram)

∴ ΔABE ~ ΔCFB (By AA criterion of similarity)

10.



In ΔRPQ and ΔRTS, we have

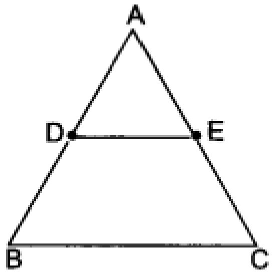
∠RPQ = ∠RTS (Given)

∠PRQ = ∠TRS = ∠R (Common)

∴ ΔRPQ ~ ΔRTS (By AA criterion of similarity)

Long Answers:

1.



Given: A ΔABC in which D is the mid-point of AB and DE is drawn parallel to BC, which meets AC at E.

To prove: AE = EC

Proof: In ΔABC, DE || BC

∴ By Basic Proportionality Theorem, we have

$\frac{AD}{DB} = \frac{AAND}{ANDC}$ ... (i)

Now, since D is the mid-point of AB

⇒ AD = BD ... (ii)

From (i) and (ii), we have

$\frac{AD}{DB} = \frac{AAND}{ANDC}$

$\Rightarrow 1 = \frac{AAND}{ANDC}$

Hence, E is the mid-point of AC.

2. **Given:** ABCD is a trapezium, in which AB || DC and its diagonals intersect each other at point O.

To prove: $\frac{AO}{BO} = \frac{CO}{DO}$

Construction: Through O, draw OE || AB i.e., OE || DC.

Proof: In ΔADC, we have OE || DC (Construction)

∴ By Basic Proportionality Theorem, we have

$$\frac{AE}{ED} = \frac{AO}{CO} \quad ...(i)$$

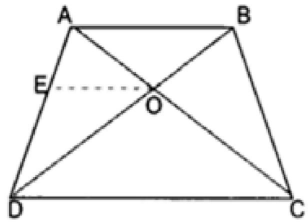
Now, in ΔABD, we have OE || AB (Construction)

∴ By Basic Proportionality Theorem, we have

$$\frac{ED}{AE} = \frac{DO}{BO} \Rightarrow \frac{AE}{ED} = \frac{BO}{DO} \quad ...(ii)$$

From (i) and (ii), we have

$$\frac{AO}{CO} = \frac{BO}{DO} \Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$$



3. In ΔABD and ΔPQM we have

$\angle B = \angle Q$ ($\because \Delta ABC \sim \Delta PQR$) ... (i)

$$\frac{AB}{PQ} = \frac{BC}{QR} \qquad (\because \Delta ABC \sim \Delta PQR)$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} \Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \qquad ...(ii)$$

[Since AD and PM are the medians of ΔABC and ΔPQR respectively]

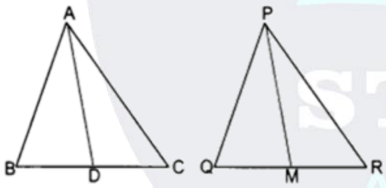
From (i) and (ii), it is proved that

$\Delta ABD \sim \Delta PQM$

(By SAS criterion of similarity)

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$$



4. In ΔEDC and ΔEBA we have

$\angle 1 = \angle 2$ [Alternate angles]

$\angle 3 = \angle 4$ [Alternate angles]

$\angle CED = \angle AEB$ [Vertically opposite angles]

$\therefore$ $\Delta EDC \sim \Delta EBA$ [By AA criterion of similarity]

$$\Rightarrow \frac{ED}{EB} = \frac{EC}{EA} \quad \Rightarrow \quad \frac{ED}{EC} = \frac{EB}{EA} \qquad ...(i)$$

It is given that $\Delta AED \sim \Delta BEC$

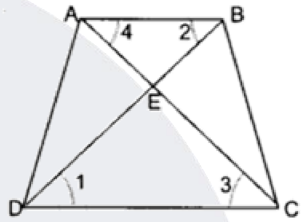
$$\therefore \frac{ED}{EC} = \frac{EA}{EB} = \frac{AD}{BC} \qquad ...(ii)$$

From (i) and (ii), we get

$$\frac{EB}{EA} = \frac{EA}{EB} \Rightarrow (EB)^2 = (EA)^2 \Rightarrow EB = EA$$

Substituting EB = EA in (ii), we get

$$\frac{EA}{EA} = \frac{AD}{BC} \Rightarrow \frac{AD}{BC} = 1 \Rightarrow AD = BC$$



Case Study Answers:

1. **Answer:**

i.	c	90º
ii.	b	SAS
iii.	b	4 : 9
iv.	a	48 cm²

Assertion Reason Answers:

1. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Coordinate Geometry 7

1. **Coordinate axes:**

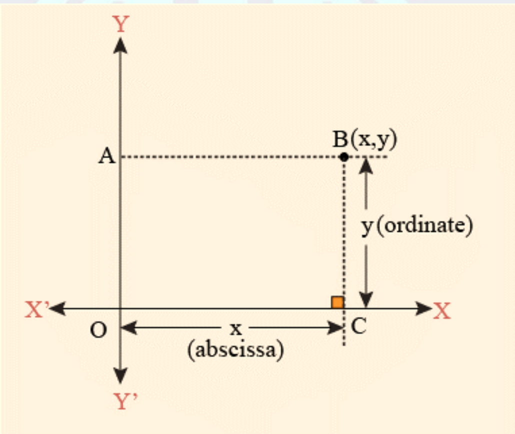
Two perpendicular number lines intersecting at point zero are called **coordinate axes**. The point of intersection is called **origin** and denoted by '*O*'. The horizontal number line is the ***x*-axis** (denoted by *X'OX*) and the vertical one is the ***y*-axis** (denoted by *Y'OY*).

2. **Cartesian plane** is a plane formed by the coordinate axes perpendicular to each other in the plane. It is also called as *xy* plane.

The axes divide the Cartesian plane into four parts called the **quadrants** (one fourth part), numbered I, II, III and IV anticlockwise from *OX*.

Points on a Cartesian Plane

A pair of numbers locate points on a plane called the coordinates. The distance of a point from the y-axis is known as abscissa or x-coordinate. The distance of a point from the x-axis is called ordinates or y-coordinate.



Representation of (x,y) on the cartesian plane

3. **Coordinates of a point:**

- The x-coordinate of a point is its perpendicular distance from *y*-axis, called **abscissa.**
- The y-coordinate of a point is its perpendicular distance from *x*-axis, called **ordinate**

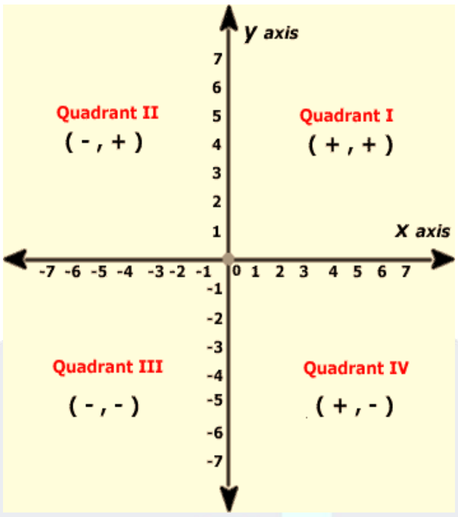
  If the abscissa of a point is *x* and the ordinate of the point is *y*, then (*x, y*) is called the **coordinates** of the point.
- The point where the *x*-axis and the *y*-axis intersect is represented by the coordinate point (0, 0) and is called the **origin**.

4. **Sign of the coordinates in the quadrants:**

Sign of coordinates depicts the quadrant in which it lies.

- The point having both the coordinates positive i.e. of the form (+, +) will lie in the first quadrant.

- The point having x-coordinate negative and y-coordinate positive i.e. of the form (-, +) will lie in the second quadrant.
- The point having both the coordinates negative i.e. of the form (-, -) will lie in the third quadrant.
- The point having x-coordinate positive and y-coordinate negative i.e. of the form (+,-) will lie in the fourth quadrant.



5. Coordinates of a point on the x-axis or y-axis:

The coordinates of a point lying on the *x*-axis are of the form (*x*, 0) and that of the point on the *y*-axis are of the form (0, *y*).

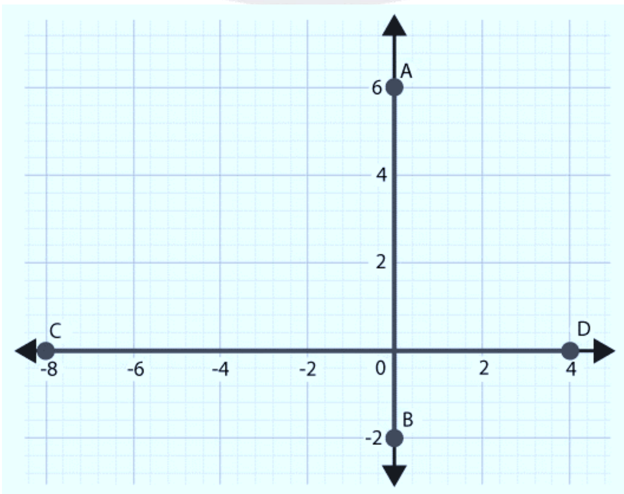
6. **Distance formula**

The distance formula is used to find the distance between two any points say $P(x_1, y_1)$ and $Q(x_2, y_2)$ which is given by: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

- The distance of a point P(x, y) from the origin O(0, 0) is $OP\sqrt{x^2 + y^2}$
- The points *A, B* and *C* are **collinear** if *AB* + *BC* = *AC*.

Distance between Two Points on the Same Coordinate Axes

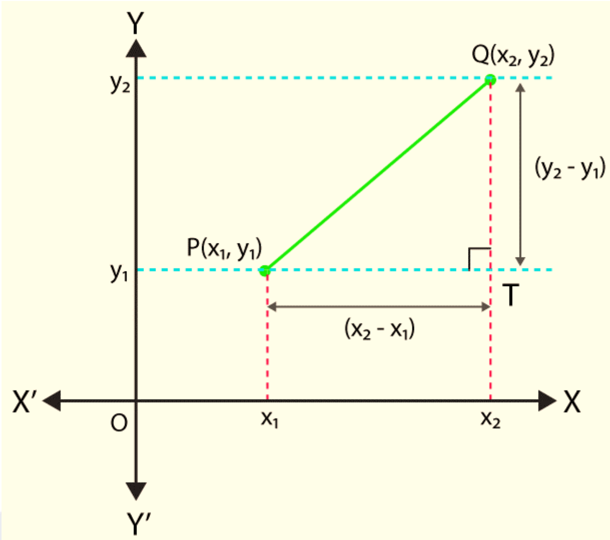
The distance between two points that are on the same axis (x-axis or y-axis), is given by the difference between their ordinates if they are on the y-axis, else by the difference between their abscissa if they are on the x-axis.



Distance AB = 6 – (-2) = 8 units

Distance CD = 4 – (-8) = 12 units

Distance between Two Points Using Pythagoras Theorem



Finding distance between 2 points using Pythagoras Theorem

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be any two points on the cartesian plane.

Draw lines parallel to the axes through P and Q to meet at T.

ΔPTQ is right-angled at T.

By Pythagoras Theorem,

$PQ^2 = PT^2 + QT^2$

$= (x_2 - x_1)^2 + (y_2 - y_1)^2$

$PQ = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

7. Determining the type of triangle using distance formula

i. Three points *A, B* and *C* are the vertices of an **equilateral triangle** if *AB = BC = CA*.

ii. The points *A, B* and *C* are the vertices of an **isosceles triangle** if *AB = BC or BC = CA or CA = AB*.

iii. Three points *A, B* and *C* are the vertices of *a* **right triangle** if the sum of the squares of any two sides is equal to the square of the third side.

8. Determining the type of quadrilateral using distance formula

For the given four points A, B, C and D, if:

i. $AB = CD, BC = DA; AC \neq BD \Rightarrow ABCD$ is aparallelogram.

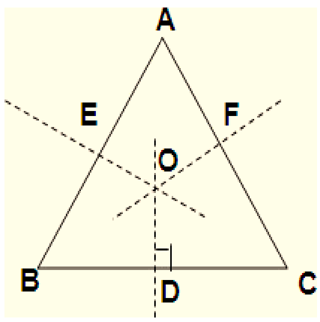
ii. $AB = BC = CD = DA; AC \neq BD \Rightarrow ABCD$ is a rhombus

iii. $AB = CD, BC = DA; AC = BD \Rightarrow ABCD$ is a rectangle

iv. $AB = BC = CD = DA; AC = BD \Rightarrow ABCD$ is a square.

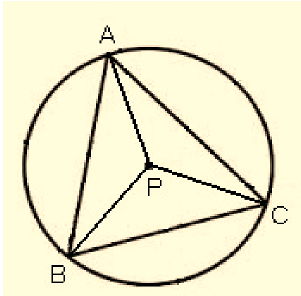
9. Circumcenter of a triangle

The point of intersection of the perpendicular bisectors of the sides of a triangle is called the **circumcentre**. In the figure, O is the circumcentre of the triangle ABC.



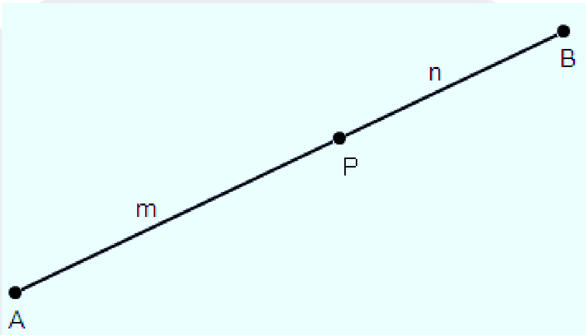
Circumcentre of a triangle is equidistant from the vertices of the triangle. That is, ***P* is the circumcentre of Δ *ABC*, if *PA = PB = PC*.**

Moreover, if a circle is drawn with *P* as centre and PA or PB or PC as radius, the circle will pass through all the three vertices of the triangle. *PA* (or *PB* or *PC*) is said to be the **circumradius** of the triangle.



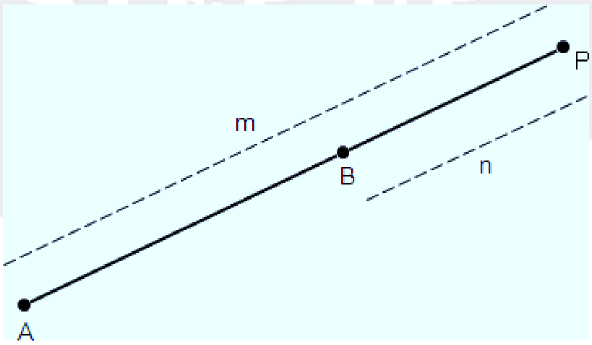
10. **Section formula**

If *P* is a point lying on the line segment joining the points *A* and *B* such that *AP: BP = m: n.* Then, we say that the **point *P* divides the line segment *AB* internally** in the ratio *m: n*.



Coordinates of a point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m: n$ internally are given by: $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+my_1}{m+n}\right)$ This is known as the **section formula.**

11. If *P* is a point lying on *AB* produced such that *AP: BP = m: n*, then point ***P* divides *AB* externally** in the ratio *m: n*.



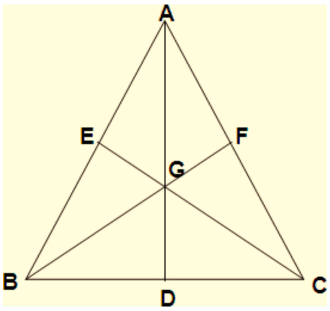
If *P* divides the line segment joining the points A (x_1, y_1) and *B* (x_2, y_2) in the ratio $m: n$ externally, then the coordinates of point *P* are given by $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-my_1}{m-n}\right)$

12. **Coordinates of Mid-point**

Mid-point divides the line segment in the ratio 1:1. Coordinates of the mid-point of a line segment joining the points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

13. **Centroid of a triangle**

The point of intersection of the three medians of a triangle is called the centroid.



In the figure, G is the centroid of the triangle ABC where AD, BF and CE are the medians through A, B and C respectively.

Centroid divides the median in the ratio of 2:1.

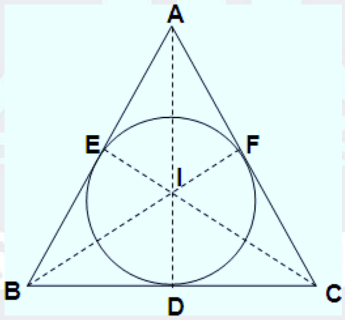
14. Coordinates of the centroid

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle ABC, then the **coordinates of the centroid** are given by $G(x, y) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

15. Incentre of a triangle

The point of intersection of all the internal bisectors of the angles of a triangle is called the **incentre**.

It is also the centre of a circle which touches all the sides of a triangle (such type of a circle is named as the incircle).



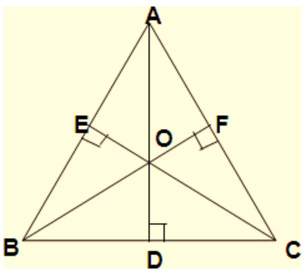
In the figure, *I* is the incentre of the triangle ABC.

16. Coordinates of incentre

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle, then the **coordinates of incentre** are given by $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$

17. Orthocentre of a triangle

The point of intersection of all the perpendiculars drawn from the vertices on the opposite sides (called altitudes) of a triangle is called the **Orthocentre** which can be obtained by solving the equations of any two of the altitudes.



In the figure, O is the orthocentre of the triangle ABC.

18. If the triangle is equilateral, the centroid, the incentre, the orthocenter and the circumcentre coincides.

19. Orthocentre, centroid and circumcentre are always collinear, whereas the centroid divides the line joining the orthocentre and the circumcentre in the ratio of 2:1.

20. Area of a triangle

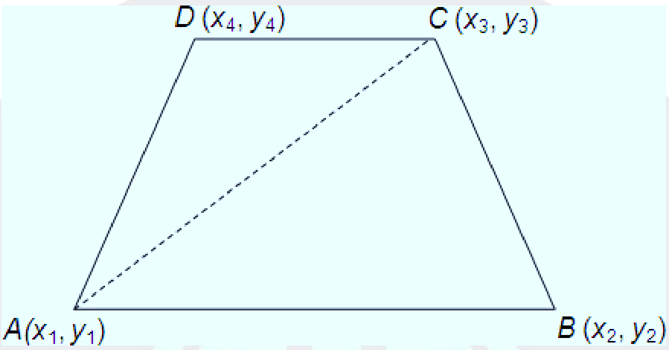
If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle, then the area of triangle ABC is given by $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

- Three given points are **collinear**, if the **area of triangle formed by these points is zero.**

21. Area of a quadrilateral

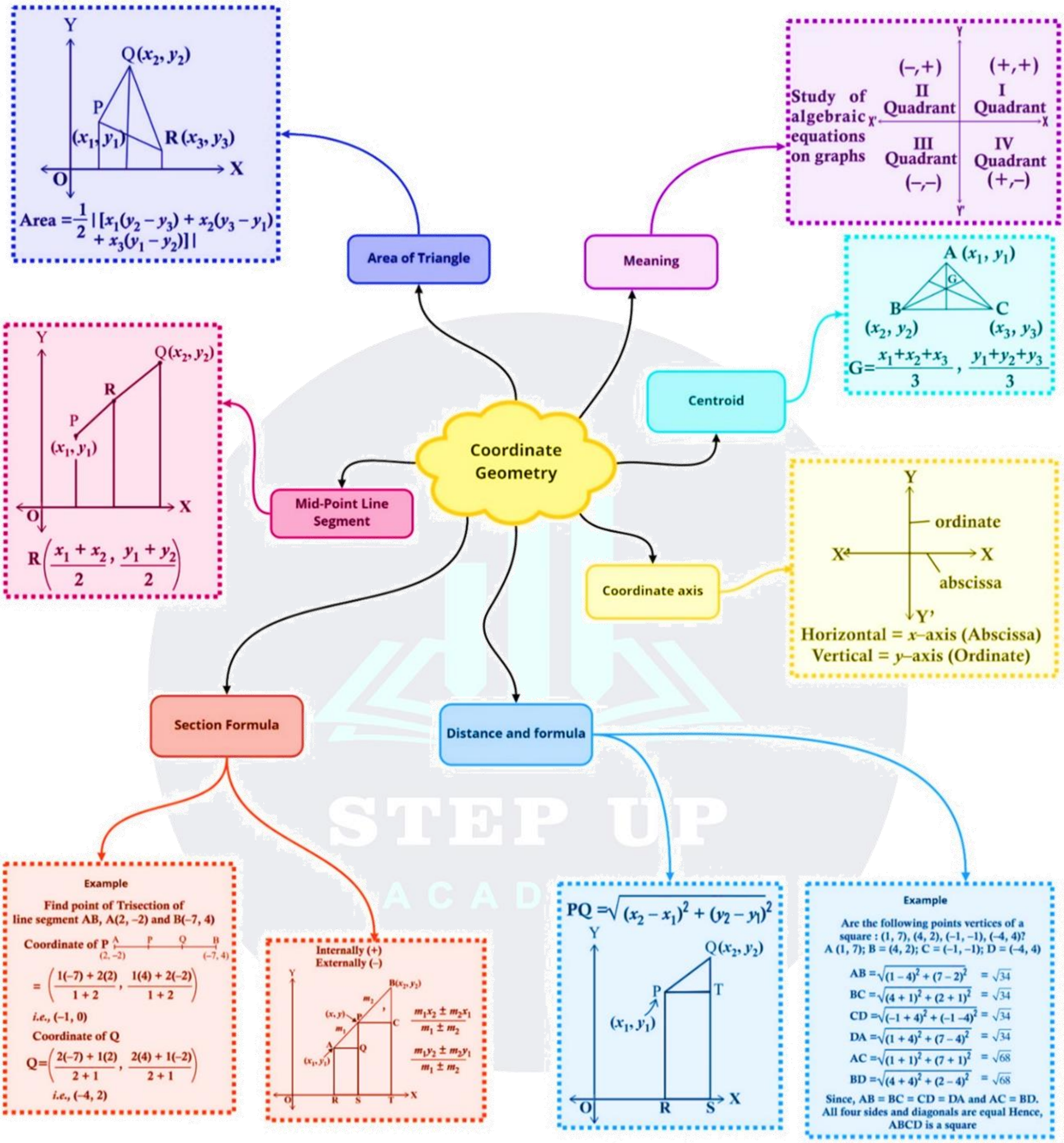
Area of a quadrilateral can be calculated by dividing in into two triangles:

Area of quadrilateral ABCD = Area of ΔABC+ Area of ΔACD



Note: To find the area of a polygon, divide it into triangular regions having no common area, then add the areas of these regions.

Class : 10th mathematics
Chapter- 7 : Coordinate Geometry



Important Questions

Multiple Choice Questions-

1. The ratio in which (4,5) divides the line segment joining the points (2,3) and (7,8) is
   (a) 2:3 (b) -3:2
   (c) 3:2 (d) -2:3
2. The values of x and y, if the distance of the point (x, y) from (-3,0) as well as from (3,0) is 4 are
   (a) $x = 1, y = 7$ (b) $x = 2, y = 7$
   (c) $x = 0, y = -\sqrt{7}$ (d) $x = 0, y = \pm\sqrt{7}$
3. The distance between the points (3,4) and (8, −6) is
   (a) $2\sqrt{5}$ units (b) $3\sqrt{5}$ units
   (c) $\sqrt{5}$ units (d) $5\sqrt{5}$ units
4. The ratio in which the x-axis divides the segment joining A(3,6) and B(12,-3) is
   (a) 1:2 (b) −2:1
   (c) 2:1 (d) −1: −1
5. The horizontal and vertical lines drawn to determine the position of a point in a Cartesian plane are called
   (a) Intersecting lines
   (b) Transversals
   (c) Perpendicular lines
   (d) X-axis and Y-axis
6. The midpoint of the line segment joining A(2a,4) and B(-2,3b) is M (1,2a + 1). The values of a and b are
   (a) 2,3 (b) 1,1
   (c) -2, -2 (d) 2,2
7. The points (1,1), (-2, 7) and (3, -3) are
   (a) vertices of an equilateral triangle
   (b) collinear
   (c) vertices of an isosceles triangle
   (d) none of these
8. The line $3x + y - 9 = 0$ divides the line joining the points (1, 3) and (2, 7) internally in the ratio
   (a) 3 : 4 (b) 3 : 2
   (c) 2 : 3 (d) 4 : 3
9. The ordinate of a point is twice its abscissa. If its distance from the point (4,3) is $\sqrt{10}$, then the coordinates of the point are
   (a) (1,2) or (3,6) (b) (1,2) or (3,5)
   (c) (2,1) or (3,6) (d) (2,1) or (6,3)
10. The mid-point of the line segment joining the points A (-2, 8) and B (-6, -4) is
    (a) (-4, -6)
    (b) (2, 6)
    (c) (-4, 2)
    (d) (4, 2)

Very Short Questions:

1. What is the area of the triangle formed by the points 0 (0, 0), A (-3, 0) and B (5, 0)?
2. If the centroid of triangle formed by points P (a, b), Q (b, c) and R (c, a) is at the origin, what is the value of a + b + c?
3. AOBC is a rectangle whose three vertices are A (0, 3), 0 (0, 0) and B (5, 0). Find the length of its diagonal.
4. Find the value of a, so that the point (3, a) lie on the line $2x - 3y = 5$.
5. Find distance between the points (0, 5) and (-5, 0).
6. Find the distance of the point (-6,8) from the origin.
7. If the distance between the points (4, k) and (1, 0) is 5, then what can be the possible values of k?
8. If the points A (1, 2), B (0, 0) and C (a, b) are collinear, then what is the relation between a and b?
9. Find the ratio in which the line segment joining the points (-3, 10) and (6, – 8) is divided by (-1, 6).
10. The coordinates of the points P and Q are respectively (4, -3) and (-1, 7). Find the abscissa of a point R on the line segment PQ such that $\frac{PR}{PQ} = \frac{3}{5}$.

Short Questions:

1. Write the coordinates of a point on x-axis which is equidistant from the points (-3, 4) and (2, 5).
2. Find the values of x for which the distance between the points P (2, -3) and Q (x, 5) is 10.
3. What is the distance between the points (10 cos 30°, 0) and (0, 10 cos 60°)?
4. If A(-1, 3), B(1, -1) and C (5, 1) are the vertices of a triangle ABC, what is the length of the median through vertex A?

5. Find the ratio in which the line segment joining the points P (3, -6) and Q (5,3) is divided by the x-axis.
6. Point P (5, -3) is one of the two points of trisection of the line segment joining the points A (7, -2) and B (1, -5). State true or false and justify your answer.
7. Show that ΔABC, where A(-2, 0), B(2, 0), C(0, 2) and APQR where P(-4, 0), Q(4, 0), R(0,4) are similar triangles.

OR

Show that ΔABC with vertices A(-2, 0), B(0, 2) and C(2, 0) is similar to ΔDEF with vertices D(-4, 0), F(4,0) and E(0, 4).

[ΔPQR is replaced by ΔDEF]

8. Point P (0, 2) is the point of intersection of y-axis and perpendicular bisector of line segment joining the points, A (-1, 1) and B (3, 3). State true or false and justify your answer.
9. Determine, if the points (1, 5), (2, 3) and (-2, −11) are collinear.
10. Find the distance between the following pairs of points:

(i) (-5, 7), (-1, 3)

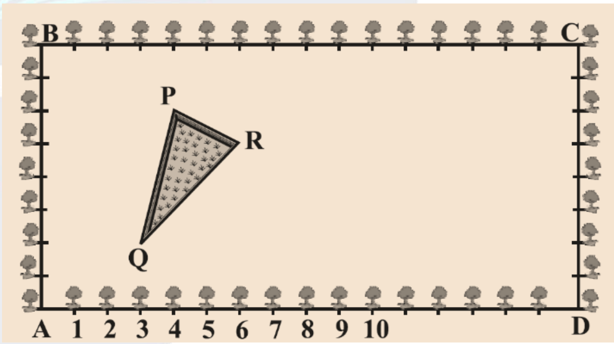
(ii) (a, b), (-a, -b)

Long Questions:

1. Find the value of 'k", for which the points are collinear: (7, -2), (5, 1), (3, k).
2. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.
3. Find the area of the quadrilateral whose vertices, taken in order, are (-4, -2), (-3, -5), (3, −2) and (2, 3).
4. A median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A (4, -6), B (3, -2) and C (5, 2).
5. Find the ratio in which the point P (x, 2), divides the line segment joining the points A (12, 5) and B (4, -3). Also find the value of x.
6. If A (4, 2), B (7, 6) and C (1, 4) are the vertices of a ΔABC and AD is its median, prove that the median AD divides into two triangles of equal areas.
7. If the point A (2, -4) is equidistant from P (3, 8) and Q (-10, y), find the values of y. Also find distance PQ.
8. The base BC of an equilateral triangle ABC lies on y-axis. The coordinates of point Care (0, -3). The origin is the mid-point of the base. Find the coordinates of the points A and B. Also find the coordinates of another point D such that BACD is a rhombus.
9. Prove that the area of a triangle with vertices (t, t-2), (t + 2, t + 2) and (t + 3, t) is independent of t.
10. The area of a triangle is 5 sq units. Two of its vertices are (2, 1) and (3, -2). If the third vertex is $\left(\frac{7}{2}, y\right)$, find the value of y.

Case Study Questions:

1. The Class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Sapling of Gulmohar is planted on the boundary of the plot at a distance of 1m from each other. There is a triangular grassy lawn inside the plot as shown in Fig. The students have to sow seeds of flowering plants on the remaining area of the plot.



i. Considering A as the origin, what are the coordinates of A?

a. (0, 1)

b. (1, 0)

c. (0, 0)

d. (-1, -1)

ii. What are the coordinates of P?

a. (4, 6)

b. (6, 4)

c. (4, 5)

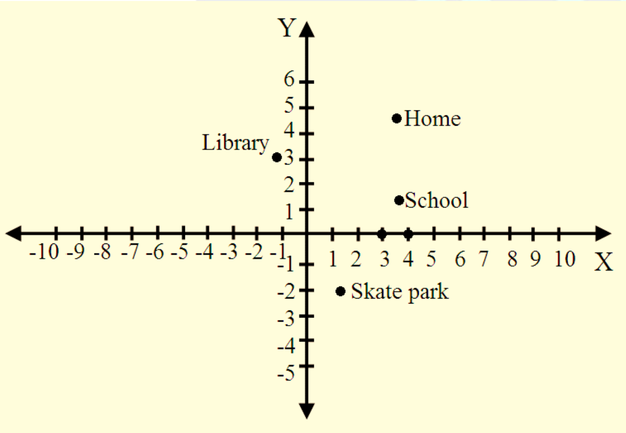
d. (5, 4)

iii. What are the coordinates of R?
 a. (6, 5)
 b. (5, 6)
 c. (6, 0)
 d. (7, 4)

iv. What are the coordinates of D?
 a. (16, 0)
 b. (0, 0)
 c. (0, 16)
 d. (16, 1)

v. What are the coordinates of P if D is taken as the origin?
 a. (12, 2)
 b. (-12, 6)
 c. (12, 3)
 d. (6, 10)

2. Two brothers Ramesh and Pulkit were at home and have to reach School. Ramesh went to Library first to return a book and then reaches School directly whereas Pulkit went to Skate Park first to meet his friend and then reaches School directly.



i. How far is School from their Home?
 a. 5m
 b. 3m
 c. 2m
 d. 4m

ii. What is the extra distance travelled by Ramesh in reaching his School?
 a. 4.48 metres
 b. 6.48 metres
 c. 7.48 metres
 d. 8.48 metres

iii. What is the extra distance travelled by Pulkit in reaching his School? (All distances are measured in metres as straight lines).
 a. 6.33 metres
 b. 7.33 metres
 c. 5.33 metres
 d. 4.33 metres

iv. The location of the library is:
 a. (-1, 3)
 b. (1, 3)
 c. (3, 1)
 d. (3, -1)

v. The location of the Home is:
 a. (4, 2)
 b. (1, 3)
 c. (4, 5)
 d. (5, 4)

Assertion Reason Questions-

1. Directions: Each of these questions contains two statements: Assertion [A] and Reason [R]. Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes [a], [b], [c] and [d] given below.
 a. A is true, R is true; R is a correct explanation for A.
 b. A is true, R is true; R is not a correct explanation for A.
 c. A is true; R is False.
 d. A is false; R is true.

2. Directions: Each of these questions contains two statements: Assertion [A] and Reason [R]. Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes [a], [b], [c] and [d] given below.
 a. A is true, R is true; R is a correct explanation for A.
 b. A is true, R is true; R is not a correct explanation for A.
 c. A is true; R is False.
 d. A is false; R is true.

Answer Key

Multiple Choice Questions:

1. (a) 2:3
2. (d) x = 0, y = ± √7
3. (d) 5√5 units
4. (c) 2:1
5. (d) X-axis and Y-axis
6. (d) 2,2
7. (b) collinear
8. (a) 3 : 4
9. (a) (1,2) or (3,6)
10. (c) (-4, 2)

Very Short Answers:

1. Area of ΔOAB

$= \frac{1}{2}[0(0-1) - 3(0-0) + 5(0-0)] = 0$

$\Rightarrow$ Given points are collinear

2.

Centroid of ΔPQR = $\left(\frac{a+b+c}{3}, \frac{b+c+a}{3}\right)$

Given $\left(\frac{a+b+c}{3}, \frac{b+c+a}{3}\right) = (0,0)$

$\Rightarrow$ a + b + c = 0

3. Length of diagonal

$AB = \sqrt{(5-0)^2 + (0-3)^2}$

$= \sqrt{25+9} = \sqrt{34}$

4. Since (3, a) lies on the line 2x – 3y = 5

Then 2(3) – 3(a) = 5

– 3a = 5 – 6

– 3a = −1

$\Rightarrow$ a = $\frac{1}{3}$

5. Here $x_1 = 0$, $y_1 = 5$, $x_2 = -5$ and $y_2 = 0$)

$\therefore \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-5-0)^2 + (0-5)^2}$

$= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$ *units*

6. Here $x_1 = -6$, $y_1 = 8$

$x_2 = 0$, $y_2 = 0$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left[0-(-6)^2\right] + (0-8)^2}$

$= \sqrt{(6)^2 + (-8)^2} = \sqrt{36+64}$

$= \sqrt{100} = 10$ *units*

7. Using distance formula

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ = distance given

$\sqrt{(4-1)^2 + (k-0)^2} = 5$

$9 + k^2 = 25 \quad \Rightarrow \quad k^2 = 16$

$\Rightarrow \quad k = \pm 4$

8. Points A, B and C are collinear

$\Rightarrow$ 1(0 – b) + 0 (b – 2) + a(2 – 0) = 0

$\Rightarrow$ -b + 2a = 0 or 2a = b

9. In Fig. 6.6, let the point P(−1, 6) divides the line joining A(−3, 10) and B (6, −8) in the ratio k : 1

then, the coordinates of P are $\left(\frac{6k-3}{k+1}, \frac{-8k+10}{k+1}\right)$

But, the coordinates of P are (−1, 6)

$\therefore \quad \frac{6k-3}{k+1} = -1 \quad \Rightarrow \quad 6k - 3 = -k - 1$

$\Rightarrow \quad 6k + k = 3 - 1 \quad \Rightarrow \quad 7k = 2$

$\Rightarrow \quad k = \frac{2}{7}$

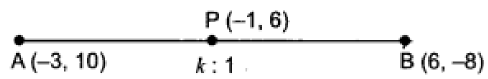


Fig. 6.6

Hence, the point P divides AB in the ratio 2 : 7.

10.

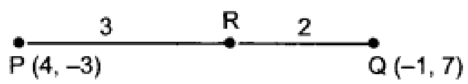


Fig. 6.7

$\frac{PQ}{PR} = \frac{5}{3} \quad \Rightarrow \quad \frac{PQ - PR}{PR} = \frac{5-3}{3}$

$$\Rightarrow \quad \frac{PQ}{PR} = \frac{2}{3}$$

i.e., R divides PQ in the ratio 3 : 2

Abscissa of R = $\frac{3\times(-1)+2\times4}{3+2} = \frac{-3+8}{5} = 1$

Short Answer:

1. Let the required point be (x, 0).

   Since, (x, 0) is equidistant from the points (-3, 4) and (2, 5).

$$\therefore \quad \sqrt{(-3-x)^2+(4-0)^2} = \sqrt{(2-x)^2+(5-0)^2}$$

$$\Rightarrow \quad \sqrt{9+x^2+6x+16} = \sqrt{4+x^2-4x+25}$$

$$\Rightarrow \quad x^2+6x+25 = x^2-4x+29$$

$$\Rightarrow \quad 10x = 4 \quad \text{or} \quad x = \frac{4}{10} = \frac{2}{5}$$

$$\therefore \quad \text{Required point is } \left(\frac{2}{5}, 0\right).$$

2. Distance between the given points

$$= \sqrt{(x-2)^2+(5+3)^2}$$

$$\Rightarrow \quad 10 = \sqrt{x^2+4-4x+64}$$

$$\Rightarrow \quad 100 = x^2-4x+68$$

$$\Rightarrow \quad x^2-4x-32 = 0$$

$$\Rightarrow \quad x^2-8x+4x-32 = 0$$

$$\Rightarrow \quad (x-8)(x+4) = 0 \quad \Rightarrow \quad x = 8, -4$$

3. Distance between the given points

$$= \sqrt{\left(0-10\cos 30^o\right)^2+\left(10\cos 60^o-0\right)^2}$$

$$= \sqrt{100\cos^2 30^o+100\cos^2 60^o}$$

$$= \sqrt{100\left[\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2\right]}$$

$$= \sqrt{100\left(\frac{3}{4}+\frac{1}{4}\right)} = \sqrt{100} = 10\,units$$

4. Coordinates of the mid-point of BC

$$= \left(\frac{1+5}{2}, \frac{-1+1}{2}\right) = (3, 0)$$

∴ Length of the median through A

$$= \sqrt{(3+1)^2+(0-3)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5\ units.$$

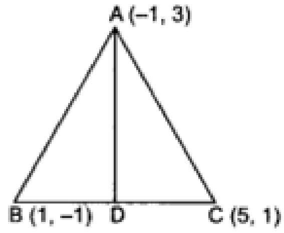


Fig. 6.8

5. Let the required ratio be λ : 1

   Then, the point of division is $\left(\frac{5\lambda+3}{\lambda+1}, \frac{3\lambda-6}{\lambda+1}\right)$

   Given that this point lies on the x-axis

$$\therefore \quad \frac{3\lambda-6}{\lambda+1} = 0 \quad \text{or} \quad 3\lambda = 6 \quad \text{or} \quad \lambda = 2$$

   Thus, the required ratio is 2 : 1.

6. Points of trisection of line segment AB are given by

$$= \left(\frac{2\times1+1\times7}{3}, \frac{2\times(-5)+1\times(-2)}{3}\right)$$

and $= \left(\frac{1\times1+2\times7}{3}, \frac{1\times(-5)+2\times(-2)}{3}\right)$

$$= \left(\frac{9}{3}, \frac{-12}{3}\right) \text{ and } \left(\frac{15}{3}, \frac{-9}{3}\right) \text{ or } (3, -4) \text{ and } (5, -3)$$

∴ Given statement is true.

7.

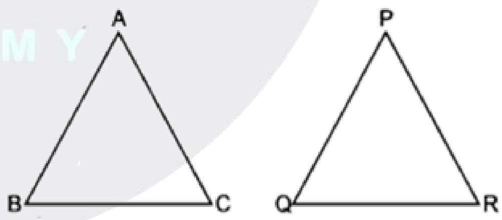


Fig. 6.9

$$AB = \sqrt{(2+2)^2+0} = \sqrt{16} = 4$$

$$BC = \sqrt{(0-2)^2+(2-0)^2} = \sqrt{8} = 2\sqrt{2}$$

$$CA = \sqrt{(-2-0)^2+(0-2)^2} = \sqrt{8} = 2\sqrt{2}$$

$$PQ = \sqrt{(4+4)^2+0} = \sqrt{64} = 8$$

$$AR = \sqrt{(0-4)^2+(4-0)^2} = \sqrt{32} = 4\sqrt{2}$$

$$RP = \sqrt{(-4-0)^2+(0-4)^2} = \sqrt{32} = 4\sqrt{2}$$

$$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{1}{2} \Rightarrow \Delta ABC \sim \Delta PQR$$

8. The point P (0, 2) lies on y-axis

Also, $AP = \sqrt{(0+1)^2 + (2-1)^2} = \sqrt{2}$

$BP = \sqrt{(0-3)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$

AP ≠ BP

∴ P(0, 2) does not lie on the perpendicular bisector of AB. So, given statement is false.

9. Let A (1, 5), B (2, 3) and C (-2, -11) be the given points. Then we have

$AB = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{1+4} = \sqrt{5}$

$BC = \sqrt{(-2-2)^2 + (-11-3)^2}$

$= \sqrt{16+196} = \sqrt{4\times 53} = 2\sqrt{53}$

$AC = \sqrt{(-2-1)^2 + (-11-5)^2}$

$= \sqrt{9+256} = \sqrt{265}$

Clearly, AB + BC ≠ AC

∴ A, B, C are not collinear.

10. (i) Let two given points be A (−5, 7) and B (−1, 3).

Thus, we have $x_1 = -5$ and $x_2 = -1$

$y_1 = 7$ and $y_2 = 3$

$\therefore\ AB = \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$

⇒

$AB = \sqrt{(-1+5)^2 - (3-7)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$ *units.*

Long Answer:

1. Let the given points be

A (x_1, y_1) = (7, −2), B (x_2, Y_2) = (5, 1) and C (x_3, y_3) = (3, k)

Since these points are collinear therefore area (ΔABC) = 0

⇒ $12\,[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

⇒ $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

⇒ 7(1 – k) + 5(k + 2) + 3(-2 -1) = 0

⇒ 7 – 7k + 5k + 10 – 9 = 0

⇒ −2k + 8 = 0

⇒ 2k = 8 ⇒ k = 4

Hence, given points are collinear for k = 4.

2. Let A (x_1, y_1) = (0, -1), B (x_2, y_2) = (2, 1), C (x_3, y_3) = (0, 3) be the vertices of ΔABC.

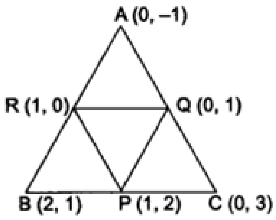
Now, let P, Q, R be the mid-points of BC, CA and AB, respectively.

So, coordinates of P, Q, R are

$P = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$

$Q = \left(\frac{0+0}{2}, \frac{3-1}{2}\right) = (0, 1)$

$R = \left(\frac{2+0}{2}, \frac{1-1}{2}\right) = (1, 0)$



Therefore,

$ar(\Delta PQR) = \frac{1}{2}[1(1-0) + 0(0-2) + 1(2-1)]$

$= \frac{1}{2}(1+1) = 1$ *sq.units*

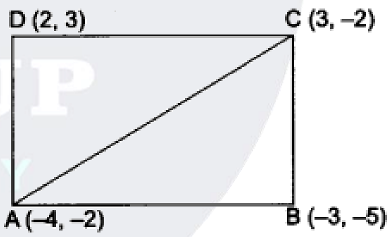
Now,

$ar(\Delta ABC) = \frac{1}{2}[0(1-3) + 2(3+1) + 0(-1-1)]$

$= \frac{1}{2}[0+8+0] = \frac{8}{2} = 4$ *sq.units*

Ratio of ar (ΔPQR) to the ar (ΔABC) = 1 : 4.

3.



Let A(4, -2), B(-3, -5), C(3, -2) and D(2, 3) be the vertices of the quadrilateral ABCD.

Now, area of quadrilateral ABCD

= area of ΔABC + area of ΔADC

$= \frac{1}{2}[-4(-5+2) - 3(-2+2) + 3(-2+5)]$

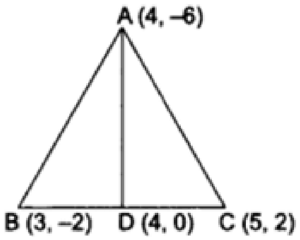
$+ \frac{1}{2}[-4(-2-3) + 3(3+2) + 2(-2+2)]$

$= \frac{1}{2}[12 - 0 + 9] + \frac{1}{2}[20 + 15 + 0]$

$= \frac{1}{2}[21+35] = \frac{1}{2}\times 56 = 28$ *sq. units.*

4. Since AD is the median of ΔABC, therefore, D is the mid-point of BC.

Coordinates of D are $\left(\frac{3+5}{2}, \frac{-2+2}{2}\right)$ i.e., (4,0)



Now, area of ΔABD

$$=\frac{1}{2}\left[4(-2-0)+3(0+6)+4(-6+2)\right]$$

$$=\frac{1}{2}\left[-8+18-16\right]=\frac{1}{2}\times(-6)=-3$$

Since area is a measure, it cannot be negative.

Therefore, ar(ΔABD) = 3 sq. units

and area of ΔADC

$$=\frac{1}{2}\left[4(0-2)+4(2+6)+5(-6-0)\right]$$

$$=\frac{1}{2}(-8+32-30)=\frac{1}{2}(-6)=-3,$$

which cannot be negative.

∴ ar(ΔADC) = 3 sq. units

Here, ar(ΔABD) = ar(ΔADC)

Hence, the median divides it into two triangles of equal areas.

5.

Let the ratio in which point P divides the line segment be k : 1.

Then, coordinates of P : $\left(\frac{4k+12}{k+1}, \frac{-3k+5}{k+1}\right)$

Given, the coordinates of P as (x, 2)

∴ $\frac{4k+12}{k+1}=x$ ...(i)

and $\frac{-3k+5}{k+1}=x$ ...(ii)

−3k +5 = 2k + 2

5k = 3 ⇒ $k=\frac{3}{5}$

Putting the value of k in (i), we have

$$\frac{4\times\frac{3}{5}+12}{\frac{3}{5}+1}=x \Rightarrow \frac{12+60}{3+5}=x$$

$$x=\frac{72}{8} \Rightarrow x=9$$

The ratio in which p divides the line segment is $\frac{3}{5}$, i.e., 3 : 5.

6. Given: AD is the median on BC.

⇒ BD = DC

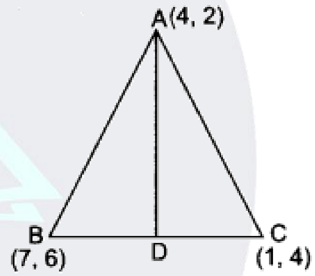
The coordinates of midpoint D are given by.

$\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$ i.e., $\left(\frac{1+7}{2}, \frac{4+6}{2}\right)$

Coordinates of D are (4, 5).

Now, Area of triangles ABD

$$=\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|=5$$



$$=\frac{1}{2}\left|4(6-5)+7(5-2)+4(2-6)\right|$$

$$=\frac{1}{2}\left|4+21-16\right|=\frac{9}{2}\,sq.units$$

Area of ΔACD $=\frac{1}{2}\left|4(4-5)+1(5-2)+4(2-4)\right|$

$$=\frac{1}{2}\left|-4+3-8\right|=\frac{1}{2}\left|-9\right|=\frac{9}{2}\,sq.units$$

Hence, AD divides ΔABC into two equal areas.

7. Given points are A(2, 4), P(3, 8) and Q(-10, y)

According to the question,

PA = QA

$$\sqrt{(2-3)^2+(-4-8)^2}=\sqrt{(2+10)^2+(-4-y)^2}$$

$$\sqrt{(-1)^2+(-12)^2}=\sqrt{(12)^2+(4+y)^2}$$

$$\sqrt{1+144}=\sqrt{144+16+y^2+8y}$$

$$\sqrt{145}=\sqrt{160+y^2+8y}$$

On squaring both sides, we get

$$145 = 160 + y^2 + 8y$$

$$y^2 + 8y + 160 - 145 = 0$$

$$y^2 + 8y + 15 = 0$$

$$y^2 + 5y + 3y + 15 = 0$$

$$y(y+5) + 3(y+5) = 0$$

$\Rightarrow \quad (y+5)(y+3) = 0$

$\Rightarrow \quad y + 5 = 0 \quad \Rightarrow \quad y = -5$

and $\quad y + 3 = 0 \quad \Rightarrow \quad y = -3$

$\therefore \quad y = -3, -5$

Now, $\quad PQ = \sqrt{(-10-3)^2 + (y-8)^2}$

For $\quad y = -3 \quad PQ = \sqrt{(-13)^2 + (-3-8)^2}$

$$= \sqrt{169 + 121} = \sqrt{290} \text{ units}$$

and for $y = -5 \quad PQ = \sqrt{(-13)^2 + (-5-8)^2}$

$$= \sqrt{169 + 169} = \sqrt{338} \text{ units}$$

Hence, values of y are -3 and -5, PQ = $\sqrt{290}$ and $\sqrt{338}$ units.

8. ∵ O is the mid-point of the base BC.

∴ Coordinates of point B are (0, 3). So,

BC =6 units Let the coordinates of point A be (x, 0).

Using distance formula,

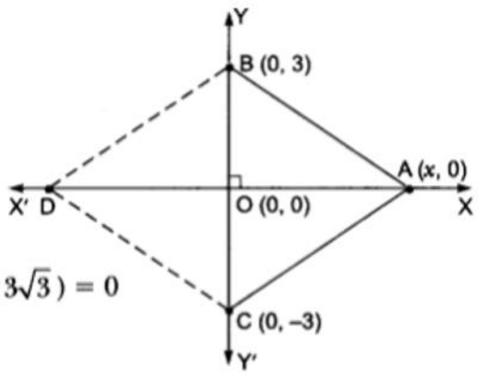


Fig. 6.30

$$AB = \sqrt{(0-x)^2 + (3-0)^2} = \sqrt{x^2 + 9}$$

$$BC = \sqrt{(0-0)^2 + (-3-3)^2} = \sqrt{36}$$

Also, $\quad$ AB = BC (∵ ΔABC is an equilateral triangle)

$$\sqrt{x^2 + 9} = \sqrt{36}$$

$$x^2 + 9 = 36$$

$x^2 = 27 \quad \Rightarrow \quad x^2 - 27 = 0$

$x^2 - 3\sqrt{3} = 0 \quad \Rightarrow \quad (x + 3\sqrt{3})(x - 3\sqrt{3}) = 0$

$x = -3\sqrt{3} \quad or \quad x = 3\sqrt{3}$

$\Rightarrow \quad x = \pm 3\sqrt{3}$

∴ $\quad$ Coordinates of point A = (x, 0) = (3√3, 0)

Since BACD is a rhombus.

∴ $\quad$ AB = AC = CD = DB

∴ $\quad$ Coordinates of point D = (-3√3, 0).

9. Area of a triangle = $\frac{1}{2}$ [$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$]

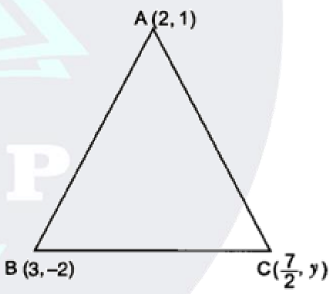
Area of the triangle = $\frac{1}{2}$[t + 2 – t) + (t + 2) (t – t + 2) + (t + 3) (t – 2 – t – 2)]

= $\frac{1}{2}$ [2t + 2t + 4 – 4t – 12] = 4 sq. units

which is independent of t.

Hence proved.

10.



Given: $\quad$ ar(ΔABC) = 5 sq. units

$$\Rightarrow \quad \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right| = 5$$

$$\Rightarrow \quad \frac{1}{2}\left|2(-2-y) + 3(y-1) + \frac{7}{2}(1+2)\right| = 5$$

$$\Rightarrow \quad -4 - 2y + 3y - 3 + \frac{7}{2} + 7 = 10$$

$$\Rightarrow \quad y + \frac{7}{2} = 10 \quad \Rightarrow \quad y = 10 - \frac{7}{2}$$

$$\Rightarrow \quad y = \frac{13}{2}$$

Case Study Answer:

1. **Answer:**

It can be observed that the coordinates of point P, Q and R are (4, 6), (3, 2), and (6, 5) respectively.

i	c	(0, 0)
ii	a	(4, 6)
iii	a	(6, 5)
iv	a	(16, 0)
v	b	(-12, 6)

2. **Answer:**

i. (b) Distance between home and school,

$$HS=\sqrt{(4-4)^2+(3-5)^2}=3m$$

ii. (c) Now, $HL=\sqrt{(-1-4^2)+(3-5)^2}=\sqrt{25+4}=\sqrt{29}$

$$LS=\sqrt{[4-(-1)]^2+(2-3)^2}=\sqrt{25+1}=\sqrt{26}$$

Thus, HL + LS = $\sqrt{29}+\sqrt{26}=10.48m$

So, extra distance covered by Ramesh is = HL + LS – HS = 10.48 – 3 = 7.48m

iii. (d) Now, $HP=\sqrt{(3-4)^2+(0-5)^2}=\sqrt{1+25}=\sqrt{26}$

$$PS=\sqrt{[4-3]^2+(2-0)^2}=\sqrt{1+4}=\sqrt{5}$$

Thus, HP + PS = $\sqrt{26}+\sqrt{5}$ = 7.33m

So, extra distance covered by pulkit is

= HP + PS – HS = 7.33 – 3 = 4.33m

iv. (a) (-1, 3)

v. (c) (4, 5)

Assertion Reason Answer:

1. (a) A is true, R is true; R is a correct explanation for A.
2. (d) A is false; R is true.

Introduction to Trigonometry | 8

1. Meaning (Definition) of Trigonometry

The word trigonometry is derived from the Greek words '*tri*' meaning three, '*gon*' meaning sides and 'metron' meaning measure.

Trigonometry is the study of relationships between the sides and the angles of the triangle.

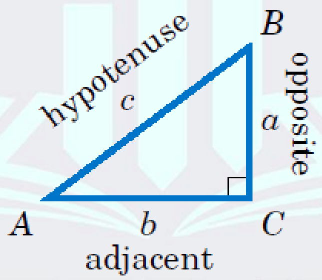
2. Positive and negative angles

Angle measured in anticlockwise direction is taken as positive angle whereas the angle measured in clockwise direction is taken as negative angle.

3. Trigonometric Ratios

Ratio of the sides of a right triangle with respect to the acute angles is called the **trigonometric ratios** of the angle.

Trigonometric ratios of the acute angle A in right triangle ABC are given as follows:



i. $\sin \angle A = \frac{\text{side opposite to} \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{p}{h}$

ii. $\cos \angle A = \frac{\text{side adjacent to} \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{b}{h}$

iii. $\tan \angle A = \frac{\text{side opposite to} \angle A}{\text{side adjacent to} \angle A} = \frac{BC}{AB} = \frac{p}{b}$

iv. $\text{cosec} \angle A = \frac{\text{hypotenuse}}{\text{side opposite to} \angle A} = \frac{AC}{BC} = \frac{h}{p}$

v. $\sec \angle A = \frac{\text{hypotenuse}}{\text{side adjacent to} \angle A} = \frac{AC}{AB} = \frac{h}{b}$

vi. $\cot \angle A = \frac{\text{side adjacent to} \angle A}{\text{side opposite to} \angle A} = \frac{AB}{BC} = \frac{b}{p}$

4. Important facts about Trigonometric ratios

- Trigonometric ratios of an acute angle in a right triangle represents the relation between the angle and the sides.
- The ratios defined above can be rewritten as sin A, cos A, tan A, cosec A, sec A and cot A.
- Each trigonometric ratio is a real number and it has not unit.
- All the trigonometric symbols i.e., cosine, sine, tangent, cotangent, secant and cosecant, have no literal meaning.

- $(\sin\theta)^n$ is generally written as $\sin^n\theta$, n being a positive integer. Similarly, other trigonometric ratios can also be written.
- The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.

5. Pythagoras theorem:

It states that "in a right triangle, square of the hypotenuse is equal to the sum of the squar es of the other two sides".

Pythagoras theorem can be used to obtain the length of the side of a right angled triangle when the other two sides are already given.

6. Relation between trigonometric ratios:

The ratios cosec A, sec A and cot A are the reciprocals of the ratios sin A, cos A and tan A respectively as given:

i. $cosec\theta = \frac{1}{\sin\theta}$

ii. $sec\theta = \frac{1}{\cos\theta}$

iii. $\tan\theta = \frac{\sin\theta}{\cos\theta}$

iv. $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

7. Values of Trigonometric ratios of some specific angles:

∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

- The value of sin A or cos A never exceeds 1, whereas the value of sec A or cosec A is always greater than 1 or equal to 1.
- The value of $\sin\theta$ increases from 0 to 1 when θ increases from 0^o to 90^o.
- The value of $\cos\theta$ decreases from 1 to 0 when θ increases from 0^o to 90^o.
- If one of the sides and any other parts like either an acute angle or any side of a right triangle are known, the remaining sides and angles of the triangle can be obtained using trigonometric ratios.

8. Trigonometric ratios of complementary angles:

Two angles are said to complementary angles if their sum is equal to 90^o. Based on this relation, the trigonometric ratios of complementary angles are given as follows:

i. $\sin(90^\circ - A) = \cos A$

ii. $\cos(90^\circ - A) = \sin A$

iii. $\tan(90^\circ - A) = \cot A$

iv. $\cot(90^\circ - A) = \tan A$

v. $\sec(90^\circ - A) = \text{cosec } A$

vi. $\text{cosec}(90^\circ - A) = \sec A$

Note: $\tan 0^\circ = 0 = \cot 90^\circ$, $\sec 0^\circ = 1 = \text{cosec } 90^\circ$, $\sec 90^\circ$, $\text{cosec } 0^\circ$, $\tan 90^\circ$ and $\cot 0^\circ$ are not defined.

9. Definition of Trigonometric Identity

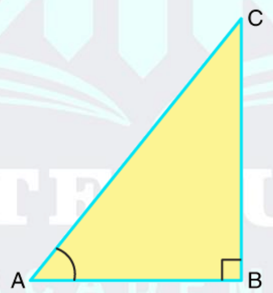
An equation involving trigonometric ratios of an angle, say θ, is termed as a **trigonometric identity** if it is satisfied by all values of θ.

10. Basic trigonometric identities

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta;\ 0 \le \theta < 90^\circ$
- $1 + \cot^2\theta = \text{cosec}^2\theta;\ 0 \le \theta < 90^\circ$

11. Opposite & Adjacent Sides in a Right Angled Triangle

In the ΔABC right-angled at B, BC is the side opposite to ∠A, AC is the hypotenuse and AB is the side adjacent to ∠A.



12. Trigonometric Ratios

For the right ΔABC, right-angled at ∠B, the trigonometric ratios of the ∠A are as follows:

sin A = opposite side/hypotenuse = BC/AC

cos A = adjacent side/hypotenuse = AB/AC

tan A = opposite side/adjacent side = BC/AB

cosec A = hypotenuse/opposite side = AC/BC

sec A = hypotenuse/adjacent side = AC/AB

cot A = adjacent side/opposite side = AB/BC

13. Visualization of Trigonometric Ratios Using a Unit Circle

Draw a circle of the unit radius with the origin as the centre. Consider a line segment OP joining a point P on the circle to the centre which makes an angle θ with the x-axis. Draw a perpendicular from P to the x-axis to cut it at Q.

Sin θ = PQ/OP=PQ/1=PQ

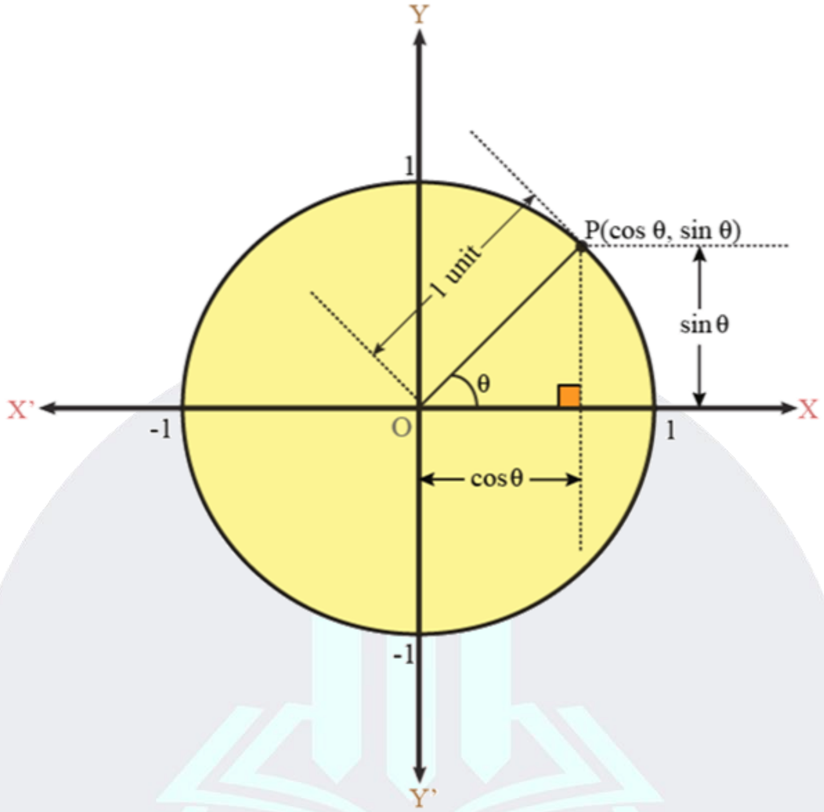
cosθ = OQ/OP=OQ/1=OQ

tanθ = PQ/OQ=sinθ/cosθ

cosecθ = OP/PQ=1/PQ

secθ = OP/OQ=1/OQ

cotθ = OQ/PQ=cosθ/sinθ



14. **Relation between Trigonometric Ratios**

cosec θ = 1/sin θ

sec θ = 1/cos θ

tan θ = sin θ/cos θ

cot θ = cos θ/sin θ = 1/tan θ

15. **Range of Trigonometric Ratios from 0 to 90 degrees**

For $0^\circ \le \theta \le 90^\circ$,

$0 \le \sin\theta \le 1$

$0 \le \cos\theta \le 1$

$0 \le \tan\theta < \infty$

$1 \le \sec\theta < \infty$

$0 \le \cot\theta < \infty$

$1 \le \text{cosec}\theta < \infty$

tanθ and secθ are not defined at 90°.

cotθ and cosecθ are not defined at 0°.

16. **Variation of trigonometric ratios from 0 to 90 degrees**

As θ increases from 0° to 90°

sin θ increases from 0 to 1

cos θ decreases from 1 to 0

tan θ increases from 0 to ∞

cosec θ decreases from ∞ to 1

sec θ increases from 1 to ∞

cot θ decreases from ∞ to 0

17. Standard values of Trigonometric ratios

∠A	0^0	30^0	45^0	60^0	90^0
sin A	0	1/2	1/√2	√3/2	1
cos A	1	√3/2	1/√2	1/2	0
tan A	0	1/√3	1	√3	not defined
cosec A	not defined	2	√2	2/√3	1
sec A	1	2/√3	√2	2	not defined
cot A	not defined	√3	1	1/√3	0

18. Complementary Trigonometric ratios

In Mathematics, the complementary angles are the set of two angles such that their sum is equal to 90°. For example, 30° and 60° are complementary to each other as their sum is equal to 90°. In this article, let us discuss in detail about the complementary angles and the trigonometric ratios of complementary angles with examples in a detailed way.

If θ is an acute angle, its complementary angle is 90∘ − θ. The following relations hold true for trigonometric ratios of complementary angles.

sin (90∘ − θ) = cos θ

cos (90∘ − θ) = sin θ

tan (90∘ − θ) = cot θ

cot (90∘ − θ) = tan θ

cosec (90∘ − θ) = sec θ

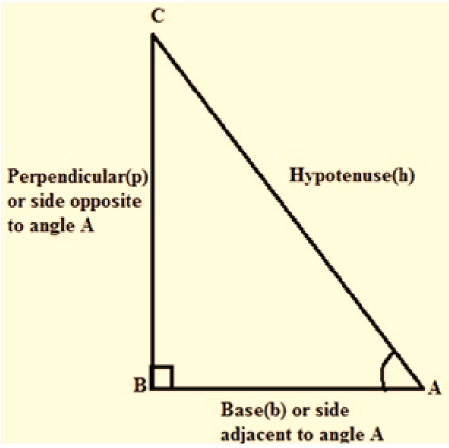
sec (90∘ − θ) = cosec θ

Finding Trigonometric Ratios of Complementary Angles

∠A and ∠C form a complementary pair.

⇒ ∠A + ∠C = 90°

The relationship between the acute angle and the lengths of sides of a right-angle triangle is expressed by trigonometric ratios. For the given right-angle triangle, the trigonometric ratios of ∠A is given as follows:



sin A = BC/AC

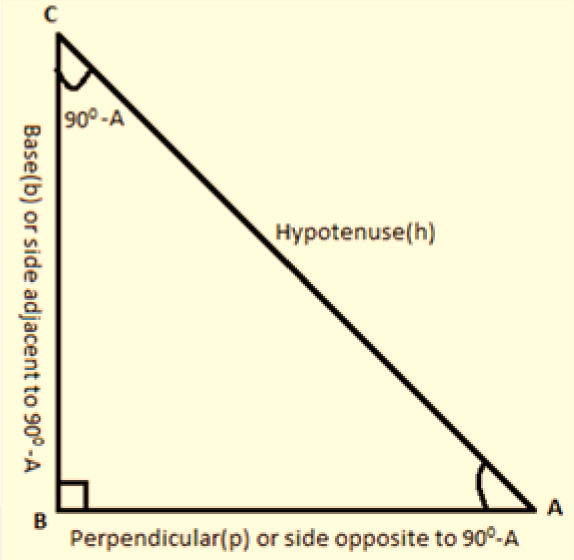
cos A = AB/AC

tan A =BC/AB

csc A = 1/sin A = AC/BC

sec A =1/cos A = AC/AB

cot A = 1/tan A = AB/BC



The trigonometric ratio of the complement of ∠A. It means that the ∠C can be given as 90° – ∠A

As ∠C = 90°– A (A is used for convenience instead of ∠A), and the side opposite to 90° – A is AB and the side adjacent to the angle 90°- A is BC as shown in the figure given above.

Therefore,

sin (90°- A) = AB/AC

cos (90°- A) = BC/AC

tan (90°- A) = AB/BC

csc (90°- A) =1/sin (90°- A) = AC/AB

sec (90°- A) = 1/cos (90°- A) = AC/BC

cot (90°- A) = 1/tan (90°- A) = BC/AB

Comparing the above set of ratios with the ratios mentioned earlier, it can be seen that;

sin (90° – A) = cos A ; cos (90°– A) = sin A

tan (90°– A) = cot A; cot (90°– A) = tan A

sec (90°– A) = csc A; csc (90°– A) = sec A

These relations are valid for all the values of A that lies between 0° and 90°.

19. Trigonometric Identities

Trigonometric Identities are useful whenever trigonometric functions are involved in an expression or an equation. Trigonometric Identities are true for every value of variables occurring on both sides of an equation. Geometrically, these identities involve certain trigonometric functions (such as sine, cosine, tangent) of one or more angles.

Sine, cosine and tangent are the primary trigonometry functions whereas cotangent, secant and cosecant are the other three functions. The trigonometric identities are based on all the six trig functions. Check Trigonometry Formulas to get formulas related to trigonometry.

Trigonometric Identities are useful whenever trigonometric functions are involved in an expression or an equation. Trigonometric Identities are true for every value of variables occurring on both sides of an equation. Geometrically, these identities involve certain trigonometric functions (such as sine, cosine, tangent) of one or more angles.

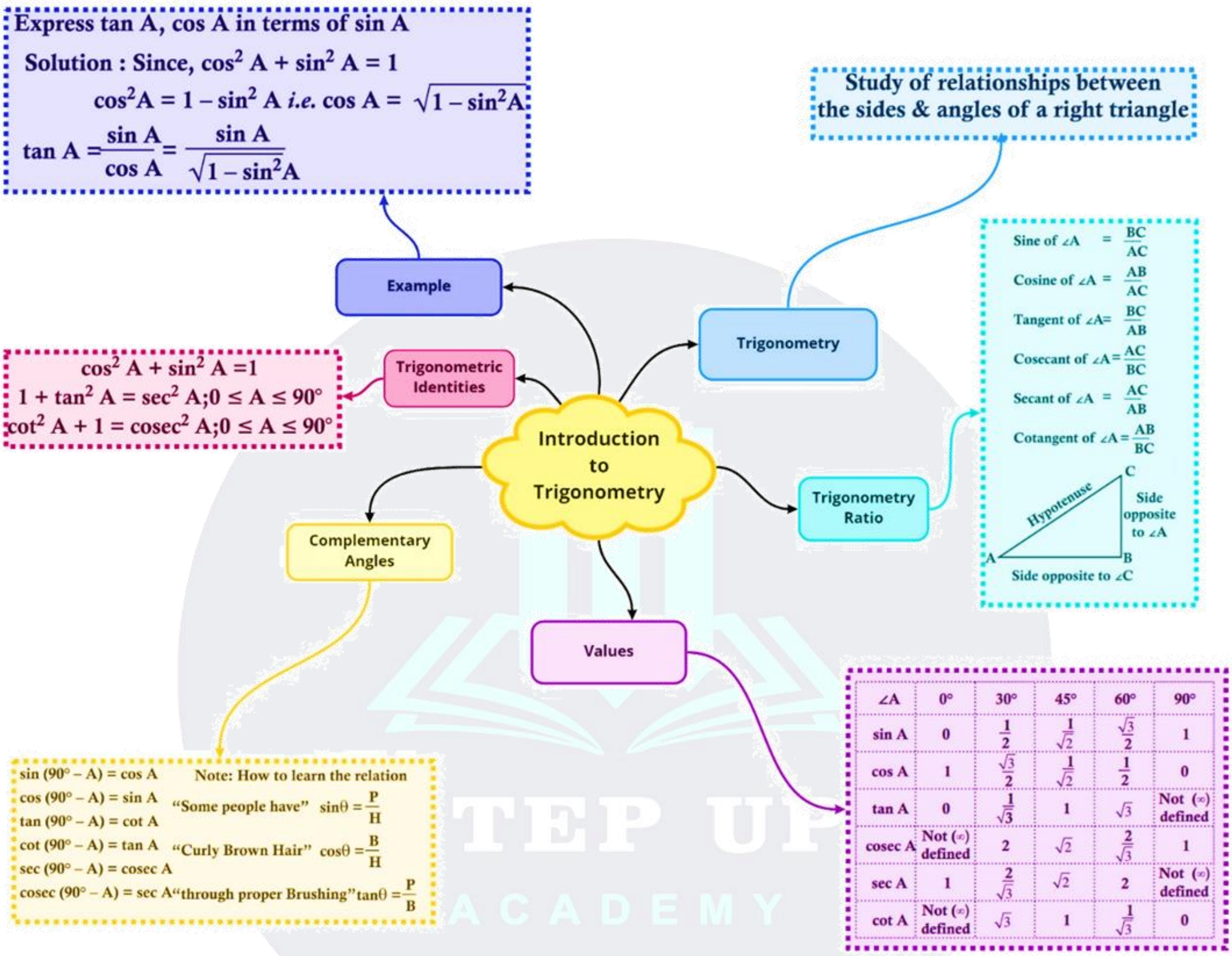
Sine, cosine and tangent are the primary trigonometry functions whereas cotangent, secant and cosecant are the other three functions. The trigonometric identities are based on all the six trig functions. Check Trigonometry Formulas to get formulas related to trigonometry.

$\sin^2\theta + \cos^2\theta = 1$

$1 + \cot^2\theta = \cos ec^2\theta$

$1 + \tan^2\theta = \sec^2\theta$

Class : 10th mathematics
Chapter- 8 : Introduction to Trigonometry



∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not (∞) defined
cosec A	Not (∞) defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not (∞) defined
cot A	Not (∞) defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Important Questions

Multiple Choice Questions:

1. If cos (α + β) = 0, then sin (α – β) can be reduced to
   (a) cos β
   (b) cos 2β
   (c) sin α
   (d) sin 2α
2. If cos (40° + A) = sin 30°, the value of A is:
   (a) 60°
   (b) 20°
   (c) 40°
   (d) 30°
3. If sin x + cosec x = 2, then $\sin^{19}x + \text{cosec}^{20}x$ =
   (a) 2^{19}
   (b) 2^{20}
   (c) 2
   (d) 2^{39}
4. If cos 9a = sin a and 9a < 90°, then the value of tan 5a is
   (a) $\frac{1}{\sqrt{3}}$
   (b) $\sqrt{3}$
   (c) 1
   (d) 0
5. (1 + tanθ + secθ) (1 + cotθ – cosecθ) is equal to
   (a) 0
   (b) 1
   (c) 2
   (d) –1
6. Ratios of sides of a right triangle with respect to its acute angles are known as
   (a) trigonometric identities
   (b) trigonometry
   (c) trigonometric ratios of the angles
   (d) none of these
7. The value of cos θ cos(90° – θ) – sin θ sin (90° – θ) is:
   (a) 1
   (b) 0
   (c) –1
   (d) 2
8. If x = a cos θ and y = b sin θ, then $b^2x^2 + a^2y^2$ =
   (a) ab
   (b) $b^2 + a^2$
   (c) a^2b^2
   (d) a^4b^4
9. If x and y are complementary angles, then
   (a) sin x = sin y
   (b) tan x = tan y
   (c) cos x = cos y
   (d) sec x = cosec y
10. sin (45° + θ) – cos (45° – θ) is equal to
   (a) 2 cos θ
   (b) 0
   (c) 2 sin θ
   (d) 1

Very Short Questions:

1. Find maximum value of $\frac{1}{\sec\theta}$, 0°≤ θ ≤ 90°.
2. Given that sin θ = $\frac{a}{b}$, find the value of tan θ.
3. If sin θ = cos θ, then find the value of 2 tan θ + $\cos^2 θ$.
4. If sin (x – 20)° = cos (3x – 10)°, then find the value of x.
5. If $\sin^2 A = \frac{1}{2}\tan^2 45°$, where A is an acute angle, then find the value of A.
6. If x = a cos θ, y = b sin θ, then find the value of $b^2x^2 + a^2y^2 – a^2b^2$.
7. If tan A = cot B, prove that A + B = 90°.
8. If sec A = 2x and tan A = 2x, find the value of $2(x^2 - \frac{1}{x^2})$.
9. In a ΔABC, if ∠C = 90°, prove that $\sin^2 A + \sin^2 B = 1$.
10. If sec 4A = cosec (A – 20°) where 4 A is an acute angle, find the value of A.

Short Questions:

1. If sin A = $\frac{3}{4}$, calculate cos A and tan A.
2. Given 15 cot A = 8, find sin A and sec A.
3. In Fig. 10.5, find tan P – cot R.

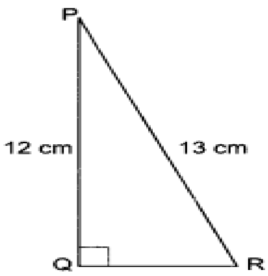


Fig. 10.5

4. If sin θ + cos θ = √3 , then prove that tan θ + cot θ = 1.

5. Prove that $\frac{1-sin\theta}{1+sin\theta}$ = (sec θ – tan θ)^2

6. $\frac{\sec^2 54^o - \cot^2 36^o}{\cos ec^2 57^o - \tan^2 33^o}$

$+2\sin^2 38^o.\sec^2 52^o - \sin^2 45^o.$

7. $\frac{2\sin 68^o}{\cos 22^o} - \frac{2\cot 15^o}{5\tan 75^o}$

$-\frac{3\tan 45^o.\tan 20^o.\tan 40^o.\tan 50^o.\tan 70^o}{5}$

8. $\frac{\sin^2 20^o + \sin^2 70^o}{\cos^2 20^o + \cos^2 70^o}$

$+\left[\frac{\sin\left(90^o - \theta\right).\sin\theta}{\tan\theta} + \frac{\cos\left(90^o - \theta\right).\cos\theta}{\cot\theta}\right]$

9. Evaluate: sin 25° cos 65° + cos 25° sin 65°.

10. Without using tables, evaluate the following:

3 cos 68°. cosec 22° – $\frac{1}{2}$ tan 43°. tan 47°. tan 12°. tan 60°. tan 78°

Long Questions:

1. In ΔPQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

2. In triangle ABC right-angled at B, if tan A = $\frac{1}{\sqrt{3}}$ find the value of:

(i) sin A cos C + cos A sin C

(ii) cos A cos C – sin A sin C.

3. If cot θ = $\frac{7}{8}$, evaluate:

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) cot^2 θ

4. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A}$ = cos^2 A – sin^2 A or not.

5. Write all the other trigonometric ratios of ∠A in terms of sec A.

6. Prove that $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan^2 A}{1-\cot^2 A}\right) = \tan^2 A$

7. Prove that:

$\tan^2 A - \tan^2 B = \frac{\cos^2 B - \cos^2 A}{\cos^2 B\cos^2 A} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}.$

8. Prove that:

$\frac{ca\sec A}{\cos ec A - 1} + \frac{ca\sec A}{\cos ec A - 1} = 2 + 2\tan^2 A = 2\sec^2 A.$

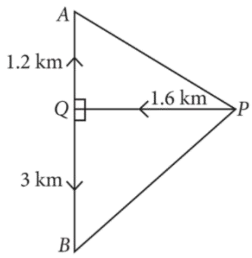
9. Prove that: (sin θ + sec θ)^2 + (cos θ + cosec θ)^2 = (1 + sec θ cosec θ)^2.

10. Prove that:

$\frac{1}{(\cos ec\, x + \cot x)} - \frac{1}{\sin x} = \frac{1}{\sin x} - \frac{1}{(\cos ec\, x - \cot x)}.$

Assertion Reason Questions:

1. Two aeroplanes leave an airport, one after the other. After moving on runway, one flies due North and other flies due South. The speed of two aeroplanes is 400km/ hr and 500km/ hr respectively. Considering PQ as runway and A and B are any two points in the path followed by two planes, then answer the following questions.



i. Find tanθ if ∠APQ = θ.

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{3}{4}$

ii. Find cot B.

(a) $\frac{3}{4}$ (b) $\frac{15}{4}$

(c) $\frac{3}{8}$ (d) $\frac{15}{8}$

iii. Find tan A.

(a) 2 (b) $\sqrt{2}$

(c) $\frac{4}{3}$ (d) $\frac{2}{\sqrt{3}}$

iv. Find sec A.

(a) 1 (b) $\frac{2}{3}$

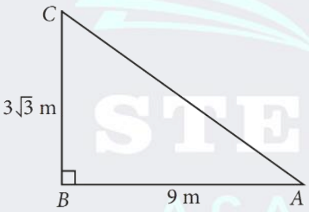
(c) $\frac{4}{3}$ (d) $\frac{5}{3}$

v. Find cosec B.

(a) $\frac{17}{8}$ (b) $\frac{12}{5}$

(c) $\frac{5}{12}$ (d) $\frac{8}{17}$

2. Three friends- Anshu, Vijay and Vishal are playing hide and seek in a park. Anshu and Vijay hide in the shrubs and Vishal have to find both of them. If the positions of three friends are at A, B and C respectively as shown in the figure and forms a right-angled triangle such that AB = 9 m, BC = $\sqrt{3}$ M and $\sqrt{3}$ m and $\angle B = 90^o$, then answer the following questions.



i. The measure of $\angle A$ is:

(a) 30^o (b) 45^o

(c) 60^o (d) None of these

ii. The measure of $\angle C$ is:

(a) 30^o (b) 45^o

(c) 60^o (d) None of these

iii. The length of AC is:

(a) $2\sqrt{3}$ m (b) $\sqrt{3}$ m

(c) $4\sqrt{3}$ m (d) $6\sqrt{3}$ m

iv. cos 2A =

(a) 0 (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

v. $\sin\left(\frac{C}{2}\right) =$

(a) 0 (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

Assertion Reason Questions:

1. **Directions:** Each of these questions contains two statements: Assertion [A] and Reason [R]. Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes [a], [b], [c] and [d] given below.

   a. A is true, R is true; R is a correct explanation for A.

   b. A is true, R is true; R is not a correct explanation for A.

   c. A is true; R is false.

   d. A is false; R is true.

   Assertion: The value of each of the trigonometric ratios of an angle does not depend on the size of the triangle. It only depends on the angle.

   Reason: In right as hypotenuse is the longest side.

   ΔABC, $\angle B = 90^o$ and $\angle A = \theta^o$ $\sin\theta = \frac{BC}{AC} < 1$ and

   $\cos\theta = \frac{AB}{AC} < 1$

2. **Directions:** Each of these questions contains two statements: Assertion [A] and Reason [R]. Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes [a], [b], [c] and [d] given below.

   a. A is true, R is true; R is a correct explanation for A.

   b. A is true, R is true; R is not a correct explanation for A.

   c. A is true; R is false.

   d. A is false; R is true.

   Assertion: Sin 60^o = Cos 30^o

   Reason: Sin 2θ = Sin θ where θ is an acute angle.

Answer Key

Multiple Choice Questions:

1. (b) cos 2β
2. (b) 20°
3. (c) 2
4. (c) 1
5. (c) 2
6. (c) trigonometric ratios of the angles
7. (b) 0
8. (c) a^2b^2
9. (d) sec x = cosec y
10. (b) 0

Very Short Answers:

1. $\frac{1}{sec\theta}$, (0° ≤ θ ≤ 90°) (Given)

∵ sec θ is in the denominator

∴ The min. value of sec θ will return max. value for $\frac{1}{sec\theta}$.

But the min. value of sec θ is sec 0° = 1.

Hence, the max. value of $\frac{1}{sec\theta°} = \frac{1}{1} = 1$

2. $\sin\theta = \frac{a}{b}$

$$\Rightarrow \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{a^2}{b^2}}$$

$$= \sqrt{\frac{b^2-a^2}{b^2}} = \frac{\sqrt{b^2-a^2}}{b}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{a/b}{\frac{\sqrt{b^2-a^2}}{b}} = \frac{a}{\sqrt{b^2-a^2}}$$

3. sin θ = cos θ (Given)

It means value of θ = 45°

Now, 2 tan θ + $\cos^2$ θ = 2 tan 45° + $\cos^2$ 45°

4. sin (x – 20)° = cos (3x – 10)°

⇒ cos [90° – (x – 20)°] = cos (3x – 10)°

By comparing the coefficient

90° – x° + 20° = 3x° – 10° = 110° + 10° = 3x° + x°

120° = 4x°

$\Rightarrow \frac{120°}{4} = 30°$

5. $\sin^2 A = 12\tan^2 45°$

$\Rightarrow \sin 2A = \frac{1}{2}(1)^2$ [∵ tan 45° = 1]

$\sin 2A = \frac{1}{2} \Rightarrow \sin A = \frac{1}{\sqrt{2}}$

Hence, ∠A = 45°

6. Given x = acos θ, y = b sin θ

$b^2x^2 + a^2y^2 - a^2b^2 = b^2(a\cos\theta)^2 + a^2(b\sin\theta)^2 - a^2b^2$

$= a^2b^2\cos^2\theta + a^2b^2\sin^2\theta - a^2b^2 = a^2b^2(\sin^2\theta + \cos^2\theta) - a^2b^2$

$= a^2b^2 - a^2b^2 = \theta$ (∵ $\sin 2\theta + \cos^2\theta = 1$)

7. We have, tan A = cot B

⇒ tan A = tan (90° – B)

A = 90° – B

[∵ Both A and B are acute angles]

⇒ A + B = 90°

8.

$$2\left(x^2 - \frac{1}{x^2}\right) = 2\left(\frac{\sec^2 A}{4} - \frac{\tan^2 A}{4}\right)$$

$$= \frac{2}{4}\left(\sec^2 A - \tan^2 A\right) = \frac{1}{2}\times 1 = \frac{1}{2}$$

9. Since ∠C = 90°

∴ ∠A + ∠B = 180° – ∠C = 90°

Now, $\sin^2 A + \sin^2 B = \sin^2 A + \sin^2(90° - A) = \sin^2 A + \cos^2 A = 1$

10. We have

sec 4 A = cosec (A – 20°)

⇒ cosec (90° – 4 A) = cosec (A – 20°)

∴ 90° – 4 A = A – 20°

⇒ 90° + 20° = A + 4 A ⇒ 110° = 5 A

∴ $A = \frac{110}{5} = 22°$

Short Answers:

1. Let us first draw a right ΔABC in which ∠C = 90°.

Now, we know that

$$\sin A = \frac{Perpendicular}{Hypotenuse} = \frac{BC}{AB} = \frac{3}{4}$$

Let BC = 3k and AB = 4k, where k is a positive number.

Then, by Pythagoras Theorem, we have

$AB^2 = BC^2 + AC^2 \Rightarrow (4k)^2 = (3k)^2 + AC^2$

$\Rightarrow 16k^2 - 9k^2 = AC \Rightarrow 7k^2 = AC^2$

$\therefore \quad AC = \sqrt{7}$

$\therefore \quad \cos A = \frac{AC}{AB} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$

And $\quad \tan A = \frac{BC}{AC} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$

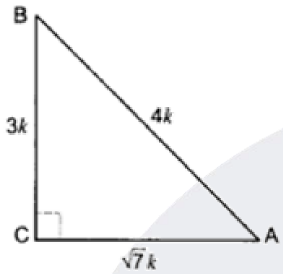


Fig. 10.3

2. Let us first draw a right ΔABC in which ∠B = 90°.

Now, we have, 15 cot A = 8

$\therefore \quad \cot A = \frac{8}{15} = \frac{AB}{BC} = \frac{Base}{Perpendicular}$

Let AB = 8k and BC = 15k

Then, $\quad AC = \sqrt{(AB)^2 + (BC)^2}$

(By Pythagoras Theorem)

$= \sqrt{(8k)^2 + (15k)^2} = \sqrt{64k^2 + 225k^2}$

$= \sqrt{289k^2} = 17k$

$\therefore \quad \sin A = \frac{Perpendicular}{Hypotenuse} = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

and $\quad \sec A = \frac{Hypotenuse}{Base} = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$

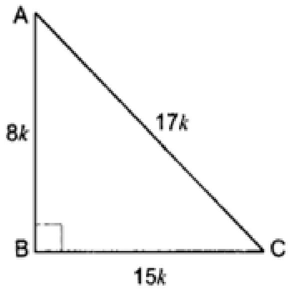


Fig. 10.4

3. Using Pythagoras Theorem, we have

$PR^2 = PO^2 + QR^2$

$\Rightarrow (13)2 = (12)^2 + QR^2$

$\Rightarrow 169 = 144 + QR^2$

$\Rightarrow QR^2 = 169 - 144 = 25$

$\Rightarrow QR = 5$ cm

Now, $\tan P = \frac{QR}{PQ} = \frac{5}{12}$ and $\cot R = \frac{QR}{PQ} = \frac{5}{12}$

$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

4. $\sin\theta + \cos\theta = \sqrt{3}$

$\Rightarrow (\sin\theta + \cos\theta)^2 = 3$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 2\sin\cos\theta = 2 \; (\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow \sin\theta.\cos\theta = 1 = \sin^2\theta + \cos^2\theta$

$\Rightarrow 1 = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\Rightarrow 1 = \tan\theta + \cot\theta = 1$

Therefore, $\tan\theta + \cot\theta = 1$

5. LHS = $\frac{1-\sin\theta}{1+\sin\theta}$

$= \frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$

[Rationalising the denominator]

$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta} = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$

$= (\sec\theta - \tan\theta)^2 = RHS$

6. We have, $\frac{\sec^2 54^o - \cot^2 36^o}{\operatorname{cosec}^2 57^o - \tan^2 33^o}$

$+ 2\sin^2 38^o.\sec^2 52^o - \sin^2 45^o.$

$= \frac{\sec^2(90^o - 36^o) - \cot^2 36^o}{\operatorname{cosec}^2(90^o - 33^o) - \tan^2 33^o}$

$+ 2\sin^2 38^o.\sec^2(90^o - 38^o) - \sin^2 45^o.$

$= \frac{\operatorname{cosec}^2 36^o - \cot^2 36^o}{\sec^2 33^o - \tan^2 33^o} + 2\sin^2 38^o.\operatorname{cosec}^2 38^o - \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{1} + 2.1 - \frac{1}{2} = 3 - \frac{1}{2} = \frac{5}{2}$

7. We have,

$\frac{2\sin 68^o}{\cos 22^o} - \frac{2\cot 15^o}{5\tan 75^o}$

$$-\frac{3\tan 45^\circ.\tan 20^\circ.\tan 40^\circ.\tan 50^\circ.\tan 70^\circ}{5}$$

$$=\frac{2\sin\left(90^\circ-22^\circ\right)}{\cos 22^\circ}-\frac{2\cot 15^\circ}{5\tan\left(90^\circ-15^\circ\right)}$$

$$-\frac{3\tan 45^\circ.\tan 20^\circ.\tan 40^\circ.\tan\left(90^\circ-40^\circ\right).\tan\left(90^\circ-20^\circ\right)}{5}$$

$$=\frac{2\cos 22^\circ}{\cos 22^\circ}-\frac{2\cot 15^\circ}{5\cot 15^\circ}$$

$$-\frac{3\tan 45^\circ.\tan 20^\circ.\tan 40^\circ.\cot 40^\circ.\cot 20^\circ}{5}$$

$$=2-\frac{2}{5}-\frac{3}{5}.1.1.1=2-\frac{2}{5}-\frac{3}{5}=2-1=1$$

8. We have

$$\frac{\sin^2 20^\circ+\sin^2 70^\circ}{\cos^2 20^\circ+\cos^2 70^\circ}$$

$$+\left[\frac{\sin\left(90^\circ-\theta\right).\sin\theta}{\tan\theta}+\frac{\cos\left(90^\circ-\theta\right).\cos\theta}{\cot\theta}\right]$$

$$=\frac{\sin^2 20^\circ+\sin^2\left(90^\circ-20^\circ\right)}{\cos^2 20^\circ+\cos^2\left(90-20^\circ\right)}$$

$$+\left[\frac{\cos\theta.\sin\theta}{\tan\theta}+\frac{\sin\theta.\cos\theta}{\cot\theta}\right]$$

$$=\frac{\sin^2 20^\circ+\cos^2 20^\circ}{\cos^2 20^\circ+\sin^2 20^\circ}+\left[\frac{\cos\theta.\sin\theta}{\frac{\sin\theta}{\cos}}+\frac{\sin\theta.\cos\theta}{\frac{\cos\theta}{\sin\theta}}\right]$$

$$=\frac{1}{1}+\left[\cos^2\theta+\sin^2\theta\right]=1+1=2.$$

9. sin 25°. cos 65° + cos 25° . sin 65°

= sin (90° – 65°). cos 65° + cos (90° – 65°). sin 65°

= cos 65° . cos 65° + sin 65°. sin 65°

= $\cos^2 65° + \sin^2 65°$ = 1.

10. We have,

3 cos 68°. cosec 22° – $\frac{1}{2}$ tan 43°. tan 47°. tan 12°. tan 60°. tan 78°.

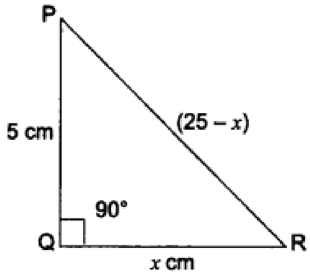
= 3 cos (90° – 22°). cosec 22° – $\frac{1}{2}$. {tan 43° . tan (90° – 43°)}. {tan 12°. tan (90° – 12°). tan 60°}

= 3 sin 22°. cosec 22° – $\frac{1}{2}$ (tan 43° . cot 43°). (tan 12°. cot 12°). tan 60°

= 3 × 1 – × 1 × 1 × √3 = $3-\frac{3}{\sqrt{2}}=\frac{6-\sqrt{3}}{\sqrt{2}}$.

Long Answers:

1.



We have a right-angled ΔPQR in which ∠Q = 90°.

Let QR = x cm

Therefore, PR = (25 – x) cm

By Pythagoras Theorem, we have

$PR^2 = PQ^2 + QR^2$

$(25 – x)^2 = 52 + x^2$

= $(25 – x)^2 – x^2 = 25$

(25 – x – x) (25 – x + x) = 25

(25 – 2x) 25 = 25 ⇒ 25 – 2x = 1

25 – 1 = 2x ⇒ 24 = 2x

∴ x = 12 cm

Hence, QR = 12 cm

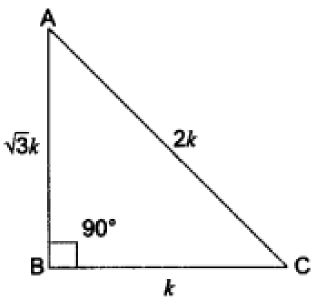
PR = (25 – x) cm = 25 – 12 = 13 cm

PQ = 5 cm

$$\therefore \quad \sin P=\frac{QR}{PR}=\frac{12}{13};\ \cos P=\frac{PQ}{PR}=\frac{5}{13};$$

$$\tan P=\frac{QR}{PQ}=\frac{12}{5}cm$$

2.



We have a right-angled ΔABC in which ∠B = 90°.

and, $\tan A=\frac{1}{\sqrt{3}}$

Now, tan A = $\frac{1}{\sqrt{3}}$ = BCAB

Let BC = k and AB = √3k

∴ By Pythagoras Theorem, we have

⇒ $AC^2 = AB^2 + BC^2$

⇒ $AC^2 = (\sqrt{3}k)^2 + (k)^2 = 3k^2 + k^2$

⇒ $AC^2 = 4k^2$

Now, $\sin A = \frac{Perpendicular}{Hypontenuse} = \frac{k}{2k} = \frac{1}{2};$

$$\cos A = \frac{Base}{Hypontenuse} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{Perpendicular}{Hypontenuse} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2};$$

$$\cos C = \frac{Base}{Hypontenuse} = \frac{k}{2k} = \frac{1}{2}$$

(i) sin A. cos C + cos A. sin C

$$= \frac{1}{2}\times\frac{1}{2} + \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1.$$

(ii) cos A. cos C − sin A. sin C

$$= \frac{\sqrt{3}}{2}\times\frac{1}{2} - \frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0.$$

3. Let us draw a right triangle ABC in which ∠B = 90° and ∠C = θ.

We have, $\cot\theta = \frac{7}{8} = \frac{Base}{Perpendicular} = \frac{BC}{AB}$ (given)

Let BC = 7k and AB = 8k

Therefore, by Pythagoras Theorem

$$AC^2 = AB^2 + BC^2$$

$$= (8k)^2 + (7k)^2 = 64k^2 + 49k^2$$

$$AC^2 = 113k^2 \quad \therefore \quad AC = \sqrt{113}k$$

$$\therefore \sin\theta = \frac{Perpendicular}{Hypontenuse} = \frac{AB}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\text{And } \cos\theta = \frac{Base}{Hypontenuse} = \frac{BC}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

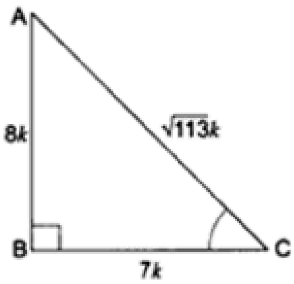


Fig. 10.9

(i) $$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2}$$

$$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}} = \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{49}{64}$$

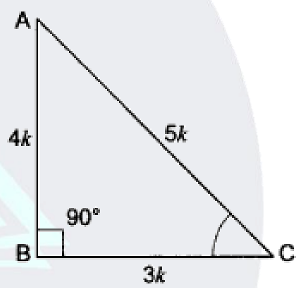
Alternate method:

(i) $$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$$

$$\cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

(ii) $$\cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}.$$

4.



Let us consider a right triangle ABC in which ∠B = 90°

Now, $\cot A = \frac{Base}{Perpendicular} = \frac{AB}{BC} = \frac{4}{3}$

Let AB = 4k and BC = 3k

∴ By Pythagoras Theorem

$AC^2 = AB^2 + BC^2$

$AC = (4k)^2 + (3k)^2 = 16k^2 + 9k^2$

$AC^2 = 25k^2$

∴ AC = 5k

Therefore, $\tan A = \frac{Perpendicular}{Base} = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

and $\sin A = \frac{Perpendicular}{Hypontenuse} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

$$\cos A = \frac{Base}{Hypontenuse} = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

Now, LHS $= \frac{1-\tan^2 A}{1+\tan^2 A}$

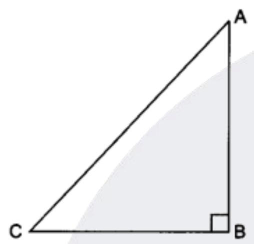
$$= \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{16-9}{16+9} = \frac{7}{25}$$

$$\text{RHS } = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Hence, $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

5.



Let us consider a right-angled ΔABC in which ∠B = 90°. For ∠A we have

$$\therefore \ \sec A = \frac{Hypontenuse}{Base} = \frac{AC}{AB}$$

$$\Rightarrow \ \frac{\sec A}{1} = \frac{AC}{AB} \qquad \Rightarrow \ AC = AB \sec A$$

Let AB = k and AC = k sec A

∴ By Pythagoras Theorem, we have

$$AC^2 = AB^2 + BC^2 \qquad \Rightarrow k^2 \sec^2 A = k^2 + BC^2$$

$$\therefore \ BC^2 = k^2 \sec^2 A - k^2$$

$$\Rightarrow BC = k\sqrt{\sec^2 A - 1}$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{k\sqrt{\sec^2 A-1}}{k \sec A} = \frac{\sqrt{\sec^2 A-1}}{\sec A}$$

$$\cos A = \frac{AB}{AC} = \frac{k}{k \sec A} = \frac{1}{\sec A}$$

$$\tan A = \frac{BC}{AB} = \frac{k\sqrt{\sec^2 A-1}}{k} = \sqrt{\sec^2 A-1}$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A-1}}$$

$$\text{cosec } A = \frac{AC}{BC} = \frac{k \sec A}{k\sqrt{\sec^2 A-1}} = \frac{\sec A}{\sqrt{\sec^2 A-1}}$$

6. $\text{LHS } = \left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \frac{\sec^2 A}{\cos ec^2 A}$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

$$\text{RHS } = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$

$$= \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2 = \left(\frac{1-\tan A}{\tan A - 1} \times \tan A\right)^2$$

$$= (-\tan A)^2 = \tan^2 A$$

LHS = RHS.

7. LHS

$$= \tan^2 A - \tan^2 B = \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$$

$$= \frac{\sin^2 A \cos^2 B - \cos^2 A \sin^2 B}{\cos^2 A \cos^2 B}$$

$$= \frac{(1-\cos^2 A)\cos^2 B - \cos^2 A(1-\cos^2 B)}{\cos^2 A \cos^2 B}$$

$$= \frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B}$$

Also $\frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B} = \frac{(1-\sin^2 B)-(1-\sin^2 A)}{\cos^2 A \cos^2 B}$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B} = RHS.$$

8. $\text{LHS } = \frac{\text{cosec } A}{(\text{cosec } A - 1)} + \frac{\text{cosec } A}{(\text{cosec } A + 1)}$

$$= \frac{\text{cosec } A(\text{cosec } A+1) + \text{cosec } A(\text{cosec } A - 1)}{(\text{cosec } A-1)(\text{cosec } A+1)}$$

$$= \frac{\text{cosec}^2 A + \text{cosec } A + \text{cosec}^2 A - \text{cosec } A}{(\text{cosec}^2 A - 1)}$$

$$= \frac{2\text{cosec}^2 A}{1+\cot^2 A - 1} = \frac{2\text{cosec}^2 A}{\cot^2 A}$$

$= 2 \text{ cosec}^2 A \tan^2 A = 2(1 + \cot^2 A). \tan^2 A$

$= 2 \tan^2 A + 2 \tan^2 A. \cot^2 A$ (∵ tan A cot A = 1)

$= 2 + 2 \tan^2 A = 2(1 + \tan^2 A) = 2 \sec^2 A$ = RHS.

9. LHS = $(\sin\theta + \sec\theta)^2 + (\cos\theta + \text{cosec }\theta)^2$

$$= \left(\sin\theta + \frac{1}{\cos\theta}\right)^2 + \left(\cos\theta + \frac{1}{\sin\theta}\right)^2$$

$$=\left(\frac{\sin\theta\cos\theta+1}{\cos\theta}\right)^2+\left(\frac{\cos\theta\sin\theta+1}{\sin\theta}\right)^2$$

$$=\frac{(\sin\theta\cos\theta+1)^2}{\cos^2\theta}+\frac{(\sin\theta\cos\theta+1)^2}{\sin^2\theta}$$

$$=(\sin\theta\cos\theta+1)^2.\left(\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}\right)$$

$$=\left(\frac{\sin\theta\cos\theta+1}{\cos\theta\sin\theta}\right)^2=\left(1+\frac{1}{\cos\theta\sin\theta}\right)^2$$

$= (1 + \sec\theta \operatorname{cosec}\theta)^2 = \text{RHS}.$

10. In order to show that,

It is sufficient to show

$$\frac{1}{\operatorname{cosec}x+\cot x}+\frac{1}{\operatorname{cosec}x-\cot x}=\frac{1}{\sin x}+\frac{1}{\sin x}$$

$$\Rightarrow \frac{1}{(\operatorname{cosec}x+\cot x)}+\frac{1}{(\operatorname{cosec}x-\cot x)}=\frac{2}{\sin x} \quad ...(i)$$

Now, LHS of above is

$$\frac{1}{(\operatorname{cosec}x+\cot x)}+\frac{1}{(\operatorname{cosec}x-\cot x)}$$

$$=\frac{(\operatorname{cosec}x-\cot x)+(\operatorname{cosec}x+\cot x)}{(\operatorname{cosec}x-\cot x)(\operatorname{cosec}x+\cot x)}$$

$$=\frac{2\operatorname{cosec}x}{\operatorname{cosec}^2x-\cot^2x} \quad \left[\because (a+b)(a-b)=a^2-b^2\right]$$

$$=\frac{2\operatorname{cosec}x}{1}=\frac{2}{\sin x}=RHS \text{ of (i)}$$

Hence, $\frac{1}{\operatorname{cosec}x+\cot x}+\frac{1}{\operatorname{cosec}x-\cot x}=\frac{1}{\sin x}+\frac{1}{\sin x}$

Or $\frac{1}{(\operatorname{cosec}x+\cot x)}-\frac{1}{\sin x}=\frac{1}{\sin x}-\frac{1}{(\operatorname{cosec}x-\cot x)}.$

Case Study Answers:

1. **Answer:**

i. (d) $\frac{3}{4}$

Solution:

In $\Delta APQ,\ \tan\theta=\frac{AQ}{PQ}=\frac{1.2}{1.6}=\frac{3}{4}$

ii. (d) $\frac{3}{4}$

Solution:

In $\Delta PBQ,\ \cot B=\frac{QB}{PQ}=\frac{3}{1.6}=\frac{15}{8}$

iii. (c) $\frac{4}{3}$

Solution:

In $\Delta APQ,\ \tan A=\frac{PQ}{AQ}=\frac{1.6}{1.2}=\frac{4}{3}$

iv. (d) $\frac{5}{3}$

Solution:

We have, $\tan^2A+1=\sec^2A$

$$\Rightarrow \sqrt{\left(\frac{4}{3}\right)^2+1}=\sqrt{\frac{16}{9}+1}$$

$$=\sqrt{\frac{25}{9}}=\frac{5}{3}$$

v. (a) $\frac{17}{8}$

Solution:

Since, $\operatorname{cosec}B=\sqrt{\cot^2B+1}$

$$\Rightarrow \sqrt{\left(\frac{15}{8}\right)^2+1}=\frac{17}{8}$$

2. **Answer:**

i. (a) 30^o

Solution:

We have, AB = 9 cm, BC = $\sqrt{3}$ m in ΔABC, we have

$$\tan A=\frac{BC}{AB}=\frac{3\sqrt{3}}{9}=\frac{1}{\sqrt{3}}$$

$\Rightarrow \tan A = \tan 30^o \Rightarrow \angle A = 30^o$

ii. (c) 60^o

Solution:

Similarly,

$$\Rightarrow \tan C=\frac{AB}{BC}=\frac{9}{3\sqrt{3}}=\sqrt{3}$$

$\Rightarrow \tan C = \tan 60^o \Rightarrow \angle C = 60^o$

iii. (d) $6\sqrt{3}$ m

Solution:

Since, $\sin A=\frac{BC}{AC}\Rightarrow \sin 30^o=\frac{BC}{AC}$

$$\Rightarrow \frac{1}{2}=\frac{3\sqrt{3}}{AC}\Rightarrow AC=6\sqrt{3}\text{ m}$$

iv. (b) $\frac{1}{2}$

Solution:

∵ $\angle A = 30^o$

$\therefore \cos 2A = \cos(2\times 30^o) = \cos 60^o = \frac{1}{2}$

v. (b) $\frac{1}{2}$

Solution:

∵ $\angle C = 60^o$

$\therefore \sin\left(\frac{C}{2}\right) = \sin\left(\frac{60^o}{2}\right) = \sin 30^o = \frac{1}{2}$

Assertion Reason Answer-

1. (b) A is true, R is true; R is not a correct explanation for A.
2. (c) A is true; R is false.

Some Applications of Trigonometry 9

1. Trigonometric Ratios

Ratio of the sides of a right triangle with respect to the acute angles is called the **trigonometric ratios** of the angle.

Trigonometric ratios of acute angle A in right triangle ABC are given below:

i. $\sin \angle A = \frac{\text{side opposite to} \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{p}{h}$

ii. $\cos \angle A = \frac{\text{side adjacent to} \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{b}{h}$

iii. $\tan \angle A = \frac{\text{side opposite to} \angle A}{\text{side adjacent to} \angle A} = \frac{BC}{AB} = \frac{p}{b}$

iv. $\text{cosec} \angle A = \frac{\text{hypotenuse}}{\text{side opposite to} \angle A} = \frac{AC}{BC} = \frac{h}{p}$

v. $\sec \angle A = \frac{\text{hypotenuse}}{\text{side adjacent to} \angle A} = \frac{AC}{AB} = \frac{h}{b}$

vi. $\cot \angle A = \frac{\text{side adjacent to} \angle A}{\text{side opposite to} \angle A} = \frac{AB}{BC} = \frac{b}{p}$

The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.

2. Relation between trigonometric ratios

The ratios cosec A, sec A and cot A are the reciprocals of the ratios sin A, cos A and tan A respectively as given:

i. $cosec\theta = \frac{1}{\sin \theta}$

ii. $sec\theta = \frac{1}{\cos \theta}$

iii. $\tan\theta = \frac{\sin \theta}{\cos \theta}$

iv. $\cot\theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$

3. Values of Trigonometric ratios of some specific angles:

∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

4. Trigonometric ratios of complementary angles

Two angles are said to complementary angles if their sum is equal to 90°. Based on this relation, the trigonometric ratios of complementary angles are given as follows:

i. $\sin(90^\circ - A) = \cos A$

ii. $\cos(90^\circ - A) = \sin A$

iii. $\tan(90^\circ - A) = \cot A$

iv. $\cot(90^\circ - A) = \tan A$

v. $\sec(90^\circ - A) = \text{cosec } A$

vi. $\text{cosec}(90^\circ - A) = \sec A$

Note: $\tan 0^\circ = 0 = \cot 90^\circ$, $\sec 0^\circ = 1 = \text{cosec } 90^\circ$, $\sec 90^\circ$, $\text{cosec } 0^\circ$, $\tan 90^\circ$ and $\cot 0^\circ$ are not defined.

5. Basic trigonometric identities:

i. $\sin^2\theta + \cos^2\theta = 1$

ii. $1 + \tan^2\theta = \sec^2\theta;\ 0 \le \theta < 90^\circ$

iii. $1 + \cot^2\theta = \text{cosec}^2\theta;\ 0 \le \theta < 90^\circ$

6. The height or length of an object or the distance between two distant objects can be determined by the help of **trigonometric ratios**.

7. Line of sight

The **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.

8. Pythagoras theorem

It states that "In a right triangle, square of the hypotenuse is equal to the sum of the square of the other two sides".

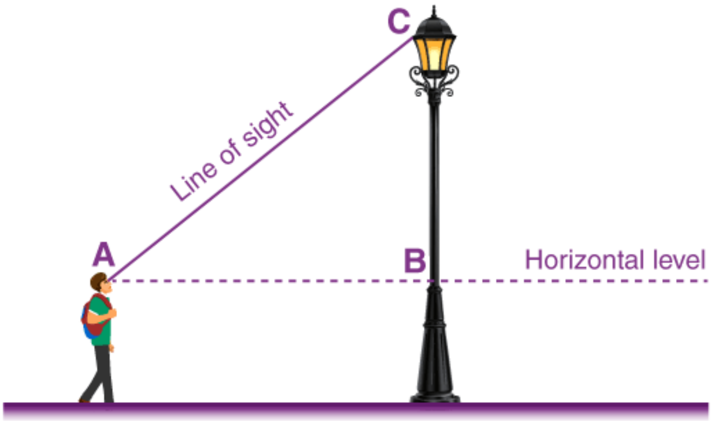
When any two sides of a right triangle are given, its third side can be obtained by using Pythagoras theorem.

9. Reflection from the water surface

In case of reflection from the water surface, the two heights above and below the ground level are equal in length.

10. Heights and Distances

Horizontal Level and Line of Sight



Line of sight and horizontal level

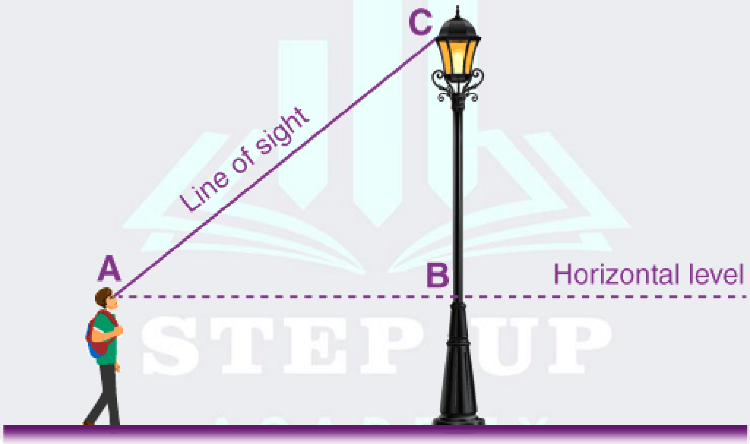
Line of sight is the line drawn from the eye of the observer to the point on the object viewed by the observer.

Horizontal level is the horizontal line through the eye of the observer.

Angle of elevation

The angle of elevation is relevant for objects above horizontal level.

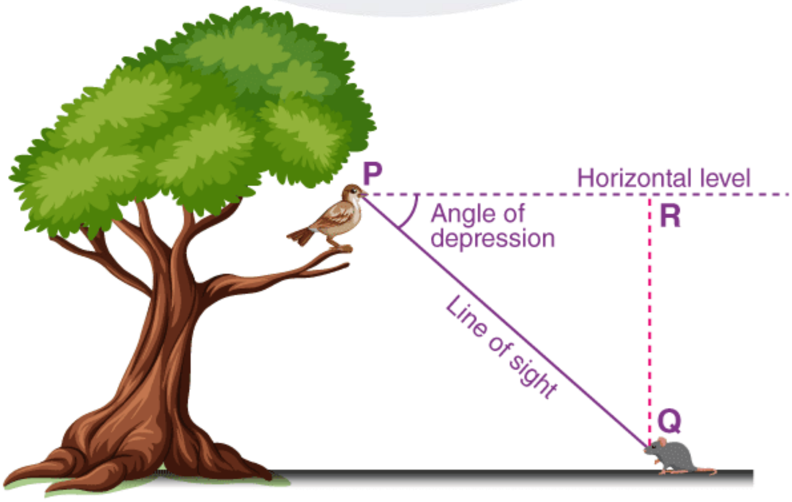
It is the angle formed by the line of sight with the horizontal level.



Angle of depression

The angle of depression is relevant for objects below horizontal level.

It is the angle formed by the line of sight with the horizontal level.



11. Calculating Heights and Distances

To, calculate heights and distances, we can make use of trigonometric ratios.

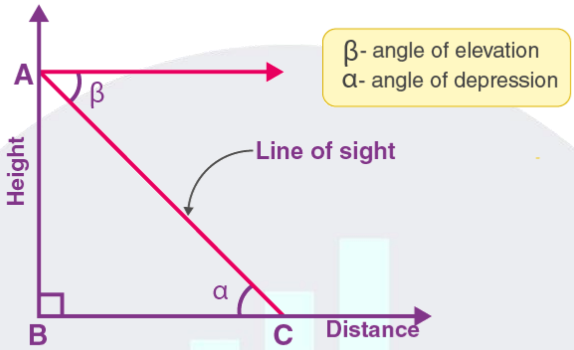
Step 1: Draw a line diagram corresponding to the problem.

Step 2: Mark all known heights, distances and angles and denote unknown lengths by variables.

Step 3: Use the values of various trigonometric ratios of the angles to obtain the unknown lengths from the known lengths.

Height and Distance in Trigonometry

The measurement of an object facing vertically is the height. Distance is defined as the measurement of an object from a point in a horizontal direction. If an imaginary line is drawn from the observation point to the top edge of the object, a triangle is formed by the vertical, horizontal and imaginary line.



From the figure, the point of observation is C. AB denotes the object's height. BC gives the distance between the object and the observer. The line of sight is given by AC. Angles alpha and beta represent the angle of elevation and depression respectively. If any of the two quantities are provided [a side or an angle], the remaining can be found using them. The law of alternate angles states that the magnitude of the angle of elevation and angle of depression are equal in magnitude. tan α = height / distance

12. Measuring the distances of Celestial bodies with the help of trigonometry

Large distances can be measured by the parallax method. The parallax angle is half the angle between two line of sights when an object is viewed from two different positions. Knowing the parallax angle and the distance between the two positions, large distances can be measured.

Solved Examples

Example 1: A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Solution:

Let A be the position of a kite at a height of 60 m above the ground.

Thus, AB = 60 m

Also, AC is the length of the string.

Angle of inclination = ∠ACB = 60

In right triangle ABC,

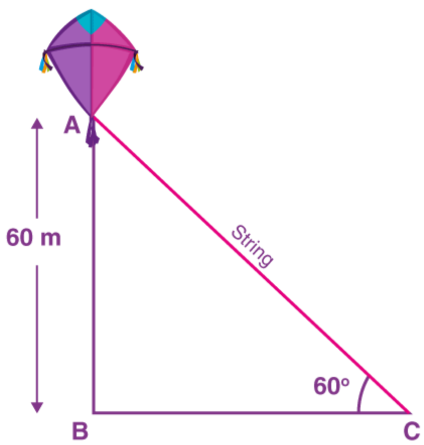
sin 60° = AB/AC

√3/2 = 60/AC

AC = (60 × 2)√/3

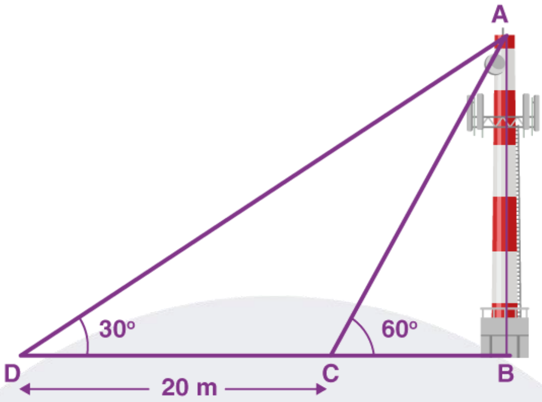
= (120 × √3)/(√3 × √3)

= (120√3)/3 = 40√3



Therefore, the length of the string is 40√3 m.

Example 2: A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° as shown in the figure. Find the height of the tower and the width of the canal.



Solution:

Given, AB is the height of the tower.

DC = 20 m (given)

In right ΔABD,

$\tan 30° = AB/BD$

$1/\sqrt{3} = AB/(20 + BC)$

$AB = (20 + BC)/\sqrt{3}$ ...(i)

In right ΔABC,

$\tan 60° = AB/BC$

$\sqrt{3} = AB/BC$

$AB = \sqrt{3}\ BC$ ...(ii)

From (i) and (ii),

$\sqrt{3}\ BC = (20 + BC)/\sqrt{3}$

$3\ BC = 20 + BC$

$2\ BC = 20$

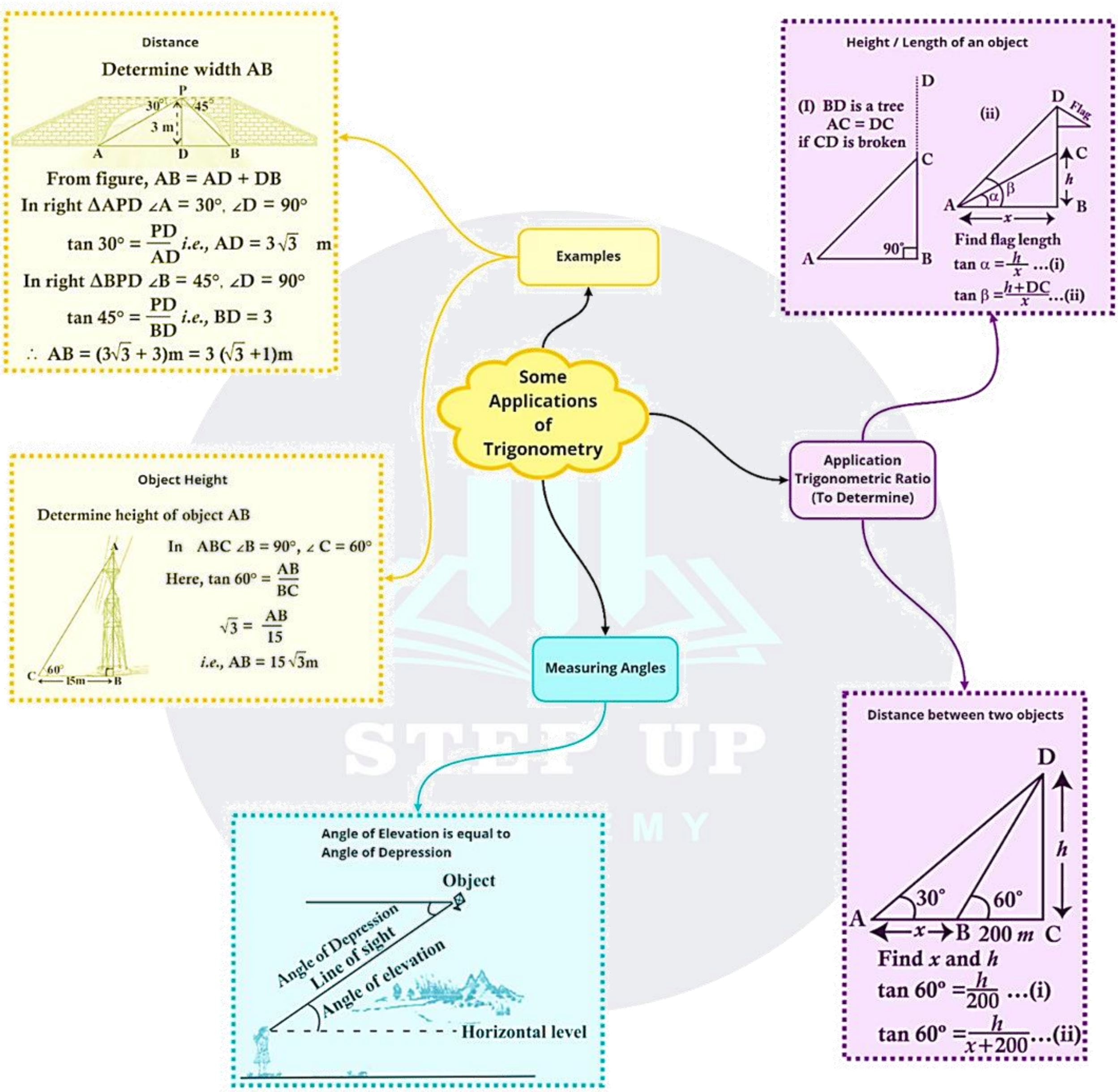
$BC = 10$

Substituting the value of BC in equation (ii),

$AB = (20 + 10)/\sqrt{3} = 30/\sqrt{3} = 10\sqrt{3}$

Therefore, the height of the tower is 10√3 m and the width of the canal is 10 m.

Class : 10th mathematics
Chapter- 9 : Some Applications of Trigonometry



Important Questions

Multiple Choice questions-

1. The tops of two poles of height 16m and 10m are connected by a wire. If the wire makes an angle of 60° with the horizontal, then the length of the wire is
   (a) 10m
   (b) 12m
   (c) 16m
   (d) 4√3m

2. A 20 m long ladder touches the wall at a height of 10 m. The angle which the ladder makes with the horizontal is
   (a) 45º
   (b) 30º
   (c) 90º
   (d) 60º

3. If the length of the shadow of a tower is √3 times that of its height, then the angle of elevation of the sun is
   (a) 30°
   (b) 45°
   (c) 60°
   (d) 75°

4. If sun's elevation is 60° then a pole of height 6 m will cast a shadow of length
   (a) 3√2 m
   (b) 6√3 m
   (c) 2√3 m
   (d) √3 m

5. The angle of elevation of top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 30°. The length of the tower is
   (a) √3 m (b) 2√3 m
   (c) 5√3m (d) 10√3 m

6. A contractor planned to install a slide for the children to play in a park. If he prefers to have a slide whose top is at a height of 1.5m and is inclined at an angle of 30° to the ground, then the length of the slide would be
   (a) 1.5m (b) 2√3m
   (c) √3m (d) 3m

7. When the length of shadow of a vertical pole is equal to √3 times of its height, the angle of elevation of the Sun's altitude is
   (a) 30°
   (b) 45°
   (c) 60°
   (d) 15°

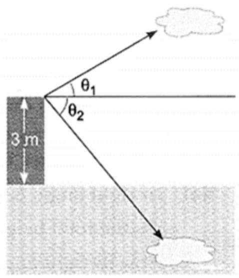
8. From a point P on the level ground, the angle of elevation of the top of a tower is 30°. If the tower is 100m high, the distance between P and the foot of the tower is
   (a) 100√3m
   (b) 200√3m
   (c) 300√3m
   (d) 150√3m

9. When the sun's altitude changes from 30° to 60°, the length of the shadow of a tower decreases by 70m. What is the height of the tower?
   (a) 35 m
   (b) 140 m
   (c) 60.6 m
   (d) 20.2 m

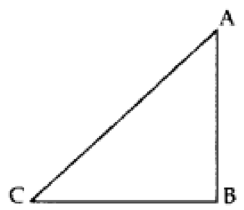
10. The _________ of an object is the angle formed by the line of sight with the horizontal when the object is below the horizontal level.
   (a) line of sight
   (b) angle of elevation
   (c) angle of depression
   (d) none of these

Very Short Questions:

1. If a man standing on a platform, 3 meters above the surface of a lake observes a cloud and its reflection in the lake, then the angle of elevation of the cloud is equal to the angle of depression of its reflection.



2. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then calculate the height of the wall.
3. In the given figure, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. Find the Sun's altitude.



4. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.
5. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun?
6. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find x : y
7. The height of a tower is 12 m. What is the length of its shadow when 10 Sun's altitude is 45°?
8. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°

Short Questions:

1. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.
2. A tree breaks due to storm and the broken part bends, so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.
3. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.
4. Determine the height of a mountain if the elevation of its top at an unknown distance from the base is 30° and at a distance 10 km further off from the mountain, along the same line, the angle of elevation is 15˚. (Use tan 15° = 0.27)
5. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it is 60°. Find the height of the tower.
6. From a point P on the ground, the angle of elevation of the top of a 10m tall building is 30°. A flag is hosted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45o. Find the length of the flagstaff and the distance of the building from the point P. (You may take $\sqrt{3}$ = 1.732).
7. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?
8. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.
9. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.
10. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 3 m from the banks, find the width of the river.

Long Questions:

1. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60°, respectively. Find the height of the tower.

2. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the bottom of the pedestal is 45°. Find the height of the pedestal.

3. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

OR

From the top of a 7 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower. (Use $\sqrt{3}$ = 1.732]

4. At a point, the angle of elevation of a tower is such that its tangent is $\frac{5}{12}$ On walking 240 m to the tower, the tangent of the angle of elevation becomes $\frac{3}{4}$. Find the height of the tower.

5. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.

6. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

7. In Fig. ABDC is a trapezium in which AB || CD. Line segments RN and LM are drawn parallel to AB such that AJ = JK = KP. If AB = 0.5 m and AP = BQ = 1.8 m, find the lengths of AC, BD, RN and LM.

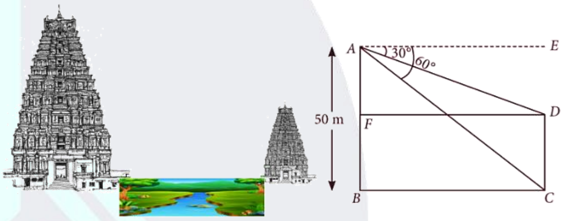
8. Two poles of equal heights are standing opposite to each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

9. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.

10. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40 metres away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river.

Case Study Answers:

1. There are two temples on each bank of a river. One temple is 50m high. A man, who is standing on the top of 50m high temple, observed from the top that angle of depression of the top and foot of other temple are 30º and 60º respectively. Take $\sqrt{3} = 1.73$



Based on the above information, answer the following questions.

i. Measure of ∠ADF is equal to:
 a. 45°
 b. 60°
 c. 30°
 d. 90°

ii. Measure of ∠ACB is equal to:
 a. 45°
 b. 60°
 c. 30°
 d. 90°

iii. Width of the river is:
 a. 28.90m
 b. 26.75m
 c. 25m
 d. 27m

iv. Height of the other temple is:
 a. 32.5m
 b. 35m
 c. 33.33m
 d. 40m

v. Angle of depression is always:

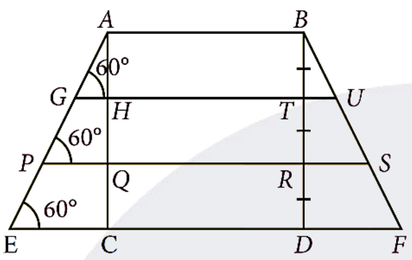
a. Reflex angle.

b. Straight.

c. An obtuse angle.

d. An acute angle.

2. Aditi purchase a wooden bar stool for her living room with square A top of side 2m and having height of 6m above the ground. Also each leg is inclined at an angle of 60º to the ground as shown in the figure (not drawn to scale).



Based on the above information, answer the following questions. Take $\sqrt{3} = 1.73$

i. Find the length of the each leg.

a. 5.9m

b. 6.93m

c. 7.3m

d. 8.2m

ii. Find the length of GH.

a. 0.53m

b. 1m

c. 1.15m

d. 2.73m

iii. The length of second step is:

a. 4.3m

b. 4.99m

c. 5.68m

d. 6.78m

iv. The length of PQ =

a. 1.56m

b. 2.31m

c. 3.34m

d. 5.68m

v. The length of first step is:

a. 4.78m

b. 5.34m

c. 6.62m

d. 7.82m

Assertion Reason Questions-

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both A and R are true and R is the correct explanation of A.

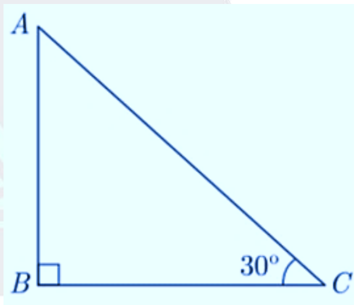
(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

Assertion: In the figure, if BC = 20 m, then height AB is 11.56 m

Reason: $\tan\theta = \frac{AB}{BC} = \frac{perpendicular}{base}$ where θ is the angle ∠ACB.



2. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

Assertion: If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45^0

Reason: According to pythagoras theorem, $h^2 = 1^2 + b^2$, where h = hypotenuse, 1 = length and b = base.

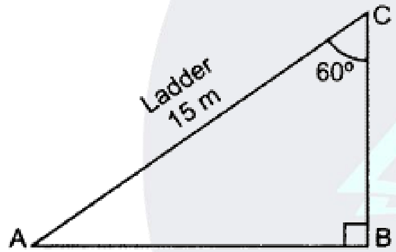
Answer Key

Multiple Choice questions:

1. (d) $4\sqrt{3}$ m
2. (b) 30º
3. (a) 30°
4. (c) $2\sqrt{3}$ m
5. (d) $10\sqrt{3}$ m
6. (d) 3m
7. (a) 30°
8. (a) $100\sqrt{3}$m
9. (c) 60.6 m
10. (c) angle of depression

Very Short Answer:

1. False, θ1 ≠ θ1
2.



$\angle BAC = 180° - 90° - 60° = 30°$

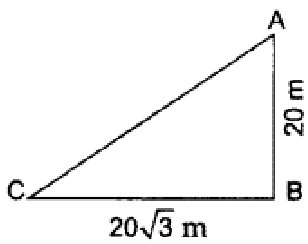
$\sin 30° = \frac{BC}{AC}$

$\frac{1}{2} = \frac{BC}{15}$

2BC = 15

$BC = \frac{15}{2} m$

3. AB = 20 m, BC = $20\sqrt{3}$ m,



θ = ?

In ΔABC,

$\frac{AB}{BC} = \tan\theta$

$\frac{20}{20\sqrt{3}} = \tan\theta$

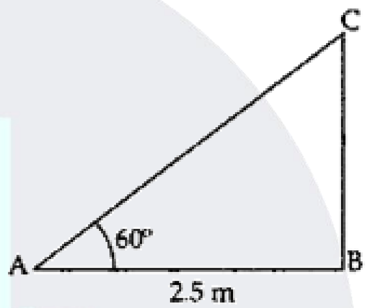
$\frac{1}{\sqrt{3}} = \tan\theta$

$\tan\theta = \tan 30^o \Rightarrow \quad \theta = 30^o$

4. Let AC be the ladder

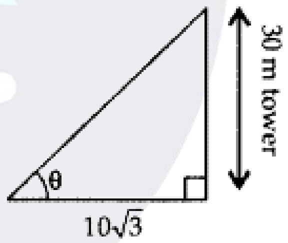
$\cos 60^o = \frac{AB}{AC}$

$\frac{1}{2} = \frac{2.5}{AC}$



∴ Length of ladder, AC = 5 m 2.5 m

5. Let required angle be θ.



$\tan\theta = \frac{30}{10\sqrt{3}}$

$\tan\theta = \sqrt{3}$

$\Rightarrow \tan\theta = \tan 60°$ ∴ $\theta = 60°$

6. When base is same for both towers and their heights are given, i.e., x and y respectively Let the base of towers be k.

$\tan 30^o = \frac{x}{k}$

$x = k\tan 30^o = \frac{k}{\sqrt{3}}$ ...(i)

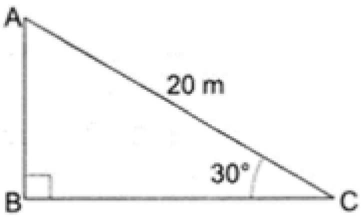
$\tan 60^o = \frac{y}{k}$

$y = k\tan 60^o = k\sqrt{3}$ ...(ii)

From equations (i) and (ii),

$$\frac{x}{y}=\frac{\frac{k}{\sqrt{3}}}{k\sqrt{3}}=\frac{k}{\sqrt{3}}\times\frac{1}{k\sqrt{3}}=\frac{1}{3}=1:3$$

7.



Let AB be the tower

Then, ∠C = 45°, AB = 12 m

$$\tan 45^o=\frac{AB}{BC}=\frac{12}{BC}$$

$$\Rightarrow\ 1=\frac{12}{BC}\quad\Rightarrow\quad BC = 12\text{ m}$$

∴ The length of the shadow is 12 m.

8. Let AB be the vertical pole and AC be the long rope tied to point C.

In right ΔABC, we have

$$\sin 30^o=\frac{AB}{BC}\Rightarrow\frac{1}{2}=\frac{AB}{20}$$

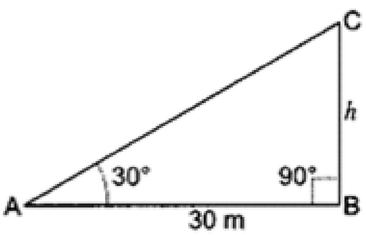
$$\Rightarrow\ \frac{20}{2}=AB\quad\Rightarrow\quad AB = 10\text{ m}$$

Therefore, height of the pole is 10 m.

Short Answer:

1. Let BC be the tower whose height is h metres and A be the point at a distance of 30 m from the foot of the tower. The angle of elevation of the top of the tower from point A is given to be 30°.

Now, in right angle ΔCBA we have,



$$\tan 30^o=\frac{BC}{AB}=\frac{h}{30}\Rightarrow\frac{1}{\sqrt{3}}=\frac{h}{30}$$

$$\Rightarrow h=\frac{30}{\sqrt{3}}=\frac{30}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{30\sqrt{3}}{3}=10\sqrt{3}m$$

Hence, the height of the tower is 10 √3 m.

2. In right angle ΔABC, AC is the broken part of the tree.

So, the total height of tree = (AB + AC)

Now in right angle ΔABC,

$$\tan 30^o=\frac{AB}{BC}\quad\Rightarrow\quad\frac{1}{\sqrt{3}}=\frac{AB}{8}$$

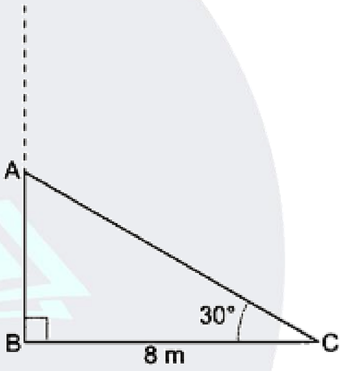
$$AB=\frac{8}{\sqrt{3}}$$

Again, $\cos 30^o=\frac{BC}{AC}$

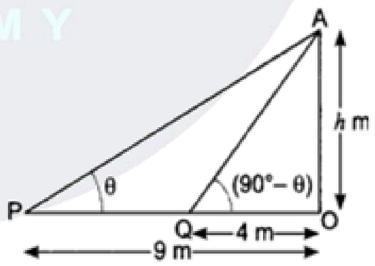
$$\Rightarrow\quad\frac{\sqrt{3}}{2}=\frac{8}{AC}\quad\Rightarrow\quad AC=\frac{16}{\sqrt{3}}$$

Hence, the height of the tree = AB + AC

$$=\frac{8}{\sqrt{3}}+\frac{16}{\sqrt{3}}=\frac{24}{\sqrt{3}}=\frac{24}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{24\sqrt{3}}{3}=8\sqrt{3}m$$



3.



$$\tan\theta=\frac{OA}{OP}=\frac{h}{9}$$

$$\Rightarrow\quad\tan\theta=\frac{h}{9}\quad...(i)$$

Again, in ΔAQO we have

$$\tan\left(90^o-\theta\right)=\frac{OA}{OQ}=\frac{h}{4}$$

$$\Rightarrow\quad\cot\theta=\frac{h}{4}\quad...(ii)$$

Multiplying (i) and (ii), we have

Let OA be the tower of height h meter and P, l be the two points at distance of 9 m and 4 m respectively from the base of the tower.

Now, we have OP = 9 m, OQ = 4 m,

Let ∠APO = θ, ∠AQO = (90° – θ)

and OA = h meter (Fig. 11.21)

Now, in ΔPOA, we have

$$\tan\theta \times \cot\theta = \frac{h}{9} \times \frac{h}{4}$$

$$\Rightarrow \quad 1 = \frac{h^2}{36} \quad \Rightarrow \quad h^2 = 36$$

$$h = \pm 6$$

Height cannot be negative.

Hence, the height of the tower is 6 meter.

4. Let AB be the mountain of height h kilometers. Let C be a point at a distance of x km, from the base of the mountain such that the angle of elevation of the top at C is 30°. Let D be a point at a distance of 10 km from C such that angle of elevation at D is of 15°.

In ΔABC,

$$\tan 60^o = \frac{AB}{BC} \quad or \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad x\sqrt{3} = h \quad \text{...(i)}$$

$$\text{In } \Delta ABD,\ \tan 30^o = \frac{AB}{BD}$$

$$\text{i.e.,} \quad \frac{1}{\sqrt{3}} = \frac{h}{x+40} \quad \text{...(ii)}$$

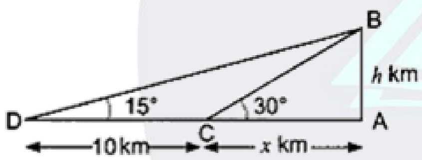
$$\tan 30^o = \frac{AB}{AC} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

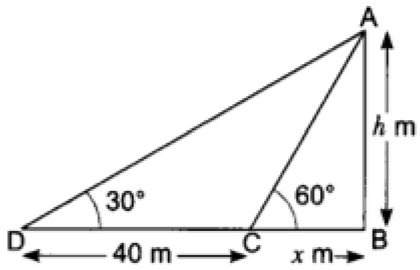
$$\Rightarrow \quad x = \sqrt{3}h$$

In ΔADB, we have

$$\tan 15^o = \frac{AB}{AD} \quad \Rightarrow \quad 0.27 = \frac{h}{x+10}$$

⇒ 0.27(x+10) = h ...(i)





Substituting x = √3h in equation (i), we get

⇒ 0.27 (√3h + 10) = h

= 0.27 × √3h + 0.27 × 10 = h

⇒ 2.7 = h – 0.27 × √3h

⇒ 27 = h (1 – 0.27 × √3)

⇒ 27 = h (1 – 0.46)

$$\Rightarrow h = \frac{2.7}{0.54} = 5$$

Hence, the height of the mountain is 5 km

5. In Fig. AB is the tower and BC is the length of the shadow when the Sun's altitude is 60°, i.e., the angle of elevation of the top of the tower from the tip of the shadow is 60° and DB is the length of the shadow, when the angle of elevation is 30°.

Now, let AB be h m and BC be x m.

According to the question, DB is 40 m longer than BC.

So, BD = (40 + x) m

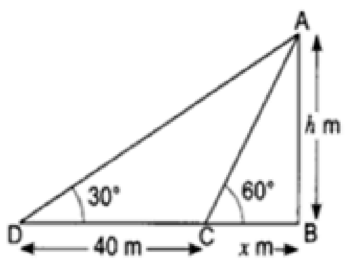
Now, we have two right triangles ABC and ABD.

$$\text{In } \Delta ABC,\ \tan 60^o = \frac{AB}{BC} \quad or \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad x\sqrt{3} = h \quad \text{...(i)}$$

$$\text{In } \Delta ABD,\ \tan 30^o = \frac{AB}{BD}$$

$$\text{i.e.,} \quad \frac{1}{\sqrt{3}} = \frac{h}{x+40} \quad \text{...(ii)}$$



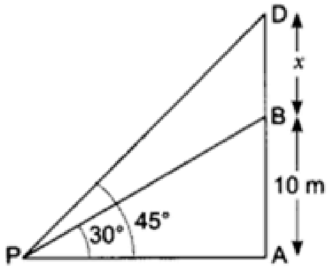
Using (i) in (ii), we get (x √3) √3 = x + 40, i.e., 3x = x + 40

i.e., x = 20

So, h = 20 √3 [From (i)]

Therefore, the height of the tower is 20 √3 m.

6.



In Fig. AB denotes the height of the building, BD the flagstaff and P the given point. Note that there are two right triangles PAB and PAD. We are required to find the length of the flagstaff, i.e., BD and the distance of the building from the point P, i.e., PA.

Since, we know the height of the building AB, we will first consider the right ΔPAB.

We have, $\tan 30^o = \dfrac{AB}{AP}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{10}{AP} \Rightarrow AP = 10\sqrt{3}$

i.e., the distance of the building from P is $10\sqrt{3}$ m = $10 \times 1.732 = 17.32$ m.

Next, let us suppose DB = x m. Then, AD = (10 + x)m.

Now, in right ΔPAD,

$$\tan 45^o = \frac{AD}{AP} = \frac{10+x}{10\sqrt{3}}$$

$$\Rightarrow 1 = \frac{10+x}{10\sqrt{3}} \quad \Rightarrow \quad 10\sqrt{3} = 10 + x$$

i.e., x = 100(√3 – 1) = 7.32

So, the length of the flagstaff is 7.32 m.

7. Let AC be a steep slide for elder children and DE be a slide for younger children. Then AB = 3 m and DB = 1.5 m.

Now, in right angle ΔDBE, we have

$$\sin 30^o = \frac{BD}{DE} = \frac{1.5}{DE}$$

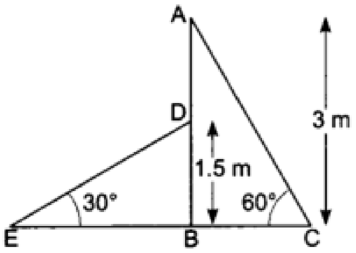
$$\Rightarrow \frac{1}{2} = \frac{1.5}{DE} \quad \therefore DE = 2 \times 1.5 = 3m$$

∴ Length of slide for younger children = 3 m

Again, in right angle ΔABC, we have

$$\sin 60^o = \frac{AB}{AC} \quad \Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{3}{AC}$$

$$\Rightarrow \quad AC = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}m$$



So, the length of slide for elder children is 2 √3 m.

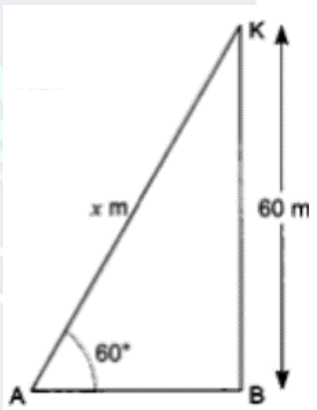
8. Let AB be the horizontal ground and K be the position of the kite and its height from the ground is 60 m and let length of string AK be x m.

∠KAB = 60°

Now, in right angle ΔABK we have

$$\sin 60^o = \frac{BK}{AK} = \frac{60}{x} \quad \Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{60}{x} \quad \Rightarrow \quad \sqrt{3}x = 120$$

$$\therefore \quad x = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{120\sqrt{3}}{3} = 40\sqrt{3}m$$



So, the length of string is 40 √3 m.

9. Let AB be the building and PQ be the initial position of the boy (Fig. 11.27) such that

∠APR = 30° and AB = 30 m

Now, let the new position of the boy be P'Q' at a distance QQ'.

Here, ∠AP'R = 60°

Now, in ΔARP, we have

$$\tan 30^o = \frac{AR}{PR} = \frac{AB - RB}{PR}$$

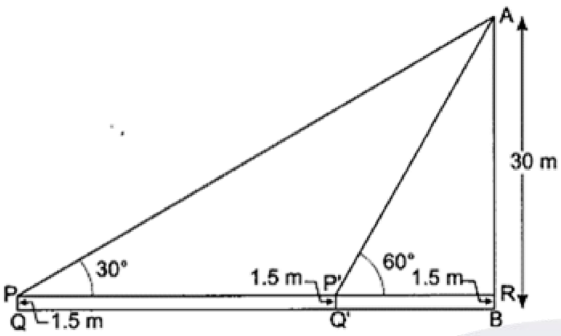
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{30 - 1.5}{PR} = \frac{28.5}{PR}$$

$$PR = 28.5 \times \sqrt{3}$$

Again, in ΔARP we have

$$\tan 60^o = \frac{AR}{P'R} \Rightarrow \sqrt{3} = \frac{28.5}{P'R}$$

$$P'R = \frac{28.5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{28.5\sqrt{3}}{3} = 9.5\sqrt{3}$$



Therefore, required distance, QQ = PP' = PR – P'R

= 28.5 √3 – 9.5 √3 = 19√3

Hence, distance walked by the boy is 19√3 m.

10. In Fig. A and B represent points on the bank on opposite sides of the river, so that AB is the width of the river. P is a point on the bridge at a height of 3 m, i.e., DP = 3m. We are interested to determine the width of the river, which is the length of the side AB of the ΔAPB.

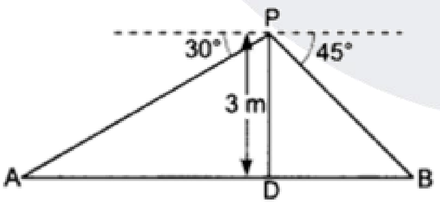
In right ΔADP, ∠A = 30°

So, $\tan 30^o = \frac{PD}{AD}$

i.e., $\frac{1}{\sqrt{3}} = \frac{3}{AD}$ or $AD = 3\sqrt{3}m$

Also, in right ΔPDB,

$$\frac{PD}{DB} = \tan 45^o \Rightarrow \frac{3}{DB} = 1$$



∴ DB = 3m

Now, AB = BD + AD = 3 + 3 √3 = 3 (1 + √3) m

Therefore, the width of the river is 3(√3 + 1) m.

Long Answer:

1. Let AB be a building of height 20 m and BC be the transmission tower of height x m and D be any point on the ground.

Here, ∠BDA = 45° and ∠ADC = 60°

Now, in ΔADC, we have

$$\tan 60^o = \frac{AC}{AD} \Rightarrow \sqrt{3} = \frac{x+20}{AD}$$

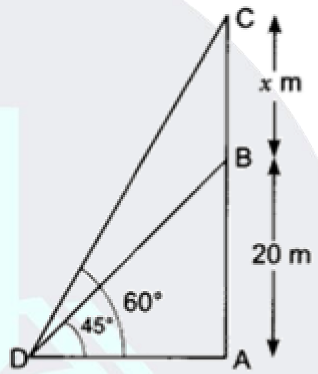
$$\Rightarrow AD = \frac{x+20}{\sqrt{3}} \quad ...(i)$$

Again, in ΔADB, we have $\tan 45^o = \frac{AB}{AD}$

$$\Rightarrow 1 = \frac{20}{AD} \Rightarrow AD = 20m \quad ...(ii)$$

Putting the value of AD in equation (i), we have

$$\Rightarrow 20 = \frac{x+20}{\sqrt{3}} \Rightarrow 20\sqrt{3} = x + 20$$



⇒ x = 20√3 – 20 = 20 (√3 – 1) = 20 (1.732 – 1)

= 20 × 0.732 = 14.64 m

Hence, the height of tower is 14.64 m.

2. Let AB be the pedestal of height h metres and BC be the statue of height 1.6 m.

Let D be any point on the ground such that,

∠BDA = 45° and ∠CDA = 60°

Now, in ΔBDA, we have

$$\tan 45^o = \frac{AB}{DA} = \frac{h}{DA} \Rightarrow 1 = \frac{h}{DA}$$

∴ DA = h

Again in ΔADC, we have

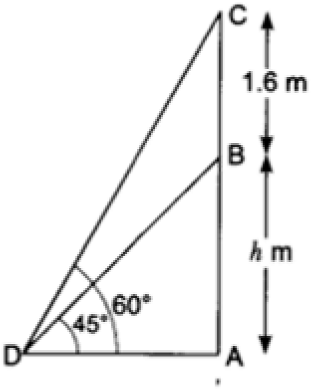
$$\tan 60^o = \frac{AC}{AD} = \frac{AB+BC}{AD}$$

$$\Rightarrow \sqrt{3} = \frac{h+1.6}{h} \quad \text{[From equation (i)]}$$

$$\Rightarrow \sqrt{3}h = h + 1.6 \Rightarrow (\sqrt{3}-1)h = 1.6$$

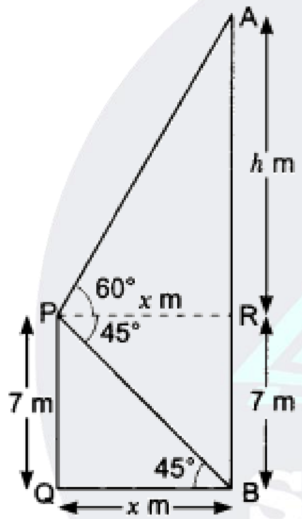
$$\therefore \quad h = \frac{1.6}{\sqrt{3}-1} = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{1.6(\sqrt{3}+1)}{3-1}$$

$$= \frac{1.6\left(\sqrt{3}+1\right)}{2} = 0.8\times\left(\sqrt{3}+1\right)m$$



Hence, height of the pedestal is 0.8 (√3 + 1) m.

3.



Let PQ be the building of height 7 metres and AB be the cable tower. Now it is given that the angle of elevation of the top A of the tower observed from the top P of building is 60° and the angle of depression of the base B of the tower observed from P is 45° (Fig. 11.38).

So, ∠APR = 60° and ∠QBP = 45°

Let QB = x m, AR = h m then, PR = x m

Now, in ΔAPR, we have

tan 60° = $\frac{AR}{PR}$

⇒ √3 = $\frac{h}{x}$

⇒ √3x = h

⇒ h = √3x ……(i)

Again, in ΔPBQ we have

tan 45° = $\frac{PQ}{QB}$

⇒ 1 = $\frac{7}{x}$

⇒ x = 7 ……(ii)

Putting the value of x in equation (i), we have

h = √3 × 7 = 7 √3

i.e., AR = 7 √3 metres

So, the height of tower = AB = AR + RB = 7 √3 + 7
= 7(√3 + 1) m.

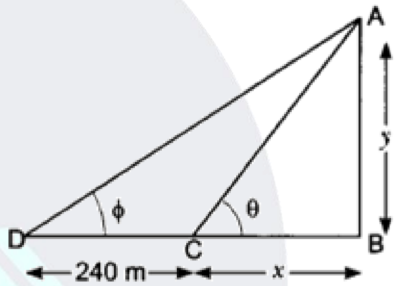
4. In the Fig. let AB be the tower, C and D be the positions of observation from where given that

$\tan\phi = \frac{5}{12}$ ...(i)

and $\tan\theta = \frac{3}{4}$ ...(ii)

Let BC = x m, AB = y m

Now in right-angled triangle ABC



$\tan\theta = \frac{y}{x}$ ...(iii)

From (ii) and (iii), we get $\frac{3}{4} = \frac{y}{x}$

⇒ $x = \frac{4}{3}y$ ...(iv)

Also in right-angled triangle ABD, we get

$\tan\phi = \frac{y}{x+240}$ ...(v)

From (i) and (v), we get

$\frac{5}{12} = \frac{y}{x+240} \Rightarrow 12y = 5x+1200$ ...(vi)

$\Rightarrow 12y = 5\times\frac{4}{3}y+1200$ (Using (iv))

$\Rightarrow 12y - \frac{20}{3}y = 1200$

$\Rightarrow \frac{36y-20y}{3} = 1200$

$\Rightarrow 16y = 3600 \Rightarrow y = \frac{3600}{16} = 225$

Hence, the height of the tower is 225 metres.

5. Let A and B be two positions of the balloon and G be the point of observation. (eyes of the girl)

Now, we have

AC = BD = BQ – DQ = 88.2 m – 1.2 m = 87 m .

∠AGC = 60°, ∠BGD = 30°

Now, in ΔAGC, we have

$$\tan 60^\circ = \frac{AC}{GC} \Rightarrow \sqrt{3} = \frac{87}{GC}$$

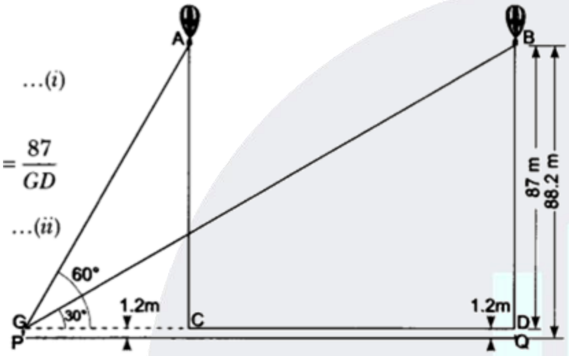
$$\Rightarrow GC = \frac{87}{\sqrt{3}} = \frac{87}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{87\sqrt{3}}{3}$$

$$\Rightarrow GC = 29 \times \sqrt{3}$$

...(i)

$= \frac{87}{GD}$

...(ii)



Hence, the balloon travels 58 √3 metres.

6. Let OA be the tower of height h, and P be the initial position of the car when the angle of depression is 30°.

After 6 seconds, the car reaches to such that the angle of depression at Q is 60°. Let the speed of the car be v metre per second. Then,

PQ = 6υ (∵ Distance = speed × time)

and let the car take t seconds to reach the tower OA from Q (Fig. 11.41). Then, OQ = υt metres.

Now, in ΔAQO, we have

$$\tan 60^o = \frac{OA}{QO}$$

$$\Rightarrow \sqrt{3} = \frac{h}{\upsilon t} \Rightarrow h = \sqrt{3}\upsilon t \qquad ...(i)$$

Now, in ΔAPQ, we have

$$\tan 30^o = \frac{OA}{PO}$$

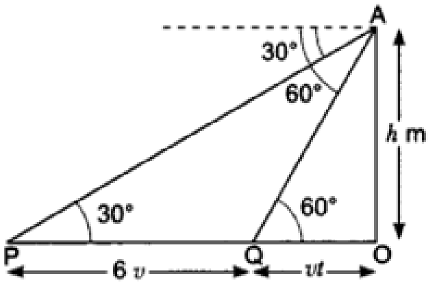
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{6\upsilon + \upsilon t} \Rightarrow \sqrt{3}h = 6\upsilon + \upsilon t \qquad ...(ii)$$

$$\sqrt{3} \times \sqrt{3}\upsilon t = 6\upsilon + \upsilon t$$

$$\Rightarrow 3\upsilon t = 6\upsilon + \upsilon t \Rightarrow 2\upsilon t = 6\upsilon$$

$$\Rightarrow t = \frac{6\upsilon}{2\upsilon} = 3$$

Now, substituting the value of h from (i) into (ii), we have



Hence, the car will reach the tower from Q in 3 seconds.

7. We have,

AP = 1.8 m

AJ = JK = KP = 0.6 m

AK = 2AJ = 1.2 m

In ΔARJ and ΔBNJ' we have

AJ = BJ, ∠ARJ = ∠BNJ = 60°

and ∠AJR = ∠BJ'N = 90°

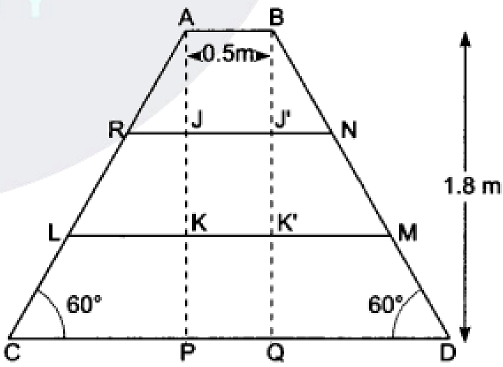
∴ ΔARJ ≅ ΔBNJ

⇒ RJ = NJ (By AAS congruence criterion)

Similarly, ΔALK ≅ ΔBMK"

⇒ LK = MK"

In ΔARJ,



$$\tan 60^o = \frac{AJ}{RJ} \Rightarrow \sqrt{3} = \frac{0.6}{RJ}$$

$$\Rightarrow RJ = \frac{0.6}{\sqrt{3}} = \frac{0.6\sqrt{3}}{3} = 0.2 \times 1.732 = 0.3464m$$

In ΔALK,

$$\tan 60^o = \frac{AK}{LK} \Rightarrow \sqrt{3} = \frac{1.2}{LK}$$

$$\Rightarrow \quad LK = \frac{1.2}{\sqrt{3}} = \frac{1.2\times\sqrt{3}}{3} = 0.4\times1.732 = 0.6928m$$

Since ΔACP ≅ ΔBDQ

So, BD = AC = 2.0784 m

Now, RN = RJ + JJ + J'N

= 2RJ + AB [∵ RJ = J' N and JJ = AB]

= 2 × 0.3464 +0.5 = 1.1928 m

Length of step LM = LK + KK + KM

= 2LK + AB [∵ LK = K M and KK = AB]

= 2 × 0.6928 + 0.5 = 1.8856 m

Thus, length of each leg = 2.0784 m = 2.1 m

Length of step RN = 1.1928 m = 1.2 m

and, length of step LM = 1.8856 m = 1.9 m

8. In ΔACP, $\sin 60^o = \frac{AP}{AC}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{1.8}{AC}$$

$$\Rightarrow \quad AC = \frac{3.6}{\sqrt{3}} = \frac{3.6\sqrt{3}}{3} = 1.2\times1.732 = 2.0784m$$

Let AB and CD be two poles of equal height h metre and let P be any point between the poles, such that

∠APB = 60° and ∠DPC = 30°

The distance between two poles is 80m.(Given)

Let AP = x m, then PC = (80 – x) m.

h'm Now, in ΔAPB, we have

$$\tan 60^o = \frac{AB}{AP} = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x} \quad \Rightarrow \quad h = \sqrt{3}x \qquad ...(i)$$

Again in ΔCPD, we have

$$\tan 60^o = \frac{AB}{AP} = \frac{h}{x}$$

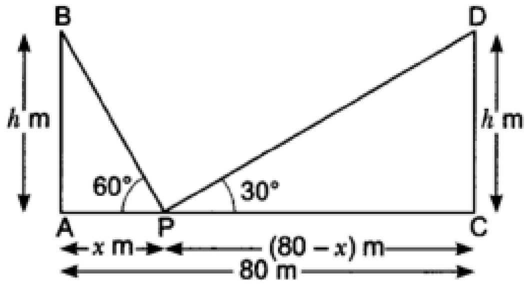
$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x \qquad ...(i)$$

$$\tan 30^o = \frac{DC}{PC} = \frac{h}{(80-x)}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{80-x} \quad \Rightarrow h = \frac{80-x}{\sqrt{3}} \quad ...(ii)$$

$$\sqrt{3}x = \frac{80-x}{\sqrt{3}} \Rightarrow 3x = 80-x$$

$$\Rightarrow \quad 4x = 80 \Rightarrow x = \frac{80}{4} = 20m$$



Now, putting the value of x in equation (i), we have

h = √3 × 20 = 20 √3

Hence, the height of the pole is 20 √3 m and the distance of the point from first pole is 20 m and that of the second pole is 60 m.

9. Let height of the tower be h metres and width of the canal be x metres, so AB = h m and BC = x m

Now in ΔABC, we have

$$\tan 60^o = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x \qquad ...(i)$$

Now, in ΔADB we have

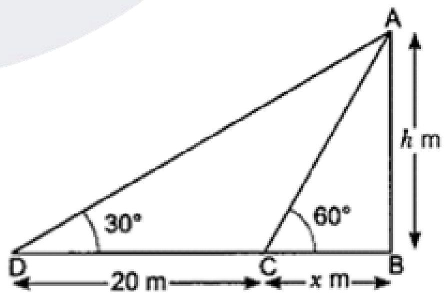
$$\tan 30^o = \frac{AB}{DB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20+x}$$

$$\Rightarrow 20 + x = \sqrt{3}h \qquad ...(ii)$$

From (i) and (ii), we have

$$20 + x = \sqrt{3}\times\sqrt{3}x \quad \Rightarrow \quad 20 + x = 3x$$

$$\Rightarrow \quad 20 = 3x - x \quad \Rightarrow x = \frac{20}{2} = 10m$$



Now, putting the value of x in equation (i), we have

h = √3 × 10 = 10√3

⇒ h = 10 73 m

Hence, height of the tower is 10 √3 m and width of the canal is 10 m.

10. Let AB be the tree of height metres standing on the bank of a river. Let C be the position of man standing on the opposite bank of the river such that BC = x m. Let D be the new position of the man. It is given that CD = 40 m and the angles of elevation of the top of the tree at C and D are 60° and 30°, respectively, i.e.,

$\angle ACB = 60°$ and $\angle ADB = 30°$.

In ΔACB, we have

$$\tan 60^o = \frac{AB}{BC} \Rightarrow \tan 60^o = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}} \quad ...(i)$$

In ΔADB, we have

$$\tan 30^o = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+40}$$

$$\Rightarrow \sqrt{3}h = x + 40 \quad ...(ii)$$

Substituting $x = \frac{h}{\sqrt{3}}$ in equation (ii), we get

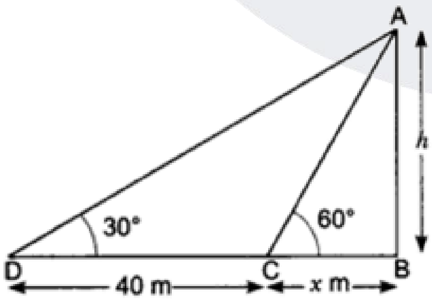
$$\sqrt{3}h = \frac{h}{\sqrt{3}} + 40 \Rightarrow \sqrt{3}h - \frac{h}{\sqrt{3}} = 40$$

$$\Rightarrow \frac{3h-h}{\sqrt{3}} = 40 \Rightarrow \frac{2h}{\sqrt{3}} = 40$$

$$\Rightarrow h = \frac{40 \times \sqrt{3}}{2}$$

$$\Rightarrow h = 20\sqrt{3} = 20 \times 1.732 = 34.64m$$

Substituting h in equation (i), we get $x = \frac{20\sqrt{3}}{\sqrt{3}}$ metres = 20 metres



Hence, the height of the tree is 34.64 m and width of the river is 20 m.

Case Study Answers:

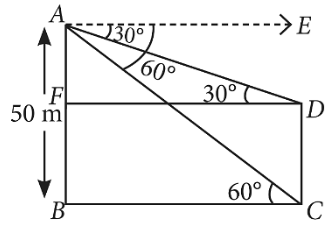
1. Answer:

i. (c) 30°

Solution:

Since, AE || FD

$\therefore \angle EAD = \angle ACB = 30°$



ii. (b) 60°

Solution:

Since, AE || BC

$\therefore \angle EAC = \angle ACB = 60°$

iii. (a) 28.90 m

Solution:

In ΔABC,

$$\tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{50}{BC}$$

$$\Rightarrow BC = \frac{50}{\sqrt{3}} = 28.90\,m$$

iv. (c) 33.33 m

Solution:

In ΔADF,

$$\tan 30° = \frac{AF}{FD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB - BF}{FD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{50 - CD}{\frac{50}{\sqrt{3}}}$$

$$\left[\because FD = BC = \frac{50}{\sqrt{3}}\right]$$

$$\Rightarrow \frac{50}{3} = 50 - CD$$

$$\Rightarrow CD = 50 - \frac{50}{3} = \frac{100}{3} = 33.33 \text{ m}$$

v. (d) An acute angle.

2. Answer:

Given, side of square top = 2m

∴ Given, side of square top = 2m

Also, AC and BD are perpendicular to the ground.Also,

So, AH= HQ = QC (By B.P.T. Theorem)

i. (b) 6.93 m

Solution:

In ΔAEC,

$$\sin 60° = \frac{AC}{AE} \Rightarrow \frac{\sqrt{3}}{2} = \frac{6}{AE}$$

$\Rightarrow AE = 6.93\text{m}$

$\therefore$ Length of each leg i.e., AE = BF = 6.93 m.

ii. (c) 1.15 m

Solution:

In ΔAGH,

$$\tan 60° = \frac{AH}{GH} \Rightarrow \sqrt{3} = \frac{2}{GH}$$

$\Rightarrow GH = 1.15\text{m}$

iii. (a) 4.3 m

Solution:

Length of second step = GH + HT + TU

= 1.15 + 2 + 1.15 = 4.3 m

iv. (b) 2.31 m

Solution:

In ΔAPQ,

$$\tan 60° = \frac{AQ}{PQ} \Rightarrow \sqrt{3} = \frac{4}{PQ}$$

$$\Rightarrow PQ = \frac{4}{\sqrt{3}}\text{m} = 2.31\text{m}$$

v. (c) 6.62 m

Solution:

Length of first step = PQ + QR + RS

= 2.31 + 2 + 2.31 = 6.62 m

Assertion Reason Answer-

1. (a) Both A and R are true and R is the correct explanation of A.
2. (b) Both A and R are true and R is not the correct explanation of A.

Circles 10

Introduction to Circle

A **circle** is the locus of a point which lies in the plane in such a manner that its distance from a fixed point in the plane is constant. The fixed point is called the **centre** and the constant distance is called the **radius** of the circle.

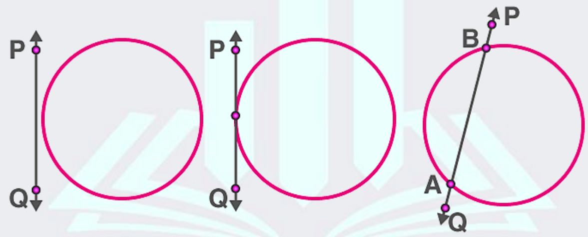
Circle and line in a plane

For a circle and a line on a plane, there can be three possibilities.

they can be non-intersecting

they can have a single common point: in this case, the line touches the circle.

they can have two common points: in this case, the line cuts the circle.

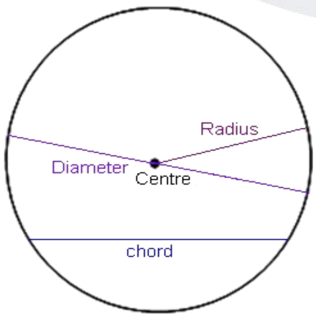


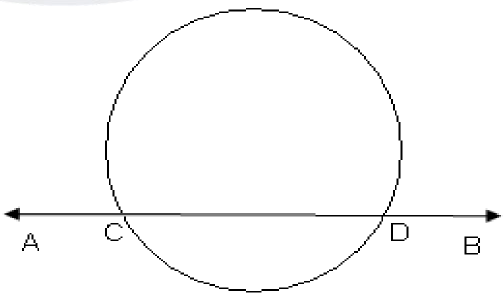
(i) Non intersecting (ii) Touching (iii) Intersecting

1. Parts of the circle

- A line segment that joins any two points lying on a circle is called the **chord** of the circle.
- A chord passing through the centre of the circle is called **diameter** of the circle.
- A line segment joining the centre and a point on the circle is called **radius** of the circle.
- A line which intersects a circle at two distinct points is called a **secant** of the circle.

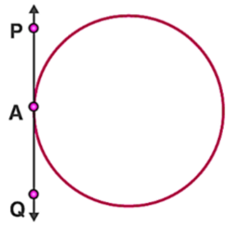
In the figure, AB is a secant to the circle.





2. Tangent to the circle

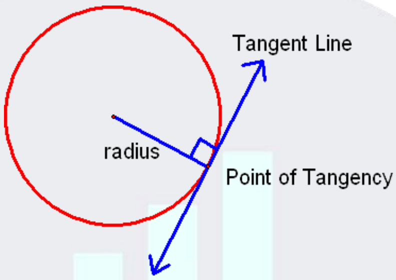
A **tangent** to the circle is a line that intersects the circle (touches the circle) at only one point. The word 'tangent' comes from the Latin word 'tangere', which means to touch. The common point of the circle and the tangent is called **point of contact.**



In the figure, AB is a tangent to the circle and P is the point of contact.

3. **Important facts about tangent**

- The tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincide.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact. This point of contact is also called as point of tangency.



- A line drawn through the end of a radius (point on circumference) and perpendicular to it is a tangent to the circle.

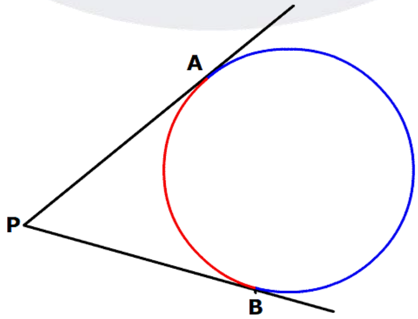
4. **Number of tangents on a circle**

- There is no tangent possible to a circle from the point (or passing through a point) lying inside the circle.
- There are **exactly two tangents** possible to a circle **through a point outside the circle**.
- At any point on the circle, there can be one and only one tangent possible.

5. **Length of the tangent**

The length of the segment of the tangent from the external point P and the point of contact with the circle is called the **length of the tangent**.

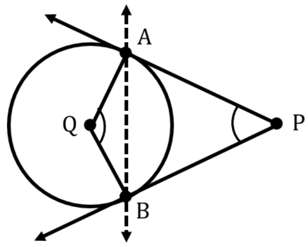
- The lengths of tangents drawn from an external point to the circle are equal.
- The figure shows two equal tangents ($PA = PB$) from an external point P.



6. **Angle between two tangents from an external point**

- The centre of a circle lies on the bisector of the angle between the two tangents drawn from an external point.

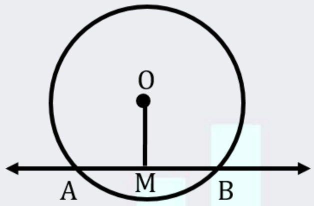
- Angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.



In the figure, angle P and angle Q are supplementary.

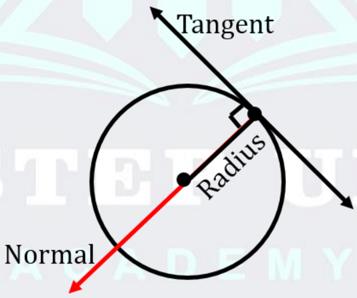
7. Perpendicular from the centre

Perpendicular drawn from the centre to any chord of the circle, divides it into two equal parts. In the figure, OM is perpendicular to AB and AM = MB.



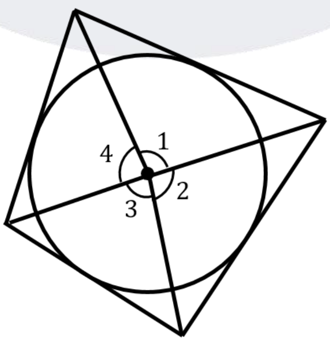
8. Normal to the circle

The line containing the radius through the point of contact is called the normal to the circle at that point.



9. Inscribed circle

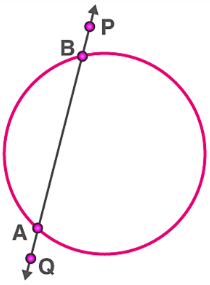
Opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.



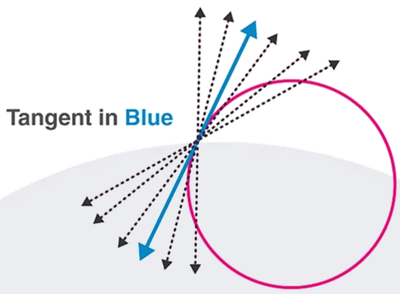
In the figure, angles 1 and 3 are supplementary. Accordingly, angles 2 and 4 are supplementary.

Secant

A secant to a circle is a line that has two points in common with the circle. It cuts the circle at two points, forming a chord of the circle.



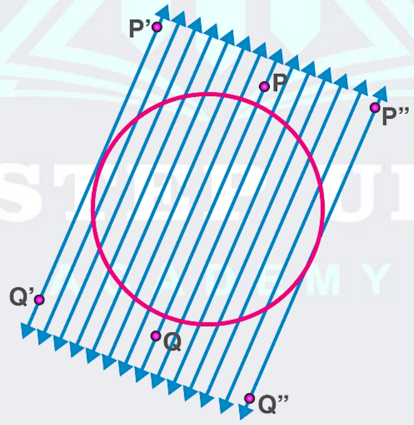
Tangent as a special case of Secant



The tangent to a circle can be seen as a special case of the secant when the two endpoints of its corresponding chord coincide.

Two parallel tangents at most for a given secant

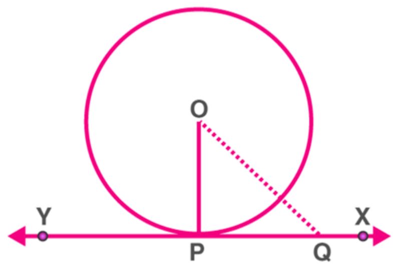
For every given secant of a circle, there are exactly two tangents which are parallel to it and touches the circle at two diametrically opposite points.



Theorems

Tangent perpendicular to the radius at the point of contact

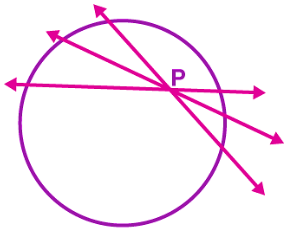
Theorem: The theorem states that "the tangent to the circle at any point is the perpendicular to the radius of the circle that passes through the point of contact".



Here, O is the centre and OP⊥XY.

The number of tangents drawn from a given point

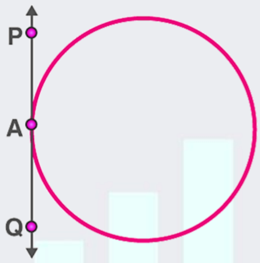
If the point is in an interior region of the circle, any line through that point will be a secant. So, no tangent can be drawn to a circle which passes through a point that lies inside it.



No tangent can be drawn to a circle from a point inside it

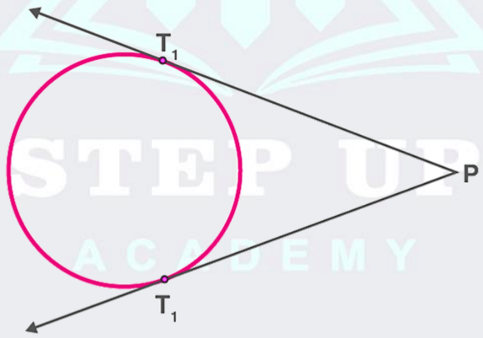
AB is a secant drawn through the point S

When a point of tangency lies on the circle, there is exactly one tangent to a circle that passes through it.



A tangent passing through a point lying on the circle

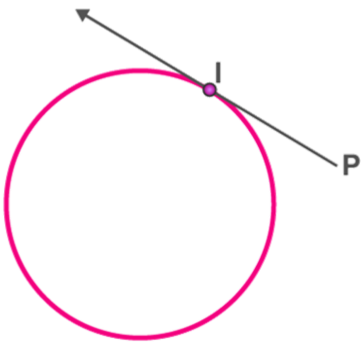
When the point lies outside of the circle, there are accurately two tangents to a circle through it



Tangents to a circle from an external point

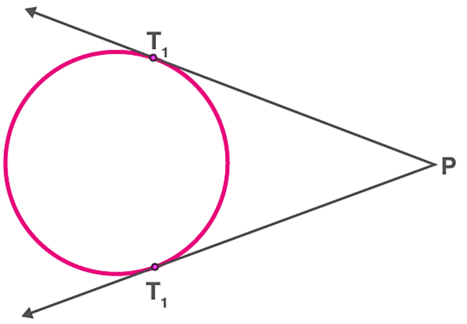
Length of a tangent

The length of the tangent from the point (Say P) to the circle is defined as the segment of the tangent from the external point P to the point of tangency I with the circle. In this case, PI is the tangent length.



Lengths of tangents drawn from an external point

Theorem: Two tangents are of equal length when the tangent is drawn from an external point to a circle.



$PT_1 = PT_2$

Thus, the two important theorems in Class 10 Maths Chapter 10 Circles are:

Theorem 10.1: The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Theorem 10.2: The lengths of tangents drawn from an external point to a circle are equal.

Interesting facts about Circles and its properties are listed below:

In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

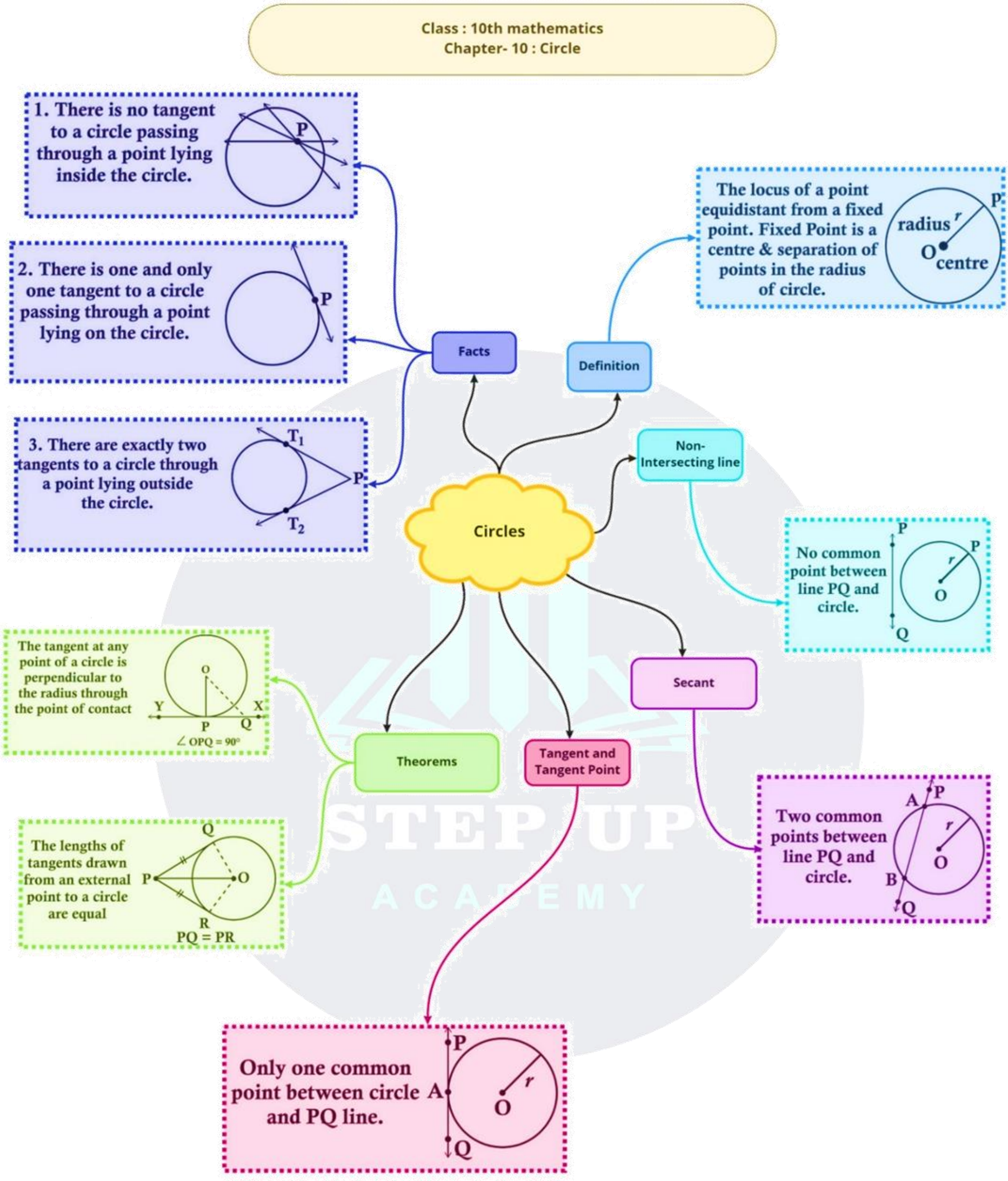
The tangents drawn at the ends of a diameter of a circle are parallel.

The perpendicular at the point of contact to the tangent to a circle passes through the centre.

The angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

The parallelogram circumscribing a circle is a rhombus.

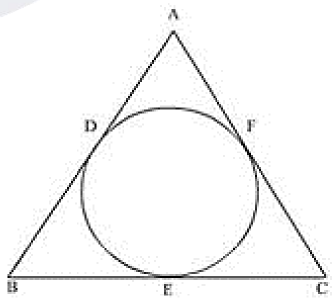
The opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.


Class : 10th mathematics
Chapter- 10 : Circle
Circles
Facts
1. There is no tangent to a circle passing through a point lying inside the circle.
P
2. There is one and only one tangent to a circle passing through a point lying on the circle.
P
3. There are exactly two tangents to a circle through a point lying outside the circle.
T1
T2
P
Definition
The locus of a point equidistant from a fixed point. Fixed Point is a centre & separation of points in the radius of circle.
radius
r
O
centre
P
Non-Intersecting line
No common point between line PQ and circle.
P
Q
r
O
Secant
Two common points between line PQ and circle.
A
B
P
Q
r
O
Theorems
The tangent at any point of a circle is perpendicular to the radius through the point of contact
O
Y
P
Q
X
∠ OPQ = 90°
The lengths of tangents drawn from an external point to a circle are equal
Q
P
O
R
PQ = PR
Tangent and Tangent Point
Only one common point between circle and PQ line.
P
A
O
r
Q

Important Questions

Multiple Choice Questions:

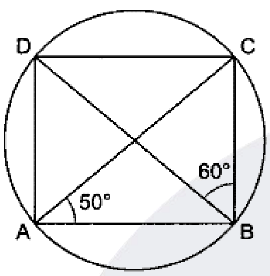
1. Two circle touch each other externally at C and AB is a common tangent to the circles. Then, ∠ACB =
   (a) 60°
   (b) 45°
   (c) 30°
   (d) 90°
2. If TP and TQ are two tangents to a circle with centre O so that ∠POQ = 110°, then, ∠PTQ is equal to
   (a) 60°
   (b) 70°
   (c) 80°
   (d) 90°
3. Tangents from an external point to a circle are
   (a) equal
   (b) not equal
   (c) parallel
   (d) perpendicular
4. Two parallel lines touch the circle at points A and B respectively. If area of the circle is 25 n cm^2, then AB is equal to
   (a) 5 cm
   (b) 8 cm
   (c) 10 cm
   (d) 25 cm
5. A line through point of contact and passing through centre of circle is known as
   (a) tangent
   (b) chord
   (c) normal
   (d) segment
6. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q
   (a) $\sqrt{119}$ cm
   (b) 13 cm
   (c) 12 cm
   (d) 8.5 cm
7. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is
   (a) 60 cm^2
   (b) 65 cm^2
   (c) 30 cm^2
   (d) 32.5 cm^2
8. At point A on a diameter AB of a circle of radius 10 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY at a distance 16 cm from A is
   (a) 8 cm
   (b) 10 cm
   (c) 16 cm
   (d) 18 cm
9. The tangents drawn at the extremities of the diameter of a circle are
   (a) perpendicular
   (b) parallel
   (c) equal
   (d) none of these
10. A circle is inscribed in a ΔABC having AB = 10cm, BC = 12cm and CA = 8cm and touching these sides at D, E, F respectively. The lengths of AD, BE and CF will be



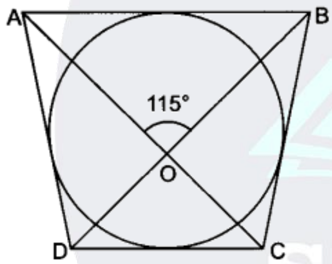
   (a) AD = 4cm, BE = 6cm, CF = 8cm
   (b) AD = 5cm, BE = 9cm, CF = 4cm
   (c) AD = 3cm, BE = 7cm, CF = 5cm
   (d) AD = 2cm, BE = 6cm, CF = 7cm

Very Short Questions:

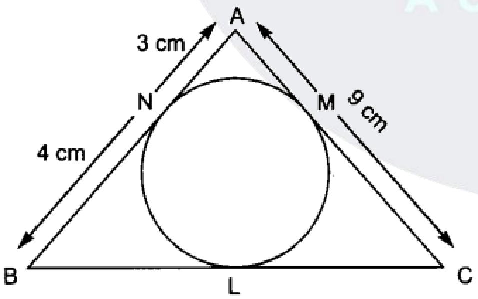
1. If a point P is 17 cm from the centre of a circle of radius 8 cm, then find the length of the tangent drawn to the circle from point P.
2. The length of the tangent to a circle from a point P, which is 25 cm away from the centre, is 24 cm. What is the radius of the circle?
3. In Fig, ABCD is a cyclic quadrilateral. If ∠BAC = 50° and ∠DBC = 60° then find ∠BCD.



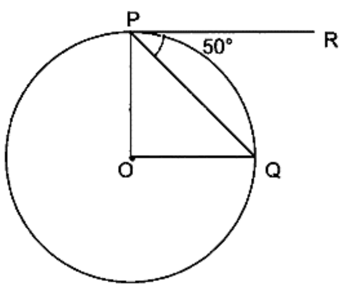
4. In Fig. the quadrilateral ABCD circumscribes a circle with centre O. If ∠AOB = 115°, then find ∠COD.



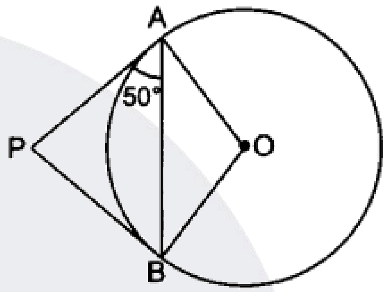
5. In Fig. AABC is circumscribing a circle. Find the length of BC.



6. In Fig. O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find ∠POQ.

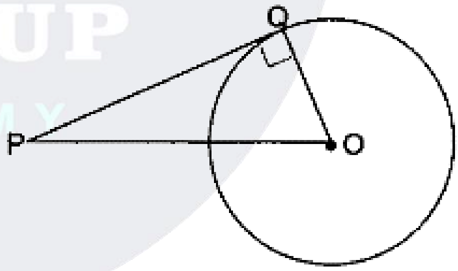


7. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.
8. If radii of two concentric circles are 4 cm and 5 cm, then find the length of each chord of one circle which is tangent to the other circle.
9. PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that ∠POR = 120° then find ∠OPQ.
10. From an external point P, tangents PA and PB are drawn to a circle with centre O. If ∠PAB = 50°, then find ∠AOB.

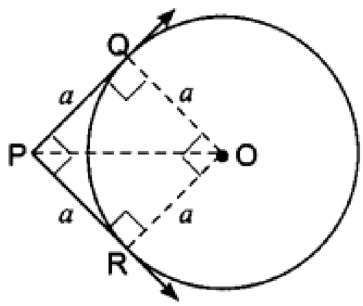


Short Questions:

1. AB is a diameter of a circle and AC is its chord such that ∠BAC = 30°. If the tangent at C intersects AB extended at D, then BC = BD.
2. The length of tangent from an external point P on a circle with centre O is always less than OP.

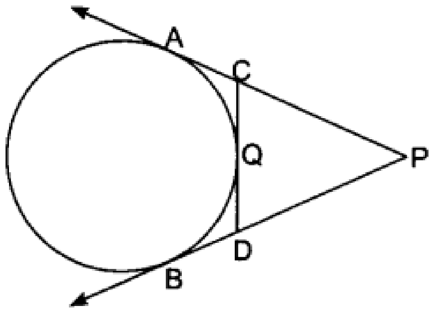


3. If angle between two tangents drawn from a point P to a circle of radius 'a' and centre 0 is 90°, then OP = a√2.

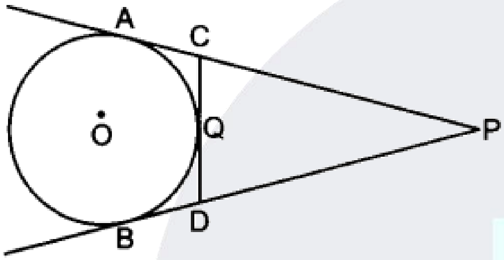


4. In Fig. PA and PB are tangents to the circle drawn from an external point P. CD is the third tangent

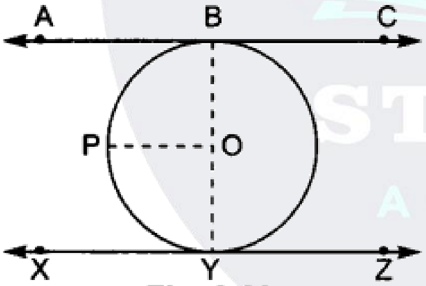
touching the circle at Q. If PA = 15 cm, find the perimeter of ΔPCD.



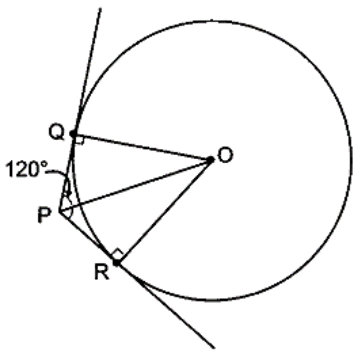
5. In Fig. PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If PA = 12 cm, QC = QD = 3 cm, then find PC + PD.



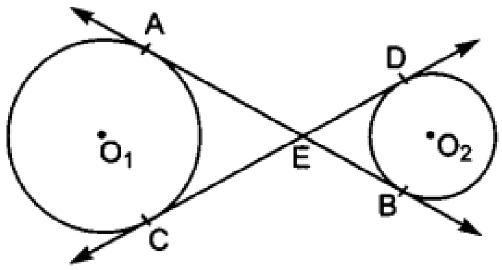
6. Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre.



7. If from an external point P of a circle with centre 0, two tangents PQ and PR are drawn such that QPR = 120°, prove that 2PQ = PO.



8. In Fig. common tangents AB and CD to two circles with centres , and 0, intersect at E. Prove that AB = CD.

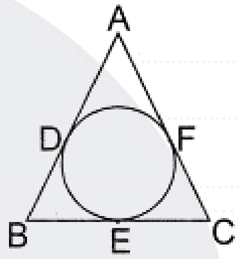


9. The incircle of an isosceles triangle ABC, in which AB = AC, touches the sides BC, CA and AB at D, E and F respectively. Prove that BD = DC.

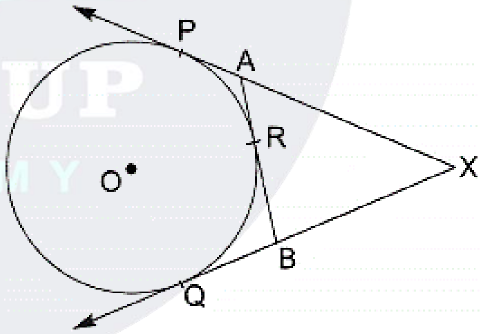
OR

In Fig. if AB = AC, prove that BE = EC.

[Note: D, E, F replace by F, D, E]

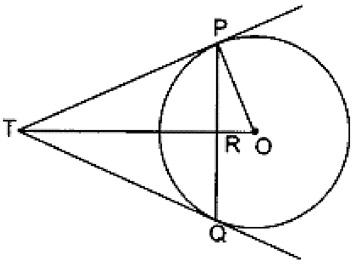


10. In Fig. XP and XQ are two tangents to the circle with centre O, drawn from an external point X. ARB is another tangent, touching the circle at R. Prove that XA + AR = XB + BR.

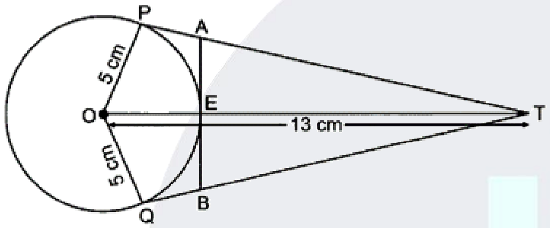


Long Questions:

1. Prove that the tangent to a circle is perpendicular to the radius through the point of contact.
2. Prove that the lengths of two tangents drawn from an external point to a circle are equal.
3. Prove that the parallelogram circumscribing a circle is a rhombus.
4. In Fig. PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at P and Q intersect at a point T. Find the length of TP.

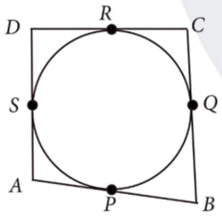


5. If PQ is a tangent drawn from an external point P to a circle with centre O and QOR is a diameter where length of QOR is 8 cm such that ∠POR = 120°, then find OP and PQ.
6. In Fig. O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



Case Study Questions:

1. In a park, four poles are standing at positions A, B, C and D around the fountain such that the cloth joining the poles AB, BC, CD and DA touches the fountain at P, Q, R and S respectively as shown in the figure.



Based on the above information, answer the following questions.

i. If O is the centre of the circular fountain, then ∠OSA
 a. 60º
 b. 90º
 c. 45º
 d. None of these

ii. Which of the following is correct?
 a. AS = AP
 b. P = BQ
 c. CQ = CR
 d. All of these

iii. If DR = 7cm and AD= 11cm, then AP =
 a. 4cm
 b. 18cm
 c. 7cm
 d. 11cm

iv. If O is the centre of the fountain, with ∠QCS=60º, then ∠QOS
 a. 60º
 b. 120º
 c. 90º
 d. 30º

v. Which of the following is correct?
 a. AB + BC = CD + DA
 b. AB + AD = BC + CD
 c. AB + CD = AD + BC
 d. All of these

2. Smita always finds it confusing with the concepts of tangent and secant of a circle. But this time she has determined herself to get concepts easier. So, she started listing down the differences between tangent and secant of a circle, along with their relation. Here, some points in question form are listed by Smita in her notes. Try answering them to clear your concepts also.

i. A line that intersects a circle exactly at two points is called:
  a. Secant
  b. Tangent
  c. Chord
  d. Both (a) and (b)

ii. Number of tangents that can be drawn on a circle is:
  a. 1
  b. 0
  c. 2
  d. Infinite

iii. Number of tangents that can be drawn to a circle from a point not on it, is:
  a. 1
  b. 2
  c. 0
  d. Infinite

iv. Number of secants that can be drawn to a circle from a point on it is:
  a. Infinite
  b. 1
  c. 2
  d. 0

v. A line that touches a circle at only one point is called:
  a. Secant
  b. Chord
  c. Tangent
  d. Diameter

Assertion Reason Questions:

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
  a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
  b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
  c. Assertion (A) is true but reason (R) is false.
  d. Assertion (A) is false but reason (R) is true.

  Assertion (A): In a circle of radius 6 cm, the angle of a sector is 60°. Then the area of the sector is 132/7 cm^2.

  Reason (R): Area of the circle with radius r is πr^2

2. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
  a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
  b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
  c. Assertion (A) is true but reason (R) is false.
  d. Assertion (A) is false but reason (R) is true.

  Assertion (A): If the circumference of a circle is 176 cm, then its radius is 28 cm.

  Reason (R): Circumference $2\pi \times$ radius.

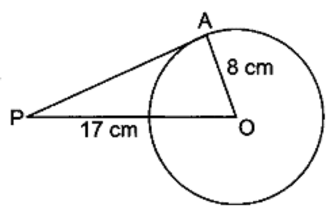
Answer Key

Multiple Choice Questions:

1. (d) 90°
2. (b) 70°
3. (a) equal
4. (c) 10 cm
5. (c) normal
6. (a) $\sqrt{119}$ cm
7. (a) 60 cm^2
8. (c) 16 cm
9. (b) parallel
10. (a) AD = 4cm, BE = 6cm, CF = 8cm

Very Short Answers:

1.



OA ⊥ PA (∵ radius is ⊥ to tangent at point of contact)

∴ In ΔOAP, we have

$PO^2 = PA^2 + AO^2$

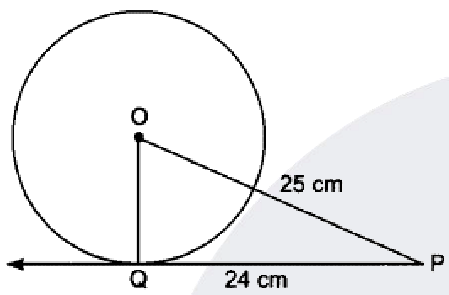
$\Rightarrow (17)^2 = (PA)^2 + (8)^2$

$(PA)^2 = 289 - 64 = 225$

$\Rightarrow PA = \sqrt{225} = 15$

Hence, the length of the tangent from point P is 15 cm.

2.



∵ OQ ⊥ PQ

∴ PQ2 + QO2 = OP2

⇒ 252 = OQ2 + 242

or OQ = √625 – √576

= √49 = 7 cm

3. Here ∠BDC = ∠BAC = 50° (angles in same segment are equal)

In ABCD, we have

∠BCD = 180° – (∠BDC + ∠DBC)

= 180° – (50° + 60°)= 70°

4. ∵ ∠AOB = ∠COD (vertically opposite angles)

∴ ∠COD = 115°

5. AN = AM = 3 cm [Tangents drawn from an external point]

BN = BL = 4 cm [Tangents drawn from an external point]

CL = CM = AC – AM = 9 – 3 = 6 cm

⇒ BC = BL + CL = 4 + 6 = 10 cm.

6. ∠OPQ = 90° – 50° = 40°

OP = OQ [Radii of a circle]

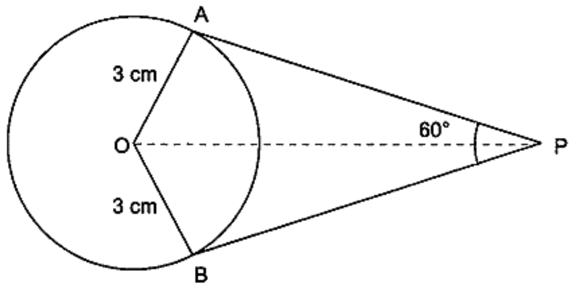
∠OPQ = ∠OQP = 40°

(Equal opposite sides have equal opposite angles)

∠POQ = 180° – ∠OPQ – ∠OQP

= 180° – 40° – 40° = 100°

7.



ΔAOP ≅ ΔBOP (By SSS congruence criterion)

$\angle APO = \angle BPO = \frac{60°}{2} = 30°$

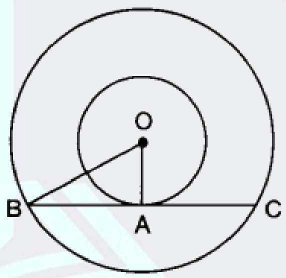
In ΔAOP, OA ⊥ AP

$\therefore \tan 30° = \frac{OA}{AP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{AP}$

$\Rightarrow AP = 3\sqrt{3}$ cm

8.



OA = 4 cm, OB = 5 cm

Also, OA ⊥ BC

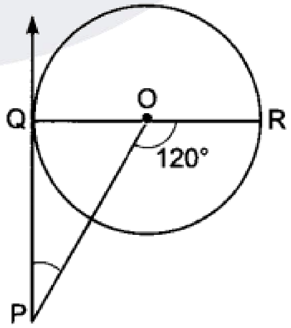
∴ OB2 = OA2 + AB2

⇒ 52 = 42 + AB2

⇒ AB = √25 – √16 = 3 cm

⇒ BC = 2 AB = 2 × 3 = 6 cm

9.



∠OQP = 90°

∠QOP = 180° – 120° = 60°

∠OPQ = 180° – ∠OQP – ∠QOP

= 180° – 90° – 60°

= 30°

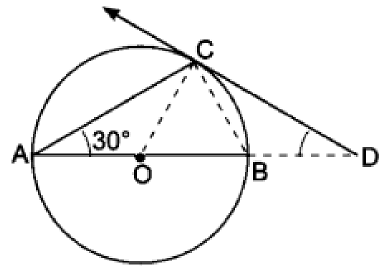
10. ∵ PA = PB ⇒ ∠BAP = ∠ABP = 50°

∴ ∠APB = 180° – 50° – 50° = 80°

∴ ∠AOB = 180° – 80° = 100°

Short Answers:

1.



True, Join OC,

∠ACB = 90° (Angle in semi-circle)

∴ ∠OBC = 180o – (90° + 30°) = 60°

Since, OB = OC = radii of same circle [Fig. 8.16]

∴ ∠OBC = ∠OCB = 60°

Also, ∠OCD = 90°

⇒ ∠BCD = 90° – 60° = 30°

Now, ∠OBC = ∠BCD + ∠BDC (Exterior angle property)

⇒ 60° = 30° + ∠BDC

⇒ ∠BDC = 30°

∵ ∠BCD = ∠BDC = 30°

∴ BC = BD

2. True, let PQ be the tangent from the external point P.

Then ΔPQO is always a right angled triangle with OP as the hypotenuse. So, PQ is always less than OP.

3. True, let PQ and PR be the tangents

Since ∠P = 90°, so ∠QOR = 90°

Also, OR = OQ = a

∴ PQOR is a square

$$\Rightarrow OP = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$$

4. ∵ PA and PB are tangent from same external point

∴ PA = PB = 15 cm

Now, Perimeter of ΔPCD = PC + CD + DP = PC + CQ + QD + DP

= PC + CA + DB + DP

= PA + PB = 15 + 15 = 30 cm

5. PA = PC + CA = PC + CQ [∵ CA = CQ (tangents drawn An from external point are equal)]

⇒ 12 = PC + 3 = PC = 9 cm

∵ PA = PB = PA – AC = PB – BD

⇒ PC = PD

∴ PD = 9 cm

Hence, PC + PD = 18 cm

6. Let the tangents to a circle with centre O be ABC and XYZ.

Construction: Join OB and OY.

Draw OP||AC

Since AB||PO

∠ABO + ∠POB = 180° (Adjacent interior angles)

∠ABO = 90° (A tangent to a circle is perpendicular to the radius through the point of contact)

90° + ∠POB = 180° = ∠POB = 90°

Similarly ∠POY = 90°

∠POB + ∠POY = 90° + 90° = 180°

Hence, BOY is a straight line passing through the centre of the circle.

7. Given, ∠QPR = 120°

Radius is perpendicular to the tangent at the point of contact.

∠OQP = 90°

⇒ ∠QPO = 60°

(Tangents drawn to a circle from an external point are equally inclined to the segment, joining the centre to that point)

In ΔAPO, $\cos 60° = \frac{PQ}{PO}$ $\Rightarrow \frac{1}{2} = \frac{PQ}{PO}$

2PQ = PO

8. AE = CE and BE = ED [Tangents drawn from an external point are equal]

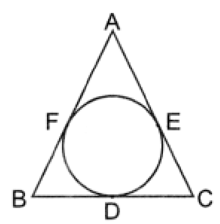
On addition, we get

AE + BE = CE + ED

∠QPO = 60°

⇒ AB = CD

9.



Given, AB = AC

We have, BF + AF = AE + CE ....(i)

AB, BC and CA are tangents to the circle at F, D and E respectively.

∴ BF = BD, AE = AF and CE = CD ....(ii)

From (i) and (ii)

BD + AE = AE + CD (∵ AF = AE)

⇒ BD = CD

10. In the given figure,

AP = AR

BR = BQ

XP = XQ [Tangent to a circle from an external point are equal]

XA + AP = XB + BQ

XA + AR = XB + BR [AP = AR, BQ = BR]

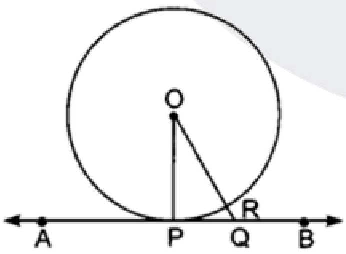
Long Answers:

1. Given: A circle C(O,r) and a tangent AB at a point P.

To Prove: OP ⊥ AB.

Construction: Take any point l, other than P, on the tangent AB. Join OQ. Suppose OQ meets the circle at R.

Proof: We know that among all line segments joining the point to a point on AB, the shortest one is perpendicular to AB. So, to prove that OP ⊥ AB it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB.



Clearly, OP = OR [Radii of the same circle]

Now, OQ = OR + RQ

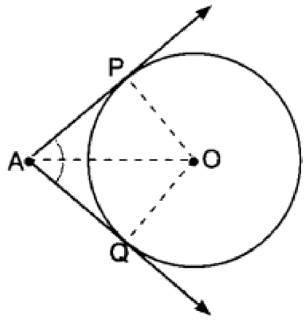
⇒ OQ > OR

⇒ OQ > OP [∵OP = OR]

Thus, OP is shorter than any other segment joining O to any point on AB.

Hence, OP ⊥ AB.

2.



Given: AP and AQ are two tangents from a point A to a circle C (O, r).

To Prove: AP = AQ

Construction: Join OP, OQ and OA.

Proof: In order to prove that AP = AQ, we shall first prove that ΔOPA ≅ ΔOQA.

Since a tangent at any point of a circle is perpendicular to the radius through the point of contact.

∴ OP ⊥ AP and OQ ⊥ AQ

⇒ ∠OPA = ∠OQA = 90°

Now, in right triangles OPA and OQA, we have

OP = OQ [Radii of a circle]

∠OPA = ∠OQA [Each 90°]

and OA = OA [Common]

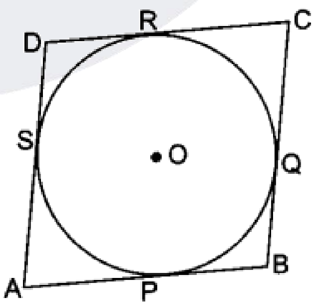
So, by RHS-criterion of congruence, we get

ΔOPA ≅ OQA

⇒ AP = AQ [CPCT]

Hence, lengths of two tangents from an external point are equal.

3.



Let ABCD be a parallelogram such that its sides touch a circle with centre O.

We know that the tangents to a circle from an exterior point are equal in length.

Therefore, we have

AP = AS [Tangents from A]

BP = BQ [Tangents from B] .... (ii)

CR = CQ [Tangents from C] .... (iii)

And DR = DS [Tangents from D] .... (iv)

Adding (i), (ii), (iii) and (iv), we have

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = AD + BC

AB + AB = BC + BC [∵ ABCD is a parallelogram ∴ AB = CD, BC = DA]

2AB = 2BC ⇒ AB = BC

Thus, AB = BC = CD = AD

Hence, ABCD is a rhombus.

4. To find: TP

$$PR = RQ = \frac{16}{2} = 8\,\text{cm}$$

[Perpendicular from the centre bisects the chord]

In ΔOPR

$$OR = \sqrt{OP^2 - PR^2}$$

$$= \sqrt{10^2 - 8^2} = \sqrt{100 - 64}$$

$$= \sqrt{36} = 6\,\text{cm}$$

Let $\angle POR$ be θ

In ΔPOR, $\tan\theta = \frac{PR}{RO} = \frac{8}{6}$

$$\tan\theta = \frac{4}{3}$$

We know, OP ⊥ TP

(Point of contact of a tangent is perpendicular to the line from the centre)

In ΔOTP, $\tan\theta = \frac{OP}{TP} \Rightarrow \frac{4}{3} = \frac{10}{TP}$

$$TP = \frac{10 \times 3}{4} = \frac{15}{2} = 7.5\,\text{cm}$$

5. Let O be the centre and QOR = 8 cm is diameter of a circle. PQ is tangent such that ∠POR = 120°

Now, $OQ = OR = \frac{8}{2} = 4$ cm

$\angle POQ = 180 - 120° = 60°$ (Linear pair)

Also OQ ⊥ PQ

Now, in right ΔPOQ

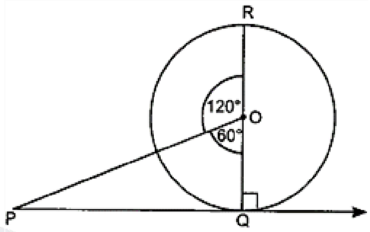
$$\cos 60° \frac{OQ}{PO}$$

$$\Rightarrow \frac{1}{2} = \frac{OQ}{PO} \quad \Rightarrow \frac{1}{2} = \frac{4}{PO}$$

⇒ PO = 8 cm.

Again, $\tan 60° = \frac{PQ}{OQ} \Rightarrow \sqrt{3} = \frac{PQ}{4}$

$$\Rightarrow PQ = 4\sqrt{3}\,\text{cm.}$$



6. In right ΔPOT

$$PT = \sqrt{OT^2 - OP^2}$$

$$PT = \sqrt{169 - 25} = 12\,\text{cm}$$

and TE = 8 cm

Let PA = AE = x

(Tangents from an external point to a circle are equal)

In right ΔAET

$TA^2 = TE^2 + EA^2$

⇒ $(12 - x)^2 = 64 + x^2$

⇒ $144 + x^2 - 24x = 64 + x^2$

⇒ $x = \frac{80}{24}$

⇒ x = 3.3 cm

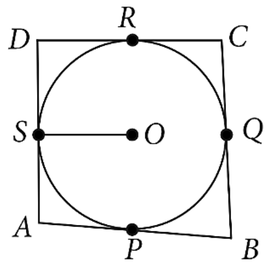
Thus, AB = 6.6 cm

Case Study Answers:

1. Answer :

i. (b) 90°

Solution:



Here, OS the is radius of circle.

Since radius at the point of contact is perpendicular to tangent So, ∠OSA=90°

ii. (d) All of these

Solution:

Since, length of tangents drawn from an external point to a circle are equal.

∴ AS = AP, BP = BQ,

CQ = CR and DR = DS

iii. (a) 4cm

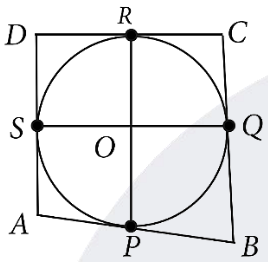
Solution:

AP = AS = AD - DS = AD - DR = 11 - 7 = 4cm.

iv. (b) 120º

Solution:

In quadrilateral OQCR,



$\angle QCR = 60^o$, (Given)

And $\angle OQC = \angle ORC = 90^o$

[Since, radius at the point of contact is perpendicular to tangent.]

∴ $\angle QCR = 360^o - 90^o - 90^o - 60^o = 120$

v. (c) AB + CD = AD + BC

Solution:

From (I), we have AS = AP, DS = DR, BQ = BP and CQ = CR

Adding all above equations, we get

AS + DS + BQ + CQ = AP + DR + BP + CR

⇒ AD + BC = AB + CD

2. Answer :

i. (a) Secant

ii. (d) Infinite

iii. (b) 2

iv. (a) Infinite

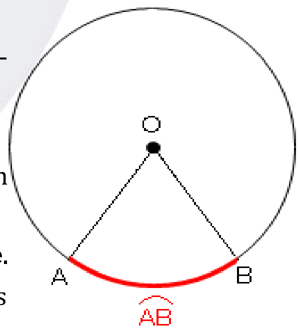
v. (c) Tangent

Assertion Reason Answers:

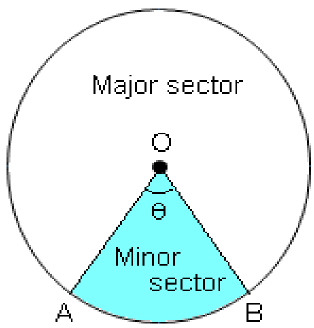
1. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
2. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Areas Related to Circles 11

1. A **circle** is a set of points in a plane that are at an equal distance from a fixed point. The fixed point is called the centre of circle and equal distance is called the radius of the circle.
2. A line segment joining the centre of the circle to a point on the circle is called its **radius.**
3. A line segment joining any two points of a circle is called a **chord.** A chord passing through the centre
4. of circle is called its **diameter.**
5. The distance around the boundary of the circle is called **the perimeter or the circumference** of the circle.
6. Circumference (perimeter) of a circle = πd or $2\pi r$, where d is he diameter, r is the radius of the circle and $\pi = \frac{22}{7}$
7. Perimeter of a semi circle or protractor = $\pi r + 2r$
8. Perimeter of a quadrant = $\frac{1}{4}$ Circumference + 2r = $\frac{\pi r}{2}$ + 2r
9. Distance moved by a wheel in 1 revolution = Circumference of the wheel.
   Number of revolutions in one minute $= \frac{Distance\ moved\ in\ 1\ minute}{Circumference}$
10. The region enclosed inside a circle is called its **area.**
11. Area of a circle = πr^2
12. Area of a semi circle= $\frac{1}{2}\pi r^2$
13. Area of a quadrant = $\frac{1}{4}$ Area of circle= $\frac{1}{4}\pi r^2$
14. Circles having the same centre but different radii are called **concentric circles.**
    Area enclosed by two concentric circles = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2) = \pi(R + r)(R - r)$
    Where, R and r are radii of two concentric circles



15. The part of the circumference between the two end points of the chord is called an **arc.** In the figure, arc $\widehat{AB}$ is shown.
16. A diameter of circle divides a circle into two equal arcs, each known as a **semi-circle.**
17. An arc of a circle whose length is less than that of a semicircle of the same circle is called a **minor arc.**
18. An arc of a circle whose length is greater than that of a semicircle of the same circle is called a **major arc.**
19. Length of an arc $= \frac{\pi r^2}{180°}$
20. The region bounded by an arc of a circle and two radii at its end points is called a **sector.**
    If the central angle of a sector is more than 180°, then the sector is called a **major sector** and if the central angle is less than 180°, then the sector is called a **minor sector.**

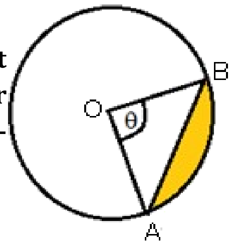


21. Perimeter of sector of angle $\theta = \frac{\pi r\theta}{180°} + 2r$
22. Area of a sector of angle $= \frac{\pi r^2\theta}{360°}$

23. Area of major sector $= \pi r^2 -$ Area of minor sector

24. A chord divides the interior of a circle into two parts, each called a segment.

The segment which is smaller than the portion of semi-circle is called the **minor segment** and the segment which is larger than the portion of semi-circle is called the **major segment**. In the circle shown, the yellow portion is the minor segment while the non-shaded portion is the major segment.



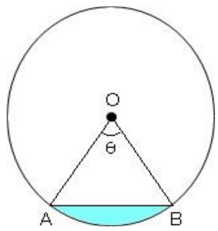
25. Perimeter of segment of angle $\theta = \frac{2\pi r\theta}{360°} + 2r\sin\frac{\theta}{2}$

26. Area of minor segment = Area of sector - Area of Δ ABC

27. Area of minor segment can also be written as:

Area of the segment ACB = Area of sector OABC – Area of Δ OAB

Area of segment ACB $= \left\{\frac{\theta}{360°} \times \pi r^2\right\} - \left\{\frac{\sin\theta}{2} + \frac{\cos\theta}{2}\right\}$

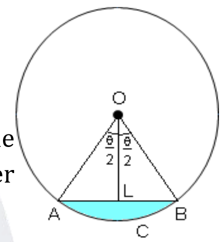


28. Area of major segment = Area of the circle – Area of minor segment

29. **Area of a Circle**

Area of a circle is πr^2, where π=22/7 or ≈3.14 (can be used interchangeably for problem-solving purposes) and r is the radius of the circle.

π is the ratio of the circumference of a circle to its diameter.



Circumference of a Circle

The perimeter of a circle is the distance covered by going around its boundary once. The perimeter of a circle has a special name: Circumference, which is π times the diameter which is given by the formula $2\pi r$

Segment of a Circle

A circular segment is a region of a circle that is "cut off" from the rest of the circle by a secant or a chord.

Sector of a Circle

A circle sector/ sector of a circle is defined as the region of a circle enclosed by an arc and two radii. The smaller area is called the minor sector and the larger area is called the major sector.

Angle of a Sector

The angle of a sector is the angle that is enclosed between the two radii of the sector.

Length of an arc of a sector

The length of the arc of a sector can be found by using the expression for the circumference of a circle and the angle of the sector, using the following formula:

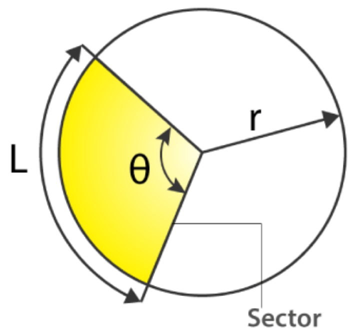
$$L = (\theta/360°) \times 2\pi r$$

Length of an arc of a sector

The length of the arc of a sector can be found by using the expression for the circumference of a circle and the angle of the sector, using the following formula:

$$L = (\theta/360°) \times 2\pi r$$

where $\angle\theta$ is the angle of this sector (minor sector in the following case) and r is its radius



Area of a Triangle

The Area of a triangle is,

Area = (1/2) × base × height

If the triangle is an equilateral then

Area = $(\sqrt{3}/4) \times a^2$ where "a" is the side length of the triangle.

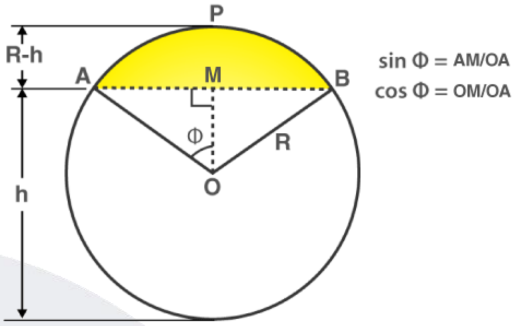
Area of a Segment of a Circle

Area of segment APB (highlighted in yellow)

= (Area of sector OAPB) – (Area of triangle AOB)

= $[(\emptyset/360°) \times \pi r^2] - [(1/2) \times AB \times OM]$

[To find the area of triangle AOB, use trigonometric ratios to find OM (height) and AB (base)]



Also, the Area of segment APB can be calculated directly if the angle of the sector is known using the following formula.

= $[(\theta/360°) \times \pi r^2] - [r^2 \times \sin\theta/2 \times \cos\theta/2]$

Where θ is the angle of the sector and r is the radius of the circle

All these formulas are tabulated as given below for quick revision.

Parameters of Circles	Formulas
Area of the sector of angle θ	$(\theta/360°) \times \pi r^2$
Length of an arc of a sector of angle θ	$(\theta/360°) \times 2\pi r$
Area of major sector	$\pi r^2 - (\theta/360°) \times \pi r^2$
Area of a segment of a circle	Area of the corresponding sector – Area of the corresponding triangle
Area of the major segment	πr^2 – Area of segment (minor segment)

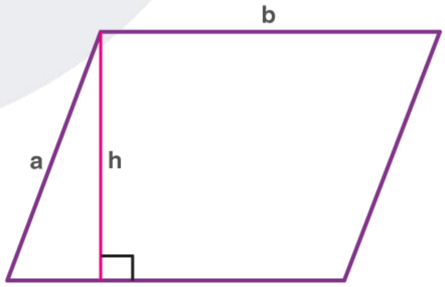
Visualizations

Areas of different plane figures

Area of a square (side l) = l^2

Area of a rectangle = l × b, where l and b are the length and breadth of the rectangle

Area of a parallelogram = b × h, where "b" is the base and "h" is the perpendicular height.



Parallelogram

Area of a trapezium = [(a + b) × h]/2,

where

a & b are the length of the parallel sides

h is the trapezium height

Area of a rhombus = pq/2, where p & q are the diagonals.

Area of Shapes

In Geometry, a shape is defined as the figure closed by the boundary. The boundary is created by the combination of lines, points and curves. Basically, there are two different types of geometric shapes such as:

Two – Dimensional Shapes

Three – Dimensional Shapes

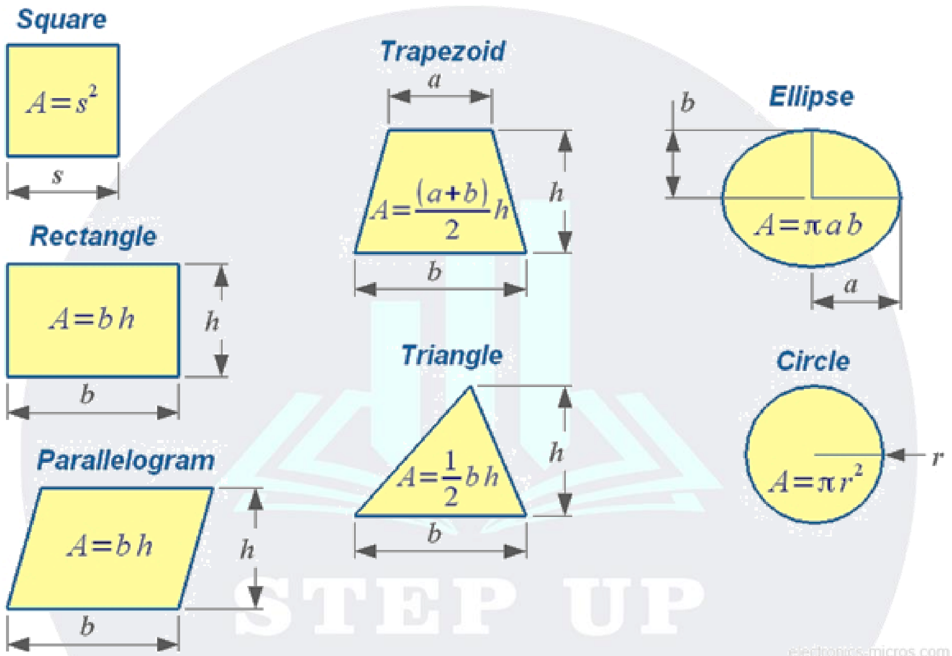
Each and every shape in the Geometry can be measured using different measures such as area, volume, surface area, perimeter and so on. In this article, let us discuss the area of shapes for 2D figures and 3D figures with formulas.

2D Shapes

The two-dimensional shapes (2D shapes) are also known as flat shapes, are the shapes having two dimensions only. It has length and breadth. It does not have thickness. The two different measures used for measuring the flat shapes are area and the perimeter. Two-dimensional shapes are the shapes that can be drawn on the piece of paper. Some of the examples of 2D shapes are square, rectangle, circle, triangle and so on.

Area of 2D Shapes Formula

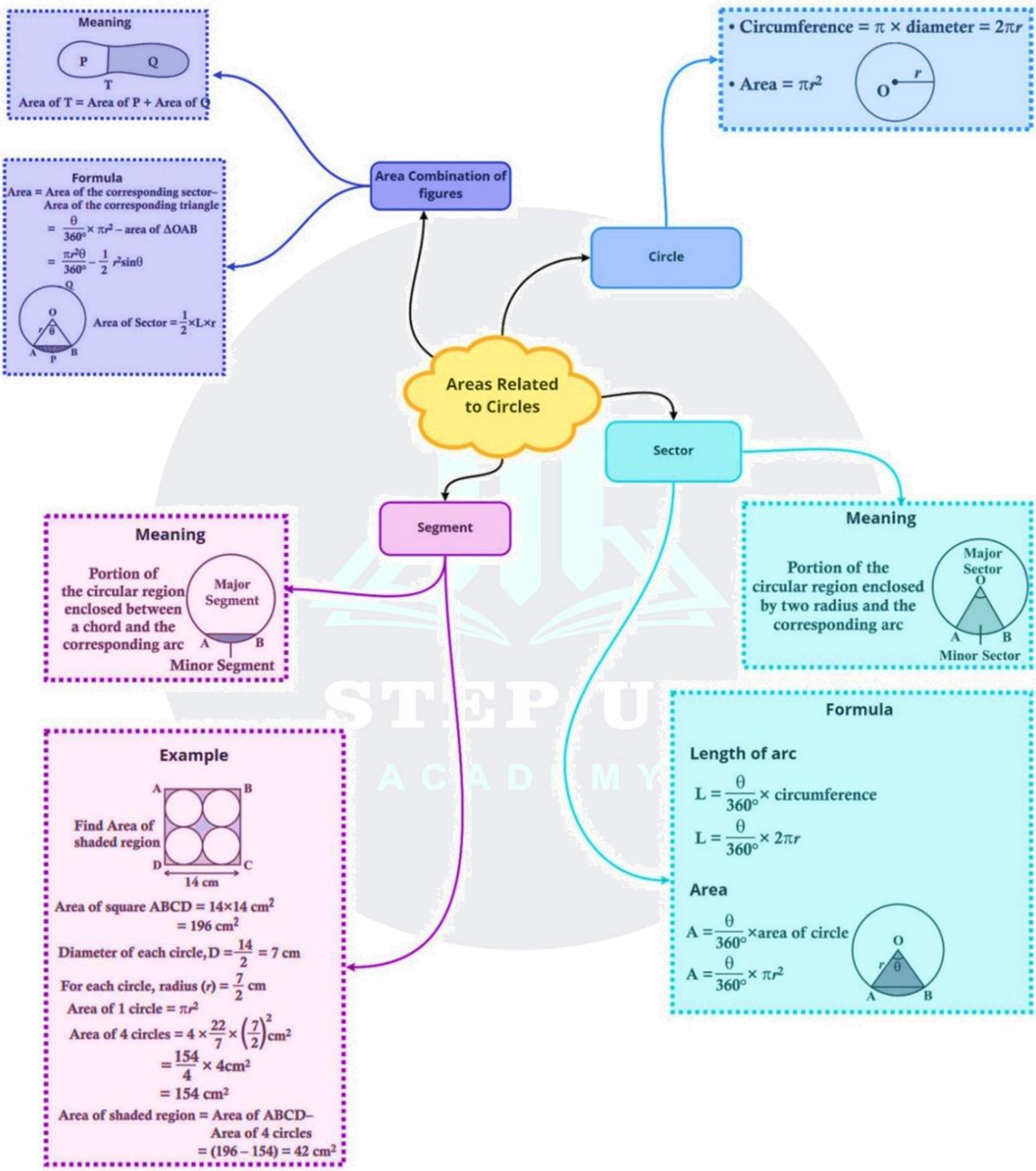
In general, the area of shapes can be defined as the amount of paint required to cover the surface with a single coat. Following are the ways to calculate area based on the number of sides that exist in the shape, as illustrated below in the fig.



Let us write the formulas for all the different types of shapes in a tabular form.

Shape	Area	Terms
Circle	π × r²	r = radius of the circle
Triangle	½ × b × h	b = base h = height
Square	a²	a = length of side
Rectangle	l × w	l = length w = width
Parallelogram	b × h	b=base h=vertical height
Trapezium	½(a+b) × h	a and b are the length of parallel sides h = height
Ellipse	πab	a = ½ minor axis b = ½ major axis

Class : 10th mathematics
Chapter- 12 : Areas Related to Circles



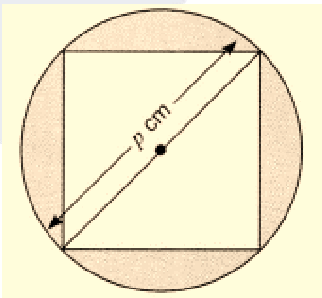
Important Questions

Multiple Choice Questions:

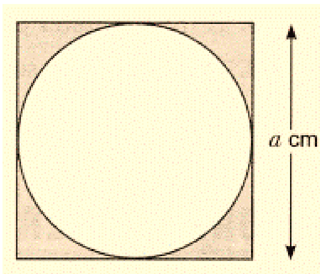
1. Perimeter of a sector of a circle whose central angle is 90° and radius 7 cm is
   (a) 35 cm
   (b) 25 cm
   (c) 77 cm
   (d) 7 cm
2. The area of a circle that can be inscribed in a square of side 10 cm is
   (a) 40π cm^2
   (b) 30π cm^2
   (c) 100π cm^2
   (d) 25π cm^2
3. The perimeter of a square circumscribing a circle of radius a units is
   (a) 2 units
   (b) 4α units
   (c) 8α units
   (d) 16α units
4. The perimeter of the sector with radius 10.5 cm and sector angle 60° is
   (a) 32 cm
   (b) 23 cm
   (c) 41 cm
   (d) 11 cm
5. In a circle of diameter 42 cm, if an arc subtends an angle of 60° at the centre, where $\pi = 227$ then length of arc is:
   (a) 11 cm
   (b) 227 cm
   (c) 22 cm
   (d) 44 cm
6. The perimeter of a sector of radius 5.2 cm is 16.4 cm, the area of the sector is
   (a) 31.2 cm^2
   (b) 15 cm^2
   (c) 15.6 cm^2
   (d) 16.6 cm^2
7. If the perimeter of a semicircular protractor is 72 cm where $\pi = 22/7$, then the diameter of protractor is:
   (a) 14 cm
   (b) 33 cm
   (c) 28 cm
   (d) 42 cm
8. If the radius of a circle is doubled, its area becomes
   (a) 2 times
   (b) 4 times
   (c) 8 times
   (d) 16 times
9. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to circumference of a circle of radius R, then
   (a) $R_1 + R_2 = R$
   (b) $R_1 + R_2 > R$
   (c) $R_1 + R_2 < R$
   (d) Can't say.
10. The perimeter of a circular and square fields are equal. If the area of the square field is 484 m^2 then the diameter of the circular field is
   (a) 14 m
   (b) 21 m
   (c) 28 m
   (d) 7 m

Very Short Questions:

1. Find the area of a square inscribed in a circle of diameter p cm.



2. Find the area of the circle inscribed in a square of side a cm.



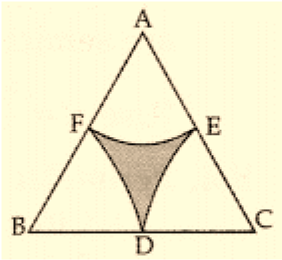
3. Find the area of a sector of a circle whose radius is and length of the arc is l.
4. Find the ratio of the areas of a circle and an equilateral triangle whose diameter and a side are respectively equal.
5. If circumference and the area of a circle are numerically equal, find the diameter of the circle.
6. The radius of a wheel is 0.25 m. Find the number of revolutions it will make to travel a distance of 11 km.
7. If the perimeter of a semi-circular protractor is 36 cm, find its diameter.
8. If the diameter of a semicircular protractor is 14 cm, then find its perimeter.
9. If a square is inscribed in a circle, what is the ratio of the areas of the circle and the square?

Short Questions:

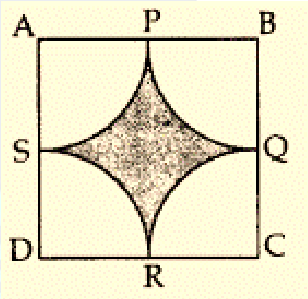
1. What is the area of the largest triangle that is inscribed in a semi circle of radius r unit?
2. What is the angle subtended at the centre of a circle of radius 10 cm by an arc of length 5π cm?
3. Difference between the circumference and radius of a circle is 37 cm. Find the area of circle.
4. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.
5. If the perimeter of a semicircular protractor is 66 cm, find the diameter of the protractor. (Take $\pi = \frac{22}{7}$).
6. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.
7. A race track is in the form of a ring whose inner circumference is 352 m, and the outer circumference is 396 m. Find the width of the track.
8. The inner circumference of a circular track [Fig. 12.10] is 220 m. The track is 7 m wide everywhere. Calculate the cost of putting up a fence along the outer circle at the rate of ₹2 per metre.
9. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Long Questions:

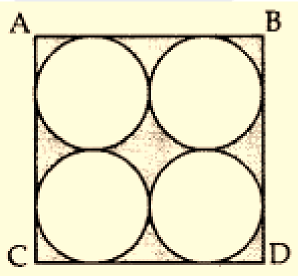
1. In Figure, arcs are drawn by taking vertices A, B and C of an equilateral triangle ABC of side 14 cm as centres to intersect the sides BC, CA and AB at BZ their respective mid-points D, E and F. Find the area of the shaded region. [Use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.73$]



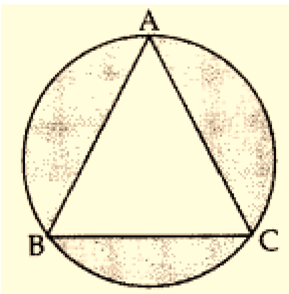
2. Find the area of the shaded region in Figure, where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA respectively of a square ABCD, where the length of each side of square is 14 cm. $\left[\text{Use } \pi = \frac{22}{7}\right]$



3. Find the area of the shaded region in Figure, where ABCD is a square of side 28 cm.

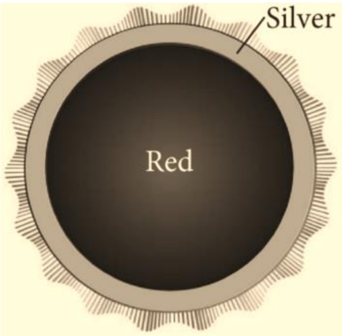


4. In Figure, an equilateral triangle has been inscribed in a circle of radius 6 cm. Find the area of the shaded region. [Use π = 3.14]



Assertion Reason Questions:

1. Principle of a school decided to give badges to students who are chosen for the post of Head boy, Head girl, Prefect, and Vice Prefect. Badges are circular in shape with two color area, red and silver, as shown in figure. The diameter of the region representing red color is 22cm and silver color is filled in l0.5 cm wide ring. Based on the above information, answer the following questions.



   i. The radius of circle representing the red region is:
      a. 9cm
      b. 10cm
      c. 11cm
      d. 12cm
   ii. Find the area of the red region.
      a. 380.28cm²
      b. 382.28cm²
      c. 384.28cm²
      d. 378.28cm²
   iii. Find the radius of the circle formed by combining the red and silver region.
      a. 20.5cm
      b. 21.5cm
      c. 22.5cm
      d. 23.5cm
   iv. Find the area of the silver region.
      a. 172.50cm²
      b. 1062.50cm²
      c. 1172.50cm²
      d. 1072.50cm²
   v. Area of the circular path formed by two concentric circles of radii r_1 and r_2 ($r_1 > r_2$) =
      a. $\pi\left(r_1^2 + r_1^2\right) sq.units$
      b. $\pi\left(r_1^2 - r_1^2\right) sq.units$
      c. $2\pi\left(r_1 - r_1\right) sq.units$
      d. $2\pi\left(r_1 + r_1\right) sq.units$

2. While doing dusting, a maid found a button whose upper face is of black color, as shown in the figure. The diameter of each of the smaller identical circles is 1414 of the diameter of the larger circle, whose radius is 16cm. Based on the above information, answer the following questions.

   i. The area of each of the smaller circle is:
      a. 40.28cm²
      b. 46.39cm²
      c. 50.28cm²
      d. 52.3cm²
   ii. The area of the larger circle is:
      a. 804.57cm²
      b. 704.57cm²
      c. 855.57cm²
      d. 990.57cm²
   iii. The area of the black color region is:
      a. 600.45cm²
      b. 603.45cm²
      c. 610.45cm²
      d. 623.45cm²
   iv. The area of a quadrant of a smaller circle is:
      a. 11.57cm²
      b. 13.68cm²
      c. 12cm²
      d. 12.57cm²
   v. If two concentric circles are of radii 2cm and 5cm, then the area between them is:
      a. 60cm²
      b. 63cm²
      c. 66cm²
      d. 68cm²

Case Study Answers:

1. **Directions:** Each of these questions contains two statements: Assertion [A] and Reason [R]. Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes [a], [b], [c] and [d] given below.
   a. A is true, R is true; R is a correct explanation for A.
   b. A is true, R is true; R is not a correct explanation for A.
   c. A is true; R is false.
   d. A is false; R is true.

   Assertion: If the circumference of a circle is 176 cm, then its radius is 28 cm.

   Reason: Circumference = 2π × radius
2. **Directions:** Each of these questions contains two statements: Assertion [A] and Reason [R]. Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes [a], [b], [c] and [d] given below.
   a. A is true, R is true; R is a correct explanation for A.
   b. A is true, R is true; R is not a correct explanation for A.
   c. A is true; R is false.
   d. A is false; R is true.

   Assertion: If a wire of length 22 cm is bent is the shape of a circle, then area of the circle so formed is 40 cm.

   Reason: Circumference of the circle = length of the wire.

Answer Key

Multiple Choice Questions:

1. (b) 25 cm
2. (d) 25π cm^2
3. (c) 8α units
4. (a) 32 cm
5. (c) 22 cm
6. (c) 15.6 cm^2
7. (c) 28 cm
8. (b) 4 times
9. (a) $R_1 + R_2 = R$
10. (c) 28 m

Very Short Answers:

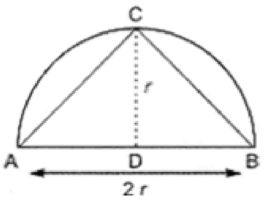
1. Diagonal of the square = p cm
   ∴ $p^2 = side^2 + side^2$
   ⇒ $p^2 = 2side^2$
   or $side^2 = \frac{p^2}{2}$ cm^2 = area of the square
2. Diameter of the circle = a
   $$\Rightarrow \text{Radius} = \frac{a}{2} \Rightarrow \text{Area} = \pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}\, cm^2$$
3. Area ola sector ola circle with radius r
   $$= \frac{\theta}{360^\circ}\times \pi r^2 = \frac{\theta}{360^\circ}\times 2\pi\frac{r}{2} = \frac{1}{2}\ lr.\ sq.\ units$$
   $$\left(\because l = \frac{2\pi r\theta}{360^\circ}\right)$$
4. Given, 2r = a ⇒ $\frac{r}{a} = \frac{1}{2}$
   $$\frac{\text{Area of circle}}{\text{Area of equilateral triangle}}$$
   $$= \frac{\pi r^2}{\frac{\sqrt{3}}{4}a^2} = \frac{4\pi}{\sqrt{3}}\left(\frac{r}{a}\right)^2 = \frac{4\pi}{3}\times\frac{1}{4} = \frac{\pi}{\sqrt{3}}$$
5. Given, $2\pi r = \pi r^2$
   ⇒ $2r = r^2$
   ⇒ r(r – 2) = 0 or r = 2
   i.e. d = 4 units
6. $$\text{Number of revolutions} = \frac{11\times 1000}{2\times\frac{22}{7}\times 0.25} = 7000.$$
7. Perimeter of a semicircular protractor = Perimeter of a semicircle
   = (2r + πr) cm
   ⇒ 2r + πr = 36
   $$\Rightarrow r\left(2 + \frac{22}{7}\right) = 36$$
   ⇒ r = 7cm
   Diameter 2r = 2 × 7 = 14 cm.
8. Perimeter of a semicircle = πr + 2r
   = $\frac{22}{7}$ × 7 + 2 × 7 = 22 + 14 = 36cm
9. Let radius of the circle be r units.
   Then, diagonal of the square = 2r

$\Rightarrow$ Slide of the square $= \dfrac{2r}{\sqrt{2}} = \sqrt{2}r$

$\therefore \dfrac{\text{Area of the circle}}{\text{Area of the square}} = \dfrac{\pi r^2}{(\sqrt{2}r)^2} = \dfrac{\pi r^2}{2r^2} = \pi : 2$

Short Answers:

1.



Area of largest ΔABC = $\frac{1}{2}$ × AB × CD

$\frac{1}{2}$ × 2r × r = r^2 sq. units

2. Are length of a circle of radius $r = \dfrac{\theta}{360°} \times 2\pi$

$\Rightarrow 5\pi = \dfrac{360°}{4} \times 2\pi \times 10$

or $\dfrac{360°}{4} = \dfrac{5\pi}{20\pi} = \dfrac{1}{4} \Rightarrow \theta \dfrac{360°}{4} = 90°$

3. Given 2π r – r = 37

or r (2π – 1) = 37

$r = \dfrac{37}{2\pi - 1} = \dfrac{37}{2 \times \frac{22}{7} - 1} = \dfrac{37 \times 7}{37} = 7$

So, Area of circle $= \pi r^2$

$= \dfrac{22}{7} \times 7 \times 7 = 154\, cm^2$

4. Let r be the radius of required circle. Then, we have

$\pi r^2 = p(8)^2 + p(6)^2$

$\Rightarrow \quad \pi r^2 = 64p + 36p$

$\Rightarrow \quad pr^2 = 100p$

$\therefore \quad r^2 = 100pp = 100$

$\Rightarrow \quad r = 10$cm

Hence, radius of required circle is 10 cm.

5. Let the radius of the protractor be r cm. Then,

Perimeter = 66 cm

= πr + 2r = 66 [∴ Perimeter of a semicircle = πr + 2r]

$\Rightarrow \quad r\left(\dfrac{22}{7} + 2\right) = 66 \quad \Rightarrow \quad \dfrac{36}{7} r = 66$

$\Rightarrow \quad r = \dfrac{66 \times 7}{36} = \dfrac{77}{6} cm$

$\therefore$ Diameter of the protractor

$= 2r = 2 \times \dfrac{77}{6} = \dfrac{77}{3} = 25\dfrac{2}{3} cm$

6. Let the radius of the circle be r cm. Then,

Diameter = 2r cm and Circumference = 2πr cm

According to question,

Circumference = Diameter + 16.8

⇒ 2πr = 2r + 16.8

⇒ 2 × $\frac{22}{7}$ × r = 2r + 16.8

⇒ 44r = 14r + 16.8 × 7

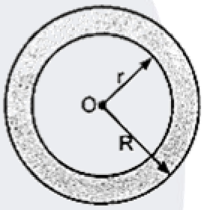
⇒ 44r – 14r = 117.6 or 30r = 117.6

⇒ r = $\frac{117.6}{30}$ = 3.92

Hence, radius = 3.92 cm.

7. Let the outer and inner radii of the ring be R m and r m respectively. Then,

2πR = 396 and 2πr = 352



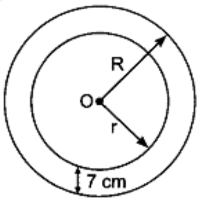
$\Rightarrow \quad 2 \times \dfrac{22}{7} \times R = 396 \;\text{ and }\; 2 \times \dfrac{22}{7} \times r = 352$

$\Rightarrow \quad R = 396 \times \dfrac{7}{22} \times \dfrac{1}{2} \;\text{ and }\; r = 352 \times \dfrac{7}{22} \times \dfrac{1}{2}$

$\Rightarrow \quad R = 63\,m \;\text{ and }\; r = 56\,m$

Hence, width of the track = (R – r) m = (63 – 56) m = 7 m

8.



Let the inner and outer radii of the circular track berm and R m respectively. Then,

Inner circumference = 2πr = 220 m

$\Rightarrow \quad 2 \times \dfrac{22}{7} \times r = 220 \;\Rightarrow\; r = \dfrac{220 \times 7}{2 \times 22} = 35\,m$

Since the track is 7 m wide everywhere. Therefore,

R = Outer radius = r + 7 = (35 + 7)m = 42 m

∴ Outer circumference = $2\pi R = 2 \times \frac{22}{7} \times 42$m = 264m

Rate of fencing = ₹ 2 per metre

∴ Total cost of fencing = (Circumference × Rate) = ₹(264 × 2) = ₹ 528

9. The diameter of a wheel = 80 cm.

radius of the wheel = 40 cm.

Now, distance travelled in one complete revolution of wheel = $2\pi \times 40 = 80\pi$

Since, speed of the car is 66 km/h

So, distance travelled in 10 minutes = $\frac{66 \times 100000 \times 10}{60}$

= 11 × 100000 cm = 1100000 cm.

So, Number of complete revolutions in 10 minutes

$$= \frac{1100000}{80\pi} = \frac{1100000}{8 \times \frac{22}{7}}$$

$$= \frac{110000 \times 7}{8 \times 22} = \frac{70000}{16} = 4375$$

Long Answers:

1. ∠ABC = ∠BAC = ∠ACB = 60° ... [equilateral Δ]

Let θ = 60°, $r = \frac{14}{2} = 7\,cm$

Area of shaded region

= ar(ΔABC) – 3(ar of sector)

$$= \frac{\sqrt{3}}{2}(side)^2 - 3 \cdot \frac{\theta}{360}\pi^2$$

...[Area of equilateral $\Delta = \frac{\sqrt{3}}{4} side^2$]

$$= \frac{1.73}{4} \times 14 \times 14 - 3 \times \frac{60}{360} \times \frac{22}{7} \times 7 \times 7$$

= 84.77 – 77 = 7.77 cm^2

2. Side = 14 cm, radius, r = $\frac{14}{2}$ = 7 cm

Area of the shaded region

= ar (square) – 4 (ar of quadrant)

$$= (side)^2 - 4\left(\frac{1}{4}\pi r^2\right)$$

$$= (14)^2 - \frac{22}{7} \times 7 \times 7$$

= 196 – 154 = 42 cm^2

3. Here r = $\frac{28}{4}$ = 7 cm

Area of the shaded region

= ar(square) – 4(circle)

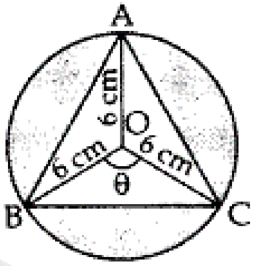
= $(side)^2 - 4(\pi r^2)$

= $(28)^2 - 4 \times \frac{22}{7} \times 7 \times 7$ = 784 – 616 = 168 cm^2

4. Here θ = $\frac{360}{3}$ = 120°, r = 6 cm

Area of shaded region

= 3(ar of minor segment) = 3[ar(minor sector) – ar(ΔABC)]



$$= 3\left[\frac{\theta}{360°}\pi^2 - r^2 \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right]$$

$$= 3\left[\frac{120°}{360°}(3.14) \times 6^2 - 6^2 \sin\left(\frac{120°}{2}\right)\cos\left(\frac{120°}{2}\right)\right]$$

$$= 3 \times 6^2\left[\frac{3.14}{3} - \sin 60° \cos 60°\right]$$

$$= 3(36)\left[\frac{3.14}{3} - \frac{\sqrt{3}}{2} \times \frac{1}{2}\right]$$

$$= 108\left[\frac{12.56 - 3(1.73)}{12}\right] \quad ...\left[\sqrt{3} = 1.73\right.$$

= 9(12.56 – 5.19) = 9 (7.37) = 66.33 cm^2

Case Study Answers:

1. **Answer:**

i. (c) 11cm

Solution:

Radius of circle representing red region

$= \frac{22}{7} = 11\,cm$ [∵ Diameter = 22 cm (Given)]

ii. 380.28cm^2

Solution:

Area of red region πr^2

$= \frac{22}{7} \times 11 \times 11 = 380.28\,cm^2$

iii. (b) 21.5cm

Solution:

Radius of circle formed by combining red and silver region = Radius of red region + width of silver sign.

= (11+ 10.5)cm = 21.5cm

iv. (d) $1072.50cm^2$

Solution:

Area of silver region = Area of combined region - Area of red region.

$$=\frac{22}{7}\times 21.5\times 21.5-380.28$$

= 1452.78 - 380.28 = $1072.50cm^2$

v. (b) $\pi\left(r_1^2-r_1^2\right)$ sq. units

Solution:

Area of circular path formed by two concentric circles

2. Answer:

Let r and R be the radii of each smaller circle and larger circle, respectively.

We have, $d=\frac{1}{4}D$

$$\Rightarrow r=\frac{1}{4}R \quad \Rightarrow r=\frac{1}{4}\times 16 \quad \Rightarrow r=4\,cm.$$

i. (c) $50.28cm^2$

Solution:

Area of smaller circle πr^2

$$=\frac{22}{7}\times 4\times 4=50.28\,cm^2$$

ii. $804.57cm^2$

Solution:

Area of larger circle πR^2

$$=\frac{22}{7}\times 16\times 16=804.57\,cm^2$$

iii. (b) $603.45cm^2$

Solution:

Area of the black color region = Area of larger circle - Area of 4 smaller circles.

= 804.57 - 4 × 50.28 = $603.45cm^2$

iv. (d) $12.57cm^2$

Solution:

Area of quadrant of a smaller circle

$$=\frac{1}{4}\times 450.2=12.57\,cm^2$$

v. (c) $66cm^2$

Solution:

Area between two concentric circles

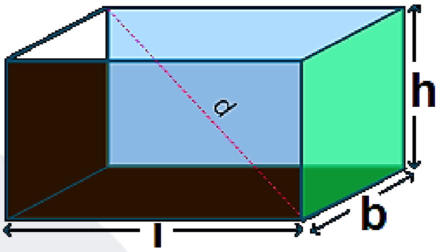
$$=\pi(R^2-r^2)=\frac{22}{7}(5^2-2^2)$$

$$=\frac{22}{7}(25-4)=\frac{22}{7}\times 21=66\,cm^2$$

Assertion Reason Answers:

1. (a) A is true, R is true; R is a correct explanation for A.
2. (d) A is false; R is true

Surface Areas and Volumes 12



1. **Surface area** of a solid is the sum of the areas of all its faces.
2. The space occupied by a solid object is the **volume** of that object.
3. If l, b, h denote respectively the length, breadth and height of a **cuboid**, then: Lateral surface area or Area of four walls = $2(\ell + b)\,h$

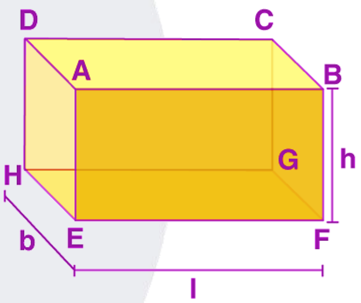
   Total surface area = $2(\ell b + bh + h\ell)$ Volume = $\ell \times b \times h$

   Diagonal of a cuboid =

   Surface Area and Volume of Cuboid

   A cuboid is the region covered by its six rectangular faces. The surface area of a cuboid is equal to the sum of the areas of its six rectangular faces.

   Surface area of the cuboid

   Consider a cuboid whose dimensions are $l \times b \times h$, respectively.

   Cuboid with length l, breadth b and height h

   The total surface area of the cuboid (TSA) = Sum of the areas of all its six faces

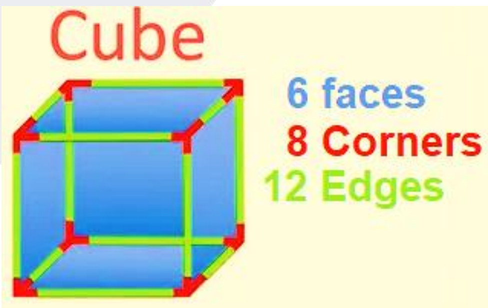
   TSA (cuboid) = $2(l \times b) + 2(b \times h) + 2(l \times h) = 2(lb + bh + lh)$

   Lateral surface area (LSA) is the area of all the sides apart from the top and bottom faces.

   The lateral surface area of the cuboid = Area of face AEHD + Area of face BFGC + Area of face ABFE + Area of face DHGC

   LSA (cuboid) = $2(b \times h) + 2(l \times h) = 2h(l + b)$

   Length of diagonal of a cuboid $= \sqrt{(l^2 + b^2 + h^2)}$

4. If the length of each edge of a **cube** is 'a' units, then:

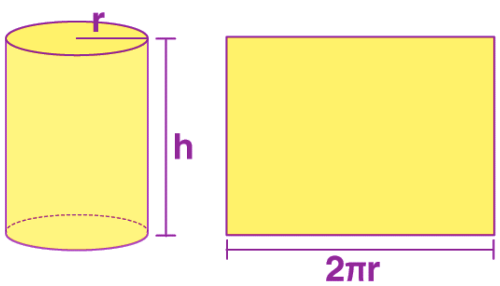
   Lateral surface area = $4 \times (\text{edge})^2$

   Total surface area = $6 \times (\text{edge})^2$

   Volume = $(\text{edge})^3$

   Diagonal of a cube $= \sqrt{3} \times$ edge

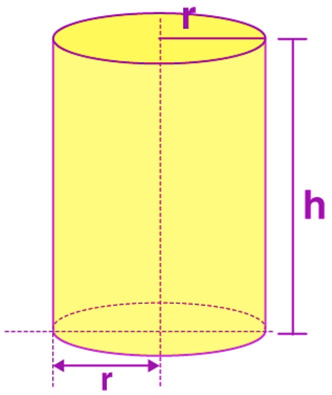
   Surface Area and Volume of Cylinder

   A cylinder is a solid shape that has two circular bases, connected with each other, through a lateral surface. Thus, there are three faces, two circular and one lateral, of a cylinder. Based on these dimensions, we can find the surface area and volume of a cylinder.

   Surface Area of Cylinder

   Take a cylinder of base radius r and height h units. The curved surface of this cylinder, if opened along the diameter (d = 2r) of the circular base can be transformed into a rectangle of length 2πr and height h units. Thus,



Transformation of a Cylinder into a rectangle.

CSA of a cylinder of base radius r and height h = 2π × r × h

TSA of a cylinder of base radius r and height h = 2π × r × h + area of two circular bases

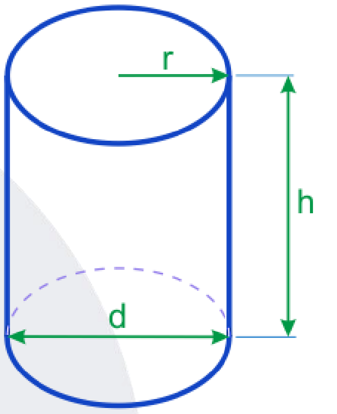
$=2\pi \times r \times h + 2\pi r^2$

$=2\pi r\,(h + r)$

Volume of a Cylinder

Volume of a cylinder = Base area × height = $(\pi r^2) \times h = \pi r^2 h$

Cylinder with height h and base radius r



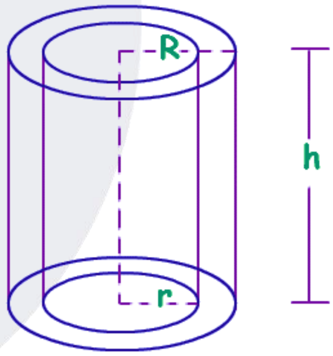
If r and h respectively denote the radius of the base and the height of a **right circular cylinder**, then: Area of each end or Base area $= \pi r^2$

Area of curved surface or lateral surface area = perimeter of the base x height $= 2\pi rh$

Total surface area (including both ends) $= 2\pi rh + \pi r^2 + 2\pi r\,(h + r)$

Volume = Area of the base × height = $\pi r^2 h$

5. If R and r respectively denote the external and internal radii of a **right circular hollow cylinder** and h denotes its height, then:



Area of each end $= \pi R^2 - \pi r^2$

Area of curved surface $= 2\pi\,(R + r)h$

Total surface area = (Area of curved surface) + 2(Area of each end)

$$= 2\pi\,(R + r)h + (\pi R^2 - \pi r^2)$$

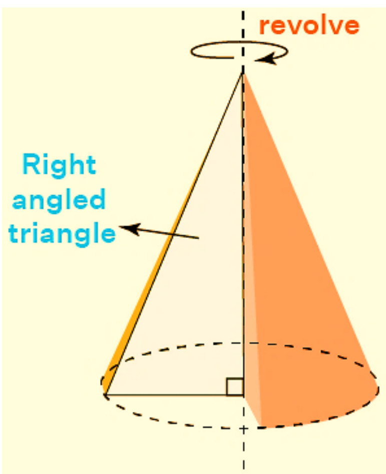
6. If r, h and l respectively denote the radius, height and slant height of a **right circular cone**, then:

Slant height $(\ell) = \sqrt{h^2+r^2}$

Area of curved surface $= \pi r\ell = \pi r\sqrt{h^2+r^2}$

Total surface area = Area of curved surface + Area of base $= \pi r\ell + \pi r^2 = \pi r(\ell + r)$

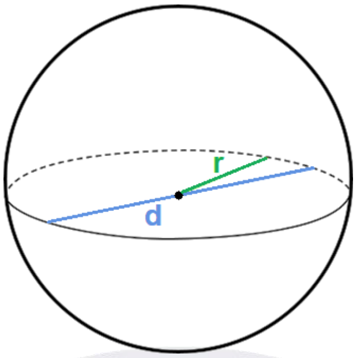
Volume $= \frac{1}{3}\pi r^2 h$



7. If r is the radius of a **sphere**, then:

Surface area $= \pi r^2$

Volume $= \frac{4}{3}\pi r^3$



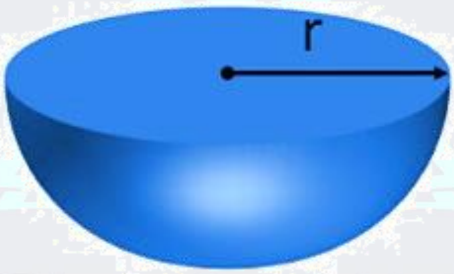
8. If r is the radius of a **hemisphere**, then:

Area of curved surface $= 2\pi r^2$

Total surface Area = Area of curved surface + Area of base

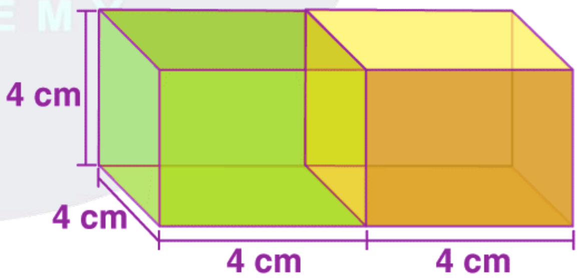
$= 2\pi r^2 + \pi r^2 = 3\pi r^2$

Volume $= \frac{2}{3}\pi r^3$



Surface Area of Combined Figures

Areas of complex figures can be broken down and analysed as simpler known shapes. By finding the areas of these known shapes, we can find out the required area of the unknown figure.



Example: 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Length of each cube = $64^{(1/3)}$ = 4cm

Since these cubes are joined adjacently, they form a cuboid whose length l = 8 cm. But height and breadth will remain the same = 4 cm.

Combination of 2 equal cubes

∴ The new surface area, TSA = 2(lb + bh + lh)

TSA = 2 (8 x 4 + 4 x 4 + 8 x 4)

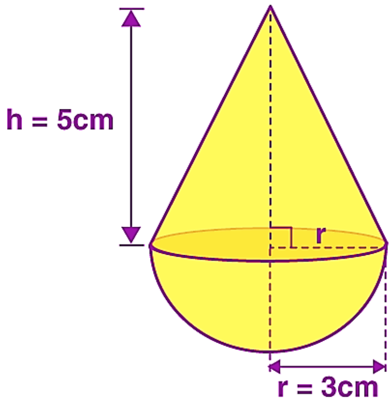
= 2(32 + 16 + 32)

= 2 (80)

TSA = 160 cm^2

Volume of Combined Solids

The volume of complex objects can be simplified by visualising them as a combination of shapes of known solids.



Example: A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 3 cm and the height of the cone is equal to 5 cm.

This can be visualised as follows:

Volume of combined solids

V(solid) = V(Cone) + V(hemisphere)

V(solid) = $(1/3)\pi r^2 h + (2/3)\pi r^3$

V(solid) = $(1/3)\pi(9)(5) + (2/3)\pi(27)$

V(solid) = 33π cm^3

Class : 10th mathematics
Chapter- 13 : Surface Areas and Volumes

Surface Areas and Volumes

Frustum of Cone

Curved Surface Area

CSA = $\pi l (r_1 + r_2)$

where $l = \sqrt{h^2 + (r_1 - r_2)^2}$

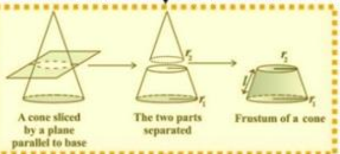
Surface Area

Sum of all of the surface areas of the faces of solid.

Volume

Quantity of 3-D space enclosed by a hollow/closed solid.

Conversion of Solids



Total Surface Area

$TSA = \pi l (r_1 + r_2) + \pi(r_1^2 + r_2^2)$

Volume

$V = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

Combination of Solids

A copper rod – Diameter 1cm, length 8cm converted into a wire of length 18m
Find the thickness of the wire.

Solution : Volume of the rod = $\pi\left(\frac{1}{2}\right)^2 \times 8\text{cm}^3$

$= 2\pi \text{ cm}^3$

Let, r is the radius of cross-section of the wire, volume = $\pi \times r^2 \times 1800 \text{ cm}^3$

$\therefore \pi \times r^2 \times 1800 = 2\pi$

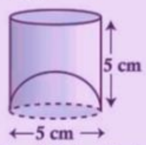
$r^2 = \frac{1}{900}$

$r = \frac{1}{30}$ cm

Thickness = Diameter of the cross-section

$= \frac{1}{15}$ cm = 0.07 cm

Given – Inner diameter of the Cylindrical glass = 5 cm
Height = 5 cm



Find – Actual capacity of Cylindrical glass

Solution :– Apparent capacity of the glass = $\pi r^2 h$

$= 3.14 \times 2.5 \times 2.5 \times 5 \text{ cm}^3$

$= 98.125 \text{ cm}^3$

Volume of hemisphere = $\frac{2}{3}\pi r^3$, if r = 2.5cm

$= \frac{2}{3} \times 3.14 \times (2.5)^3 \text{ cm}^3 = 32.71 \text{ cm}^3$

Actual capacity = Apparent capacity – Volume of hemisphere

= 98.125 – 32.71

= 65.42 cm^3

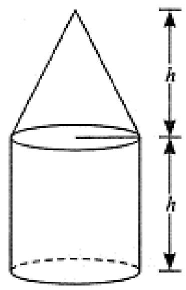
Important Questions

Multiple Choice Questions:

1. If the surface areas of two spheres are in ratio 16 : 9, then their volumes will be in the ratio:
   (a) 27 : 64
   (b) 64 : 27
   (c) 4 : 3
   (d) 3 : 4
2. A cylinder, a cone and a hemisphere are of equal base and have the same height. What is the ratio of their volumes?
   (a) 3 : 1 : 2
   (b) 3 : 2 : 1
   (c) 1 : 2 : 3
   (d) 1 : 3 : 2
3. If the area of three adjacent faces of cuboid are X, Y and Z respectively, then the volume of cuboid is:
   (a) XYZ
   (b) 3XYZ
   (c) $\sqrt{xyz}$
   (d) $\sqrt{3xyz}$
4. The volumes of two spheres are in the ratio 27 : 8. The ratio of their curved surface is:
   (a) 9 : 4
   (b) 4 : 9
   (c) 3 : 2
   (d) 2 : 3
5. The ratio of the volumes of two spheres is 8 : 27. If r and R are the radii of spheres respectively, then (R – r): r is:
   (a) 1 : 2
   (b) 1 : 3
   (c) 2 : 3
   (d) 4 : 9
6. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. The ratio of their volumes is:
   (a) 27 : 20
   (b) 20 : 27
   (c) 9 : 4
   (d) 4 : 9
7. If the radius of base of a right circular cylinder is halved, keeping the height same, the ratio of the volume of the reduced cylinder to that of the original cylinder is:
   (a) 2 : 3
   (b) 3 : 4
   (c) 1 : 4
   (d) 4 : 1
8. If the volumes of a cube is 1728 cm^3, the length of its edge is equal to:
   (a) 7 cm
   (b) 12 cm
   (c) 18 cm
   (d) 19 cm
9. The volume (in cm^3) of the largest right circular cone that can be cut off from a cube of edge 4.2 cm is: .
   (a) 9.7
   (b) 72.6
   (c) 58.2
   (d) 19.4
10. The circumference of the edge of hemispherical bowl is 132 cm. When π is taken as $\frac{22}{7}$, the capacity of bowl in cm^3 is:
   (a) 2772
   (b) 924
   (c) 19404
   (d) 9702

Very Short Questions:

1. What is the capacity of a cylindrical vessel with a hemispherical portion raised upward at the bottom?
2. A solid cone of radius r and height h is placed over a solid cylinder having same base radius and height as that of a cone. What is the total surface area of the combined solid?



3. Two identical solid hemispheres of equal base radius r cm are struck together along their bases. What will be the total surface area of the combination?
4. A solid ball is exactly fitted inside the cubical box of side a. What is the volume of the ball?
5. If two cubes of edge 5 cm each are joined end to end, find the surface area of the resulting cuboid.
6. A solid piece of iron in the form of a cuboid of dimension 49 cm × 33 cm × 24 cm is melted to form a solid sphere. Find the radius of sphere.
7. A mason constructs a wall of dimensions 270 cm × 300 cm × 350 cm with the bricks each of size 22.5 cm × 11.25 cm × 8.75 cm and it is assumed that space is covered by the mortar. Find the number of bricks used to construct the wall.
8. The radii of the ends of a frustum of a cone 40 cm high are 20 cm and 11 cm. Find its slant height.
9. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?
10. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. What is the ratio of their volumes?

Short Questions:

1. What is the ratio of the volume of a cube to that of a sphere which will fit inside it?
2. A vessel is in the form of a hollow hemisphere mounted by a hollow 7 cm cylinder. The diameter of the hemisphere is 14 cm and the total height T of the vessel is 13 cm. Find the inner surface area of the vessel.
3. Two cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.
4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.
5. The dimensions of a solid iron cuboid are 4.4 m × 2.6 m × 1.0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe.
6. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

OR

A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius on its circular face. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

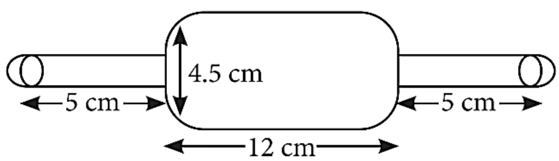
7. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Long Questions:

1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid. (Use $\pi = \frac{22}{7}$)
2. A hemispherical tank, full of water, is emptied by a pipe at the rate of $\frac{25}{7}$ litres per sec. How much time will it take to empty half the tank if the diameter of the base of the tank is 3 m?
3. Water is flowing through a cylindrical pipe, of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s. Determine the rise in level of water in the tank in half an hour.
4. 150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel.
5. From a solid cylinder of height 2.8 cm and diameter 4.2 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. (Take $\pi = \frac{22}{7}$)

Case Study Questions:

1. Arp an a is studying in X standard. While helping her mother in kitchen, she saw rolling pin made of steel and empty from inner side, with two small hemispherical ends as shown in the figure.



i. Find the curved surface area of two identical cylindrical parts, if the diameter is 2.5cm and length of each part is 5cm.

a. 475cm^2

b. 78.57cm^2

c. 877cm^2

d. 259.19cm^2

ii. Find the volume of big cylindrical part.

a. 190.93cm^3

b. 75cm^3

c. 77cm^3

d. 83.5cm^3

iii. Volume of two hemispherical ends having diameter 2.5cm, is:

a. 4.75cm^3

b. 8.18cm^3

c. 2.76cm^3

d. 75cm^3

iv. Curved surface area of two hemispherical ends, is:

a. 17.5cm^2

b. 7.9cm^2

c. 19.64cm^2

d. 15.5cm^2

v. Find the difference of volumes of bigger cylindrical part and total volume of the two small hemispherical ends.

a. 175.50cm^3

b. 182.75cm^3

c. 76.85cm^3

d. 96cm^3

2. Isha's father brought an ice-cream brick, empty cones and scoop to pour the ice-cream into cones for all the family members. Dimensions of the ice-cream brick are (30 × 25 × 10)cm? and radius of hemi-spherical scoop is 3.5cm. Also, the radius and height of cone are 3.5cm and 15cm respectively.

i. The quantity of ice-cream in the brick (in litres) is:

a. 3

b. 7.5

c. 2.5

d. 4.5

ii. Volume of hemispherical scoop is:

a. 40.6cm^3

b. 2509cm^3

c. 89.83cm^3

d. 20cm^3

iii. Volume of a cone is:

a. 148cm^3

b. 250.05cm^3

c. 145.83cm^3

d. 192.5cm^3

iv. The minimum number of scoops required to fill one cone up to brim is:

a. 2

b. 3

c. 4

d. 5

v. The number of cones that can be filled up to brim using the whole brick is:

a. 15

b. 39

c. 40

d. 42

Assertion Reason Questions:

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) Both A and R is false.

Assertion: If diameter of a sphere is decreased by 25%, then its curved surface area is decreased by 43.75%.

Reason: Curved surface area is increased when diameter decreases

2. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
   (a) Both A and R are true and R is the correct explanation of A.
   (b) Both A and R are true and R is not the correct explanation of A.
   (c) A is true but R is false.
   (d) Both A and R is false.

Assertion: The external dimensions of a wooden box are 18 cm, 10 cm and 6 cm respectively and thickness of the wood is 15 mm, then the internal volume is 765 cm^3.

Reason: If external dimensions of a rectangular box be l, b and h and the thickness of its sides be x, then its internal volume is (l – 2x) (b – 2x) (h – 2x).

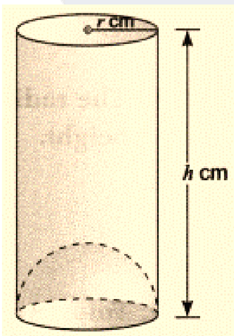
Answer Key

Multiple Choice Questions:

1. (b) 64 : 27
2. (a) 3 : 1 : 2
3. (c) $\sqrt{xyz}$
4. (a) 9 : 4
5. (a) 1 : 2
6. (b) 20 : 27
7. (c) 1 : 4
8. (b) 12 cm
9. (d) 19.
10. (a) 19404 cm^3

Very Short Answers:

1.



Capacity of the given vessel

= capacity of cylinder – capacity of hemisphere

$$=\pi r^2h-\frac{2}{3}\pi r^3=\frac{\pi r^2}{3}(3h-2r)$$

2. The total surface area of the combined solid in Fig.

= curved surface area of cone + curved surface area of cylinder + area of the base.

$$=\pi rl+2\pi rh+\pi r^2$$

$$=\pi r(l+2h+r)=\pi r\left(\sqrt{r^2+h^2}+2h+r\right)$$

3. The resultant solid will be a sphere of radius r whose total surface area is $4\pi r^2$.

4. Diameter of the solid ball = edge of the cube = a

$\therefore$ Volume of the ball $=\frac{4}{3}\pi\left(\frac{a}{2}\right)^3=\frac{4}{3}\times\frac{1}{8}\pi a^3=\frac{1}{6}\pi a^3$

5. Total length (l) = 5 + 5 = 10 cm

Breadth (b) = 5 cm, Height (h) = 5 cm

Surface Area = 2 (lb + bh + lh)

= 2(10 × 5 + 5 × 5 + 5 × 10) = 2 × 125 = 250 cm^2

6. Volume of iron piece = Volume of the sphere formed

= 49 × 33 × 24 = $\frac{4}{3}\pi r^2$

$$r^3=\frac{49\times33\times24\times3\times7}{4\times22}$$

r = 21 cm

7. Space occupied with bricks = $\frac{7}{8}$ × volume of the wall

= $\frac{7}{8}$ × 270 × 300 × 350

$$\therefore \text{Number of bricks}=\frac{\text{Space occupied with bricks}}{\text{Volume of one brick}}$$

$$=\frac{\frac{7}{8}\times270\times300\times350}{22.5\times11.25\times8.75}=11{,}200$$

8. $l=\sqrt{h^2+(r_1-r_2)^2}$

$$=\sqrt{40^2+(20-11)^2}=\sqrt{1600+81}$$

$$=\sqrt{1681}=41\,cm$$

9. As per question

Volume of hemisphere = Surface area of hemisphere

$= \frac{2}{3}\pi r^2 = 3\pi r^2 =$, units $r = \frac{9}{2}$ units.

10. Volume of a cone: Volume of a hemisphere: Volume of a cylinder

$$= \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi^3 : \pi r^2 h$$

$$= \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 \qquad (\because r = h)$$

= 1 : 2 : 3

Short Answers:

1. Let edge of the cube be 'a'.

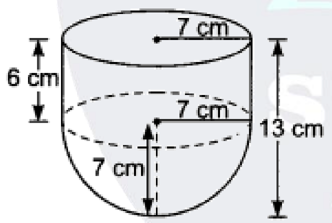
Then, diameter of the sphere that will fit inside the given cube = a

∴ Volume of the cube : Volume of the sphere.

$$= a^3 : \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = a^3 : \frac{4}{3} \times \frac{1}{8}\pi a^3$$

$$= a^3 : \frac{1}{6}\pi a^3 = 6 : \pi$$

2.



Here, radius of hemisphere = radius of cylinder = r cm = 7 cm

and height of cylinder, h = (13 – 7) cm = 6 cm

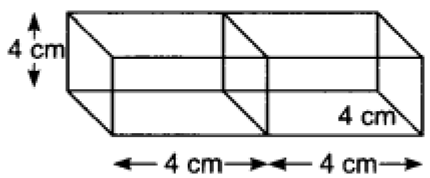
Now, inner surface area of the vessel

= Curved surface area of the cylindrical part + Curved surface area of hemispherical part = (2πrh + 2πr2) = 2πr (h + r)

$= 2 \times \frac{22}{7} \times 7\ (6 + 7)$

$= 2 \times 22 \times 13 = 572\ cm^2$

3.



Let the length of each edge of the cube of volume 64 cm3 be x cm.

Then, Volume = 64 cm^3

$\Rightarrow x^2 = 64$

$\Rightarrow x^2 = 43$

$\Rightarrow x = 4$ cm

4 cm The dimensions of cuboid so formed are

l = Length = (4 + 4) cm = 8 cm

b = Breadth = 4 cm and h = Height = 4 cm

∴ Surface area of the cuboid = 2 (lb + bh + lh)

= 2 (8 × 4 + 4 × 4 + 8 × 4)

= 2 (32 + 16 + 32)

= 160 cm^2

4. The greatest diameter that a hemisphere can have = 7 cm = l

Radius of the hemisphere (R) = $\frac{7}{2}$ cm

∴ Surface area of the solid after surmounting hemisphere

$= 6l^2 - \pi R^2 + 2\pi R^2 = 6l^2 + \pi R^2$

$$= 6(7)^2 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= 6 \times 49 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$= 294 + 38.5 = 332.5\ cm^2$

5. Let the length of pipe by h m.

Volume of cuboid = 4.4 × 2.6 × 1 m^2

Inner and outer radii of cylindrical pipe are 30 cm, (30 + 5) cm = 35 cm

$$\therefore \text{Volume of material used} = \frac{\pi}{100^2}(35^2 - 30^2) \times h\,m^3$$

$$= \frac{\pi}{100^2} \times 65 \times 5h$$

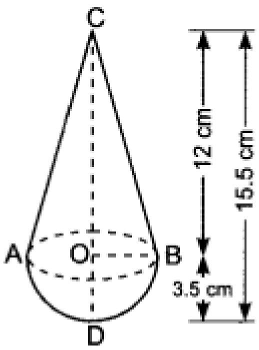
[using $a^2 - b^2 = (a + b)(a - b)$]

$$\text{Now } \frac{\pi}{100^2} \times 65 \times 5h = 4.4 \times 2.6$$

$$\Rightarrow \quad h = \frac{7 \times 4.4 \times 2.6 \times 100 \times 100}{22 \times 65 \times 5}$$

$\Rightarrow \quad h = 112$ m

6.



We have,

CD = 15.5 cm and OB = OD = 3.5 cm

Let r be the radius of the base of cone and h be the height of conical part of the toy.

Then, r = OB = 3.5 cm

h = OC = CD – OD = (15.5 – 3.5) cm = 12 cm

$$l=\sqrt{r^2+h^2}=\sqrt{3.5^2+12^2}$$

$$=\sqrt{12.25+144}=\sqrt{156.25}=12.5\,cm$$

Also, radius of the hemisphere, r = 3.5 cm

∴ Total surface area of the toy = Surface area of cone + Surface area of hemisphere

$$l=\sqrt{r^2+h^2}=\sqrt{3.5^2+12^2}$$

$$=\sqrt{12.25+144}=\sqrt{156.25}=12.5\,cm$$

Also, radius of the hemisphere, $r = 3.5\ cm$

∴ Total surface area of the toy

= Surface area of cone + Surface area of hemisphere

$$=\pi rl+2\pi r^2=\pi r(l+2r)=\frac{22}{7}\times 3.5(12.5+2\times 3.5)$$

$$=\frac{22}{7}\times 3.5\times 19.5=214.5\,cm^2$$

7. Here, we have

Edge of the cube = l = Diameter of the hemisphere

Therefore, radius of the hemisphere = $\frac{l}{2}$

∴ Surface area of the remaining solid after cutting out the hemispherical

$$\text{Depression}=6l^2-\pi\left(\frac{l}{2}\right)^2+2\pi\left(\frac{l}{2}\right)^2$$

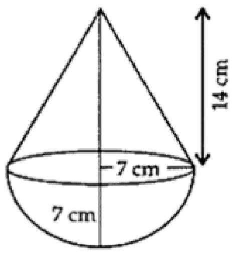
$$=6l^2+\pi\times\frac{l^2}{4}=\frac{l^2}{4}(24+\pi)$$

Long Answers:

1. Radius, r = 7 cm

Height of cone, h = 2(7) = 14 cm

Volume of solid = Vol. of hemisphere + Volume of cone



$$=\frac{2}{3}\pi r^3+\frac{1}{3}\pi r^2h$$

$$=\frac{1}{3}\pi r^2(2r+h)$$

$$=\frac{1}{3}\times\frac{22}{7}\times 7\times 7\,(2(7)+14)$$

$$=\frac{22\times 7}{3}\times 28=\frac{4312}{3}$$

$$=1437.\overline{3}\,cm^3$$

2. Here, $r=\frac{3}{2}m$

$$\frac{25}{7}\,lt=\frac{1}{1000}\times\frac{25}{7}m^3=\frac{1}{280}m^3$$

$$\ldots\left[\because 1\,lt.=\frac{1}{1000}m^3\right]$$

$$\text{Required time}=\frac{\frac{1}{2}\text{Vol. of hemispherical tank}}{\text{Vol. of pipe}}$$

$$=\frac{\frac{1}{2}\times\frac{2}{3}\times\frac{22}{7}\times\frac{3}{2}\times\frac{3}{2}\times\frac{3}{2}}{\frac{1}{280}}$$

$$=\frac{22\times 9}{7\times 8}\times\frac{280}{1}=990\text{ secs.}$$

= 16 mins. 30 secs.

3. Radius of tank, r_1 = 40 cm

Internal radius of cylindrical pipe, $r_2=\frac{2}{2}$ = 1 cm

Let the height of rises water, h_1 = ?

Length of water flow in 1 second = 0.4 m

= $\frac{4}{10}$ × 100 = 40 cm

∴ Length of water flow in 30 minutes, h2

= 40 × 60 × 30 = 72,000 cm

Volume of water in cylinder tank

= Volume of water flow from cylindrical pipe in half an hour

As $\pi r_1^2 h_1 = \pi r_2^2 h_2$

$\therefore\ 40\times 40\times h_1 = 1\times 1\times 72{,}000$

$$h_1 = \frac{72{,}000}{40\times 40} = 45\,cm$$

∴ Level of water in cylinder tank rises in half an hour, h_1 = 45 cm

4. Radius of marble, $r = \frac{1.4}{2} = \frac{7}{10}\,cm,$

Radius of cylinder, $R = \frac{7}{2} = 3.5\,cm$

No. of spherical marbles

$$= \frac{\text{Vol. of water rise in cylinder}}{\text{Vol. of one marble (sphere)}}$$

$$150 = \frac{\pi\left(\frac{7}{2}\times\frac{7}{2}\times h\right)}{\frac{4}{3}\times\pi\times\frac{7}{10}\times\frac{7}{10}\times\frac{7}{10}}$$

$$\because \left[\begin{array}{l}\text{Vol. of cylinder} = \pi r^2 h\\ \text{Vol. of sphere} = \frac{4}{3}\pi r^3\end{array}\right.$$

$$\Rightarrow\quad \frac{7}{2}\times\frac{7}{2}\times h = 150\times\frac{4}{3}\times\frac{7}{10}\times\frac{7}{10}\times\frac{7}{10}$$

$$h = \frac{50\times 2\times 2\times 4\times 7}{10\times 10\times 10} = \frac{56}{10}\,cm$$

$\therefore$ Rise in water level, $h = \frac{56}{10} = 5.6\,cm$

5. Given: $r = \frac{4.2}{2} = 2.1\,cm,$ h = 2.8 cm

$$l = \sqrt{r^2 + h^2} = \sqrt{(2.1)^2 + (2.8)^2}$$

$$= \sqrt{4.41 + 7.84} = \sqrt{12.25}$$

= 3.5 cm

T.S. area of the remaining solid

= C.S. ar. of cyl. + area of base + C.S. ar. of cone

$$= 2\pi rh + \pi r^2 + \pi rl$$

$$= \pi r(2h + r + l)$$

$$= \frac{22}{7}\times 2.1\,(5.6 + 2.1 + 3.5)$$

= 22 × 0.3 (11.2)

= 73.92 cm^2

Case Study Answers:

1. **Answer:**

i. (b) 78.57 cm^2

Solution:

Curved surface area of two identical cylindrical parts,

$$= 2\times 2\pi rh = 2\times 2\times\frac{22}{7}\times\frac{2.5}{2}\times 5$$

= 78.57 cm^2

ii. (a) 190.93 cm3

Solution:

Volume of big cylindrical part $= \pi r^2 h$

$$= \frac{22}{7}\times\frac{4.5}{2}\times\frac{4.5}{2}\times 12190.93\,cm^3$$

iii. (b) 8.18 cm^3

Solution:

Volume of two hemispherical end

$$= 2\times\frac{2}{3}\pi r^3$$

$$= \frac{2\times 2}{3}\times\frac{22}{7}\times\left(\frac{2.5}{2}\right)^3 = 8.18\,cm^3$$

iv. (c) 19.64 cm^2

Solution:

Curved surface are of two hemispherical ends,

$$= 2\times 2\pi r^2 = 2\times 2\times\frac{22}{7}\times\frac{2.5}{2}\times\frac{2.5}{2} = 19.64\,cm^3$$

v. (b) 182.75 cm^3

Solution:

Difference of volume of bigger cylinder to two small hemispherical ends = 190.93 – 8.18 = 182.75 cm^3

2. **Answer:**

i. (b) 7.5

Solution:

Quantity of the ice-cream in the brick = Volume of the brick = (30 × 25 × 10) cm^3 = 7500 cm^3

$$= \frac{7500}{1000}l \qquad [\because 1l = 1000\,cm^3]$$

= 7.5l

ii. (c) 89.83 $cm3$

Solution:

Volume of hemispherical scoop $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3}\times\frac{22}{7}\times(3.5)^3 = \frac{1886.5}{21} = 89.83\,cm^3$$

iii. (d) 192.5 cm3

Solution:

$$\text{Volume of cone } = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 15 = \frac{4042.5}{21} = 192.5\, cm^3$$

iv. (a) 2

Solution:

Number of scoops required to fill one cone

$$= \frac{\text{Volume of a cone}}{\text{Volume of a scoop}} = \frac{192.5}{89.83} = 2.14 \approx 2$$

v. (b) 39

Solution:

Number of cones that can be filled using the,

$$\text{Whole brick } = \frac{\text{Volume of brick}}{\text{Volume of 1 cone}}$$

$$= \frac{2500}{192.5} = 38.960 \approx 39$$

Assertion Reason Answers:

1. (c) A is true but R is false.
2. (a) Both A and R are true and R is the correct explanation of A.

Statistics 13

1. Three measures of central tendency are:

i. Mean

ii. Median

iii. Mode

2. The **arithmetic mean**, also called the average, is the quantity obtained by adding all the observations and then dividing by the total number of observations.

3. Arithmetic mean may be computed by anyone of the following methods:

i. Direct method

ii. Short-cut method/ Assumed mean method

iii. Step-deviation method

4. **Direct method** of finding mean:

If a variant X takes values x_1, x_2, x_3.... x_n with corresponding frequencies f_1, f_2, f_3 ,... f_n respectively, then arithmetic mean of these values is given by:

$$\overline{X} = \frac{\sum_{i=1}^{n} f_i x_i}{N} \text{ where } N = \sum_{i=1}^{n} f_1 + f_2 + f_2 \ldots \ldots . . + f_n$$

5. **Class mark** = $\frac{1}{2}$ (Upper class limit + Lower class limit)

6. **Short-cut method/ assumed mean method** of finding mean:

Let x_1, x_2....,x_n be values of a variable X with corresponding frequencies f_1,f_2, f_3...,fn respectively. Let A be the assumed mean. Then:

$$\overline{X} = A + \frac{1}{N}\left\{\sum_{i=1}^{n} f_i d_i\right\}$$

Note that in case of continuous frequency distribution, the values of x1, x2, x3 ... x_n, are taken as the mid-points or class-marks of the various classes.

7. **Step-deviation method** of finding mean:

Let x1, x2....,xn be values of a variable X with corresponding frequencies f_1,f_2, f_3,.....fn respectively. Let A be the assumed mean. Then:

$$\overline{X} = A + h\left\{\frac{1}{N}\sum_{i=1}^{n} f_i u_i\right\}$$

Here, h is generally taken as common factor of the deviations, in case of ungrouped frequency distribution. And, in case of grouped frequency distribution, h is the class width, $u_i = \frac{x_{i-A}}{h} = \frac{d_i}{h}$

Note that in case of continuous frequency distribution, the values of x1, x2, x3 ..., xn are taken as the mid-points or class-marks of the various classes.

8. The step deviation method will be convenient to apply if all the deviations (d's) have a common factor.

9. If class mark obtained, are in decimal form, then step deviation method is preferred to calculate mean.

10. **Median** is a measure of central tendency which gives the value of the middle observation in the data, arranged in order. It is that value such that the number of observations above it is equal to the number of observations below it.

11. For finding the median of a raw data, we arrange the given data in increasing or decreasing order. If n is odd, then median is the value of $\left(\frac{n+1}{2}\right)^{th}$ observation.

If n is even, then median is the arithmetic mean of the values of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations.

12. The **cumulative frequency** of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class to the frequency of the class.

13. In case of an **ungrouped frequency distribution**, we calculate the **median** by following the steps given below:

Step 1: Find the cumulative frequencies (c.f.) and obtain N $= \sum f_1$.

Step 2: Find $\frac{n}{2}$

Step 3: Look for the cumulative frequency (c. f.) just greater than $\frac{n}{2}$ and determine the corresponding value of the variable. The value so obtained is the median.

14. In case of a **continuous frequency distribution**, we calculate the **median** by following the steps:

Step 1: Find the cumulative frequencies (c.f.) and obtain N $= \sum f_1$.

Step 2: Find $\frac{N}{2}$

Step 3: Look for the cumulative frequency (c. f.) just greater than $\frac{N}{2}$ and determine the correspondingclass. This class is known as the median class. (Note that the value of the median will lie in this class)

Step 4: Use the following formula to find median:

$$\text{Mediun} = l + \left[\frac{\frac{N}{2} - cf}{f}\right] \times h$$

Here, l = lower limit of the median class

f = frequency of the median class

h = width (size) of the median class

cf = cumulative frequency of the class preceding the median class

$$N = \sum f_1 .$$

15. **Mode** is the value of the most frequently occurring observation in the data.

16. In an ungrouped frequency distribution, mode is the value of the variable having maximum frequency.

17. In a **grouped frequency distribution**, the modal class is the one with highest frequency and the **mode** can be calculated by the following formula

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

l = lower limit of the modal class

h = size of the class interval

f_1 = frequency of the modal class

f_0 =frequency of the class preceding the modal class

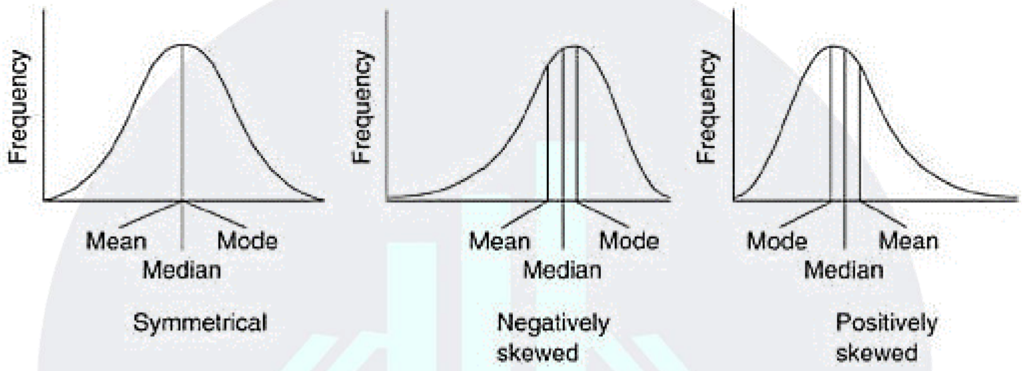
f_2 = frequency of the class succeeding the modal class

18. The most frequently used measure of central tendency is the mean, because the mean is calculated by taking into account all the observations of a given data. And it lies between the smallest and the largest value of the data.
19. The biggest drawback in considering mean is that it is affected by the extreme values. One large or small number can distort the average. In that case, median is a better measure of central tendency. While, when the most repeated value or the most wanted one is required, then mode is used.
20. When all three measures of central tendency are equal, the distribution is called **symmetrical distribution**.
21. When the values of mean, median and mode are not equal, then the distribution is known as **asymmetrical or skewed**. In this case, the distribution can be positively skewed or negatively skewed.

    Negatively skewed distributions have a few extremely low scores, while positively skewed distributions have a few extremely high scores.

    i. When the data is negatively skewed, then Mean < Median < Mode

    ii. When the positively skewed, then Mean > Median > Mode



Three measure of central values are connected by the following relation:

3 Median = Mode + 2 Mean

22. The **cumulative frequency** is the accumulated or sum of frequencies up to a particular point. A table showing the cumulative frequencies is called a **cumulative frequency distribution**.
23. There are two types of cumulative frequencies:

    i. Less than type cumulative frequency distribution: It is found by adding sequentially the frequencies of all the earlier classes including the class adjacent to which it is written. The cumulate is started from the lowest to the highest size.

    ii. More than type cumulative frequency distribution: It is obtained by finding the cumulate of frequencies starting from the highest to the lowest class.
24. A cumulative frequency distribution can be represented graphically by means of an **ogive.**
25. There are two types of ogives:

    i. **'Less than' ogive**: In a less than ogive the upper limit of a class (x axis) is plotted against its cumulative frequency (y axis) as a point on the ogive. The 'less than ogive' is a rising curve.

    ii. **'More than' ogive**: In a 'more than ogive' the lower limit of a class (x axis) is plotted against its cumulative frequency (y axis) as a point on the ogive. The 'more than ogive' is a falling curve.

Ungrouped Data

Ungrouped data is data in its original or raw form. The observations are not classified into groups.

For example, the ages of everyone present in a classroom of kindergarten kids with the teacher is as follows:

3, 3, 4, 3, 5, 4, 3, 3, 4, 3, 3, 3, 3, 4, 3, 27.

This data shows that there is one adult present in this class and that is the teacher. Ungrouped data is easy to work with when the data set is small.

Grouped Data

In grouped data, observations are organized in groups.

For example, a class of students got different marks in a school exam. The data is tabulated as follows:

Mark interval	0-20	21-40	41-60	61-80	81-100
No. of Students	13	9	36	32	10

This shows how many students got the particular mark range. Grouped data is easier to work with when a large amount of data is present.

Frequency

Frequency is the number of times a particular observation occurs in data.

Class Interval

Data can be grouped into class intervals such that all observations in that range belong to that class.

Class width = upper class limit – lower class limit

Mean

Finding the mean for Grouped Data when class Intervals are not given

For grouped data without class intervals,

$$\text{Mean} = \overline{x} = \frac{\sum x_i f_i}{\sum f_i}$$

where f_i is the frequency of i^{th} observation x_i.

Finding the mean for Grouped Data when class Intervals are given

For grouped data with class intervals,

$$\text{Mean} = \overline{x} = \frac{\sum x_i f_i}{\sum f_i}$$

Where f_i is the frequency of i^{th} class whose class mark is x_i.

Classmark = (Upper Class Limit+ Lower Class Limit)/2

Direct method of finding mean

Step 1: Classify the data into intervals and find the corresponding frequency of each class.

Step 2: Find the class mark by taking the midpoint of the upper and lower class limits.

Step 3: Tabulate the product of the class mark and its corresponding frequency for each class. Calculate their sum ($\sum x_i f_i$).

Step 4: Divide the above sum by the sum of frequencies ($\sum f_i$) to get the mean.

Assumed mean method of finding mean

Step 1: Classify the data into intervals and find the corresponding frequency of each class.

Step 2: Find the class mark by taking the midpoint of the upper and lower class limits.

Step 3: Take one of the xi's (usually one in the middle) as the assumed mean and denote it by 'a'.

Step 4: Find the deviation of 'a' from each of the x_is

$$d_i = x_i - a$$

Step 5: Find the mean of the deviations

$$\overline{d} = \frac{\sum f_i d_i}{\sum f_i}$$

Step 6: Calculate the mean as

$$\overline{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

The relation between the Mean of deviations and mean

$$d_i = x_i - a$$

Summing over all x_i, s

$$\sum d_i = \sum x_i - \sum a$$

Dividing throughout by $\sum f_i = n$, Where 'n' is the total number of observations.

$$\overline{d} = \overline{x} - a$$

Step-Deviation method of finding mean

Step 1: Classify the data into intervals and find the corresponding frequency of each class.

Step 2: Find the class mark by taking the midpoint of the upper and lower class limits.

Step 3: Take one of the x′is (usually one in the middle) as assumed mean and denote it by ′a′.

Step 4: Find the deviation of a from each of the x′is

$$d_i = x_i - a$$

Step 5: Divide all deviations –di by the class width (h) to get u′is.

$$u_i = \frac{x_i - a}{h}$$

Step 6: Find the mean of u′is

$$\overline{u} = \frac{\sum f_i u_i}{\sum f_i}$$

Step 7: Calculate the mean as

$$\overline{x} = a + h \times \frac{\sum f_i u_i}{\sum f_i} = a + h\overline{u}$$

Relation between mean of Step- Deviations (u) and mean

$$u_i = \frac{x_i - a}{h}$$

$$\overline{u} = \frac{\sum f_i \frac{x_i - a}{h}}{\sum f_i}$$

$$\overline{u} = \frac{1}{h} \times \frac{\sum f_i x_i - a\sum f_i}{\sum f_i}$$

$$\overline{u} = \frac{1}{h} \times (\overline{x} - a)$$

Important relations between methods of finding mean

- All three methods of finding mean yield the same result.
- Step deviation method is easier to apply if all the deviations have a common factor.
- Assumed mean method and step deviation method are simplified versions of the direct method.

Median

Finding the Median of Grouped Data when class Intervals are not given

Step 1: Tabulate the observations and the corresponding frequency in ascending or descending order.

Step 2: Add the cumulative frequency column to the table by finding the cumulative frequency up to each observation.

Step 3: If the number of observations is odd, the median is the observation whose cumulative frequency is just greater than or equal to (n+1)/2

If the number of observations is even, the median is the average of observations whose cumulative frequency is just greater than or equal to n/2 and (n/2)+1.

Cumulative Frequency

Cumulative frequency is obtained by adding all the frequencies up to a certain point.

Finding median for Grouped Data when class Intervals are given

Step 1: Find the cumulative frequency for all class intervals.

Step 2: The median class is the class whose cumulative frequency is greater than or nearest to n2, where n is the number of observations.

Step 3: Median = l + [(N/2 – cf)/f] × h

Where,

l = lower limit of median class,

n = number of observations,

cf = cumulative frequency of class preceding the median class,

f = frequency of median class,

h = class size (assuming class size to be equal).

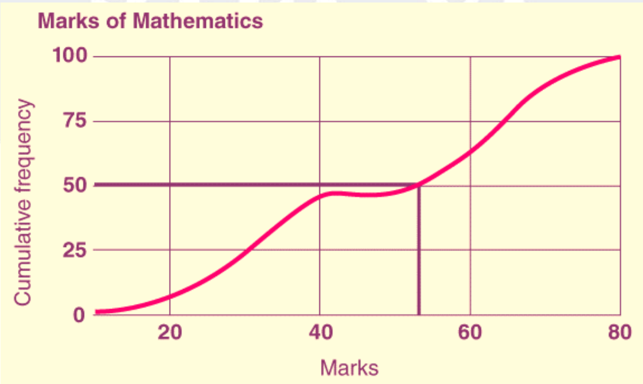
Cumulative Frequency distribution of less than type

Cumulative frequency of the less than type indicates the number of observations which are less than or equal to a particular observation.

Cumulative Frequency distribution of more than type

Cumulative frequency of more than type indicates the number of observations that are greater than or equal to a particular observation.

Visualising formula for median graphically



Median from Cumulative Frequency Curve

Step 1: Identify the median class.

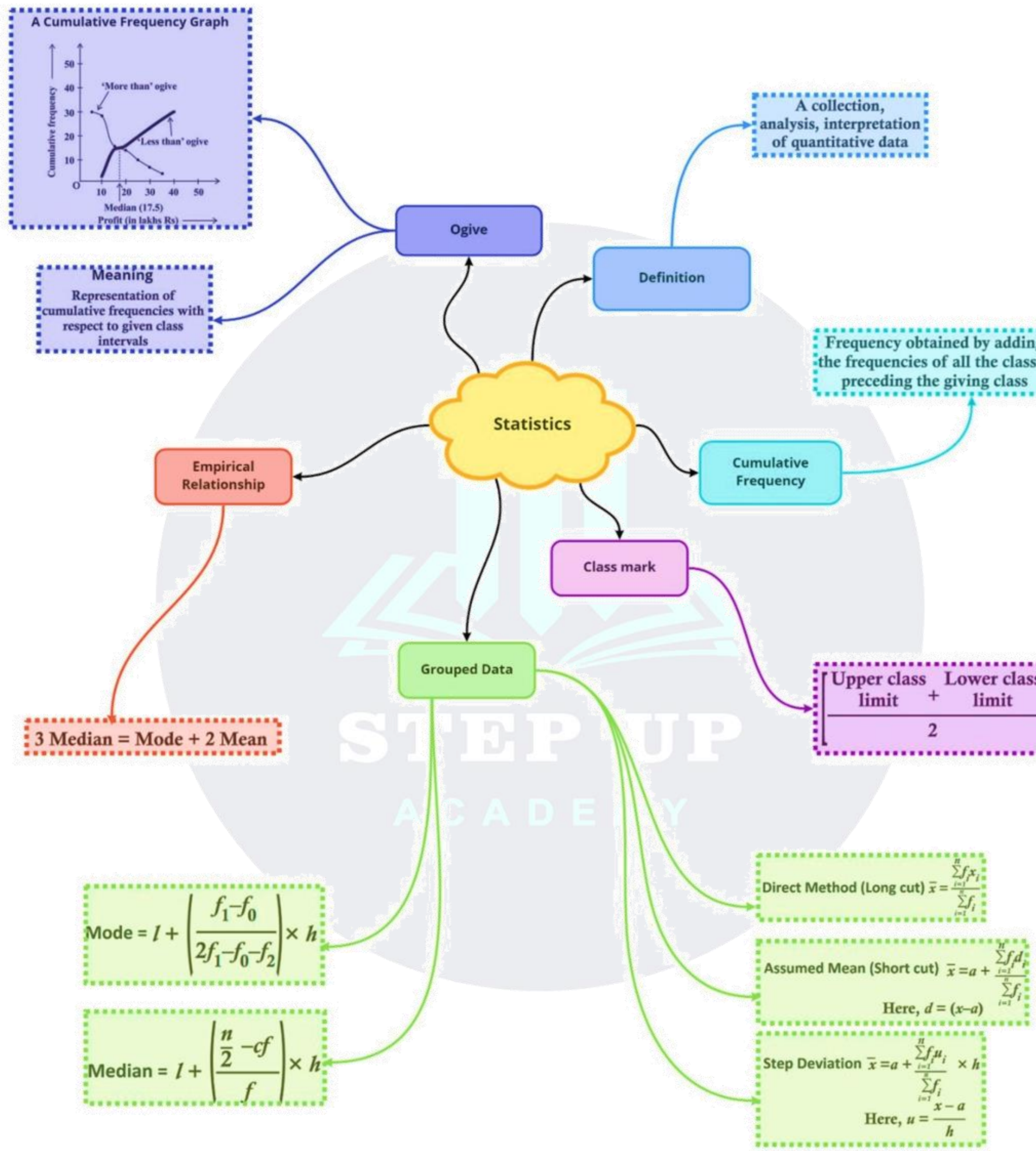
Step 2: Mark cumulative frequencies on the y-axis and observations on the x-axis corresponding to the median class.

Step 3: Draw a straight line graph joining the extremes of class and cumulative frequencies.

Step 4: Identify the point on the graph corresponding to cf = n/2

Step 5: Drop a perpendicular from this point onto the x-axis.

Class : 10th mathematics
Chapter- 14 : Statistics



Important Questions

Multiple Choice Questions:

1. The relationship between mean, median and mode for a moderately skewed distribution is
   (a) mode = median – 2 mean
   (b) mode = 3 median – 2 mean
   (c) mode = 2 median – 3 mean
   (d) mode = median – mean
2. The median of set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set
   (a) is increased by 2
   (b) is decreased by 2
   (c) is two times of the original number
   (d) Remains the same as that of the original set.
3. Mode and mean of a data are 12k and 15A. Median of the data is
   (a) 12k
   (b) 14k
   (c) 15k
   (d) 16k
4. The times, in seconds, taken by 15k athletes to run a 110 m hurdle race are tabulated below:

Class	Frequency
13.8 – 14.0	2
14.0 – 14.2	4
14.2 – 14.4	5
14.4 – 14.6	71
14.6 – 14.8	48
14.8 – 15.0	20

   The number of athletes who completed the race in less then 14.6 seconds is:
   (a) 11
   (b) 71
   (c) 82
   (d) 130
5. The abscissa of the point of intersection of the less than type and of the more than type cumulative frequency curves of a grouped data gives its
   (a) mean
   (b) median
   (c) mode
   (d) all the three above
6. While computing mean of grouped data, we assume that the frequencies are:
   (a) evenly distributed over all the classes
   (b) centered at the class marks of the classes
   (c) centered at the upper limits of the classes
   (d) centered at the lower limits of the classes
7. Mean of 100 items is 49. It was discovered that three items which should have been 60, 70, 80 were wrongly read as 40, 20, 50 respectively. The correct mean is
   (a) 48
   (b) 49
   (c) 50
   (d) 60
8. While computing mean of grouped data, we assume that the frequencies are
   (a) centered at the upper limits of the classes
   (b) centered at the lower limits of the classes
   (c) centered at the class marks of the classes
   (d) evenly distributed over all the classes
9. Which of the following cannot be determined graphically?
   (a) Mean
   (b) Median
   (c) Mode
   (d) None of these

Very Short Questions:

1. In a continuous frequency distribution, the median of the data is 21. If each observation is increased by 5, then find the new median.
2. From the following frequency distribution, find the median class:

Cost of living index	*No. of weeks*
1400-1550	8
1550-1700	15
1700-1850	21
1850-2000	8

3. Consider the following distribution, find the frequency of class 30-40.

Marks obtained	***No. of Students***
0 or more	63
10 or more	58
20 or more	55
30 or more	51
40 or more	48
50 or more	42

4. Following table shows sale of shoes in a store during one month:

Size of shoe	***No. of pairs sold***
3	4
4	18
5	25
6	12
7	5
8	1

Find the model size of the shoes sold.

5. Weekly household expenditure of families living in a housing society are shown below:

Weekly expenditure (in ₹)	***No. of families (f)***
Up to 3000	4
3000-6000	25
6000-9000	31
9000-12000	48
12000-15000	10

Find the upper limit of the modal class.

6. Find the class mark of the class 10 – 25.
7. Find the mean of the first five natural numbers.
8. A data has 13 observations arranged in descending order. Which observation represents the median of data?
9. If the mode of a distribution is 8 and its mean is also 8, then find median.
10. In an arranged sense of an even number of 2n terms which term is median?

Short Questions:

1. If xi's are the mid-points of the class intervals of a grouped data. fi's are the corresponding frequencies and is the mean, then find $\Sigma f_i\,(x_i - \bar{x})$.

2. Consider the following frequency distribution.

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

3. Find the median class of the following distribution:

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	4	8	10	12	8	4

4. Find the class marks of classes 15.5 – 18.5 and 50 – 75.

5. If the mean of the following distribution is 6, find the value of p.

x	2	4	6	10	$p + 5$
f	3	2	3	1	2

6. Find the mean of the following distribution:

x	4	6	9	10	15
f	5	10	10	7	8

7. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetime (in hours)	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	4	4	8	10	12	8

Determine the modal lifetimes of the components.

8. The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Number of Students	2	3	8	6	6	3	2

Long Questions:

1. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45-55	55-65	65-75	75-85	85-90
Number of Cities	3	10	11	8	3

2. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ₹ 18. Find the missing frequency f.

Daily pocket allowance (in ₹)	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Number of Children	7	6	9	13	f	5	4

3. The mean of the following frequency distribution is 62.8. Find the missing frequency x.

Classes	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	8	x	12	7	8

Case Study Questions:

1. A petrol pump owner wants to analyse the daily need of diesel at the pump. For this he collected the data of vehicles visited in 1hr. The following frequency distribution table shows the classification of the number of vehicles and quantity of diesel filled in them.

Diesel Filled (in liters)	3-5	5-7	7-9	9-11	11-13
Number of Vehicles	5	10	10	7	8

i. Which of the following is correct?

a. If x_i and f_i are sufficiently small, then direct method is appropriate choice for calculating mean.

b. If x_i and f_i are sufficiently large, then direct method is appropriate choice for calculating mean.

c. If x_i and f_i are sufficiently small, then assumed mean method is appropriate choice for calculating mean.

d. None of the above.

ii. Average diesel required for a vehicle is:

a. 8.15 litres
b. 6 litres
c. 7 litres
d. 5.5 litres

iii. If approximately 2000 vehicles comes daily at the petrol pump, then how much litres of diesel the pump should have?

a. 16200 litres
b. 16300 litres
c. 10600 litres
d. 15000litres

iv. The sum of upper and lower limit of median class is:

a. 22
b. 10
c. 16
d. None of this.

v. If the median of given data is 8 litres, then mode will be equal to:

a. 7.5 litres
b. 7.7 litres
c. 5.7 litres
d. 8 litres

2. A bread manufacturer wants to know the lifetime of the product. For this, he tested the lifetime of 400 packets of bread. The following tables gives the distribution of the lifetime of 400 packets.

Lifetime (in hours)	Number of packets (Cumulative frequency)
150-200	14
200-250	70
250-300	130
300-350	216
350-400	290
400-450	352
450-500	400

i. If m be the class mark and b be the upper limit of a class in a continuous frequency distribution, then lower limit of the class is:

a. $2m + \sqrt{b}$
b. 2m + b
c. m - b
d. 2m - b

ii. The average lifetime of a packet is:

a. 341hrs
b. 300hrs
c. 340hrs
d. 301hrs

iii. The median lifetime of a packet is:

a. 347hrs
b. 340hrs
c. 346hrs
d. 342hrs

iv. If empirical formula is used, then modal lifetime of a packet is:

a. 340hrs
b. 341hrs
c. 348hrs
d. 349hrs

v. Manufacturer should claim that the lifetime of a packet is:

a. 346hrs

b. 341hrs

c. 340hrs

d. 347hrs

Assertion Reason Questions:

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) Both A and R is false.

Assertion: median= ((n+1)/2)th value if n is odd

Reason: If the number of runs scored by 11 players of a cricket team of India are 5, 19, 42, 11, 50, 30, 21, 0, 52, 36, 27 then median is 30

2. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) Both A and R is false.

Assertion: if the value of mode and mean is 60 and 66 then the value of median is 64.

Reason: median = (mode + 2mean)

Answer Key

Multiple Choice Questions:

1. (b) mode = 3 median – 2 mean
2. (d) Remains the same as that of the original set.
3. (b) 14k
4. (c) 82
5. (b) median
6. (b) centred at the classmarks of the classes
7. (c) 50
8. (c) centred at the classmarks of the classes
9. (a) Mean

Very Short Answers:

1. New median = 21 + 5 = 26

2.

Cost of living index	*No. of weeks (f)*	*c.f.*
1400-1550	8	8
1550-1700	15	23
1700-1850	21	44
1850-2000	8	52
	52	

Here, n = 52; $\frac{n}{2}=\frac{52}{2}=26$

∴ Median class 1700 – 1850.

3.

Marks obtained	*No. of students (c.f.)*	*Class Interval*	*f*
0 or more	63	0-10	5
10 or more	58	10-20	3
20 or more	55	20-30	4
30 or more	51	30-40	3
40 or more	48	40-50	6
50 or more	42	50-60	42
			63

∴ Frequency of class 30 – 40 = 3

4. Maximum no. of pairs sold = 25 (size 5)

∴ Modal size of shoes = 5

5. Maximum frequency = 48

∴ Modal class = 9,000 – 12,000

Upper limit of the modal class = 12,000

6. Class mark $=\frac{\text{Upper limit}+\text{Lower limit}}{2}$

$$=\frac{10+25}{2}=\frac{35}{2}=17.5$$

7. Mean $=\frac{x_1+x_2+x_3+x_4+x_5}{5}$

$$=\frac{1+2+3+4+5}{5}=\frac{15}{5}=3$$

8. Total no. of observations = 13, which is odd

$\therefore$ The median will be $\left(\frac{n+1}{2}\right)^{th}$

term $=\left(\frac{13+1}{2}\right)^{th}=\left(\frac{14}{2}\right)^{th}=7^{th}$

i.e., 7^{th} term will be the median.

9. Mode = 8; Mean = 8; Median = ?

Relation among mean, median and mode is

3 median = mode + 2 mean

3 × median = 8 + 2 × 8

Median $=\frac{8+16}{3}=\frac{24}{3}=8$

10. No. of terms = 2n which are even

$\therefore$ The median term will be $\frac{\left[\left(\frac{n}{2}\right)^{th}+\left(\frac{n}{2}+1\right)^{th}\right]}{2}$

Put $n = 2n$

$$\frac{\left[\left(\frac{n}{2}\right)^{th}+\left(\frac{n}{2}+1\right)^{th}\right]}{2}=\left[\frac{n^{th}+(n+1)^{th}}{2}\right]$$

i.e., the mean of n^{th} and $(n + 1)^{th}$ term will be the median.

Short Answers:

1. We know mean $\left(\bar{x}\right)=\frac{\sum f_i x_i}{\sum f_i}$

$\therefore \quad \sum f_i x_i=\bar{x}\sum f_i \quad$ ...(*i*)

Now the value of $\sum f_i\left(x_i-\bar{x}\right)=\sum f_i x_i-\sum f_i\bar{x}$

$=\sum f_i\bar{x}-\sum f_i\bar{x}=0 \qquad$ [Using (*i*)]

2. Classes are not continuous, hence make them continuous by adding 0.5 to the upper limits and subtracting 0.5 from the lower limits.=

C.I.	*Frequency*	*Cumulative Frequency*
0-5.5	13	13
5.5-11.5	10	23
11.5-17.5	15	38
17.5-23.5	08	46
23.5-29.5	11	57
Total	$\sum f=57$	

Class interval can't be negative hence the first CI is starting from 0.

Now to find median class we calculate $\frac{\Sigma f}{2}=\frac{57}{2}=$ 28.5

$\therefore$ Median class = 11.5 – 17.5.

So, the upper limit is 17.5

3. First, we find the cumulative frequency

Classes	*Frequency*	*Cumulative Frequency*
0-10	4	4
10-20	4	8
20-30	8	16
30-40	10	26
40-50	12	38
50-60	8	46
60-70	4	50
Total	**50**	

Here, $\frac{n}{2}=\frac{50}{2}$

$\therefore$ Median class = 30 – 40.

4. Class mark $=\frac{\text{upper limit}+\text{lower limit}}{2}$

Class marks of 15.5 – 18.5 $=\frac{18.5+15.5}{2}=\frac{34}{2}=17$

Class marks of 50 – 75 $=\frac{75+50}{2}=\frac{125}{2}=62.5$

5. Calculation of mean

x_i	f_i	$f_i x_i$
2	3	6
4	2	8
6	3	18
10	1	10
$p + 5$	2	$2p + 10$
Total	$\sum f_i=11$	$\sum f_i x_i=2p+52$

We have, $\sum f_i=11,\ \sum f_i x_i=2p+52,\ \bar{X}=6$

$\therefore \quad Mean\left(\bar{X}\right)=\frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow \quad 6=\frac{2p+52}{11} \Rightarrow 66=2p+52$

$\Rightarrow \quad 2p = 14 \quad \Rightarrow p = 7$

6. Calculation of arithmetic mean

x_i	f_i	$f_i x_i$
4	5	20
6	10	60
9	10	90
10	7	70
15	8	120
Total	$\sum f_i=40$	$\sum f_i x_i=360$

$\therefore \quad Mean\left(\overline{X}\right)=\frac{\sum f_i x_i}{\sum f_i}=\frac{360}{40}=9$

7. Here, the maximum class frequency is 61 and the class corresponding to this frequency is 60 – 80.

So, the modal class is 60 – 80.

Here, l = 60, h = 20, f_1 = 61, f_0 = 52, f_2 = 38

$\therefore$ Mode

$$l+\frac{f_1-f_0}{2f_1-f_0-f_2}\times h=60+\frac{61-52}{2\times 61-52-38}\times 20$$

$$=60+\frac{9}{122-90}\times 20$$

$$=60+\frac{9}{32}\times 20=60+\frac{45}{8}=60+5.625=65.625$$

Hence, modal lifetime of the components is 65.625 hours.

8. Calculation of median

Weight (in kg)	Number of students (f)	Cumulative frequency (cf)
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30
Total	$\sum f_i=30$	

The cumulative frequency just greater than $\frac{n}{2}$ = 15 is 19, and the corresponding class is 55 – 60.

$\therefore$ 55 – 60 is the median class.

$$\therefore \text{Median } =l+\left(\frac{\frac{n}{2}-cf}{f}\right)\times h$$

$$=55+\left(\frac{15-13}{6}\right)\times 5=55+\frac{2}{6}\times 5=55+1.67=56.67$$

Hence, median weight is 56.67 kg.

Long Answers:

1. Here, we use step deviation method to find mean.

Let assumed mean A = 70 and class size h = 10

So, $ui \;=\; \frac{x_i-70}{10}$

Now, we have

Literacy rate (in %)	Frequency	Class mark	$u_i=\frac{x_i-70}{10}$	$f_i u_i$
45-55	3	50	−2	−6
55-65	10	60	−1	−10
65-75	11	70	0	0
75-85	8	80	1	8
85-95	3	90	2	6
Total	$\sum f_i=35$			$\sum f_i u_i=-2$

$$\therefore Mean\left(\overline{X}\right)=A+h\times\frac{\sum f_i u_i}{\sum f_i}$$

$$=70+10\times\frac{(-2)}{35}=70-0.57=69.43\%$$

2. Let the assumed mean A = 16 and class size h = 2, here we apply step deviation method.

So, $u_i=\frac{x_i-A}{h}=\frac{x_i-16}{2}$

Now, we have,

Class interval	Frequency	Class mark	$u_i=\frac{x_i-16}{2}$	$f_i u_i$
11-13	7	12	−2	−14
13-15	6	14	−1	−6
15-17	9	16	0	0
17-19	13	18	1	13
19-21	f	20	2	$2f$
21-23	5	22	3	15
23-25	4	24	4	16
Total	$\sum f_i=f+44$			$\sum f_i u_i=2f+24$

We have, Mean $\left(\overline{X}\right)$ = 18, A = 16 and h = 2

$$\therefore \quad \overline{X}=A+h\times\frac{\sum f_i u_i}{\sum f_i}$$

$$18=16+2\times\left(\frac{2f+24}{f+44}\right) \Rightarrow 2=2\times\left(\frac{2f+24}{f+44}\right)$$

$$\Rightarrow \quad 1=\frac{2f+24}{f+44} \quad \Rightarrow \quad f+44=2f+24$$

$\Rightarrow \quad f$ = 44 – 24 $\quad \Rightarrow \quad f$ = 20

Hence, the missing frequency is 20.

3. We have

Class interval	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
0-20	5	10	50
20-40	8	30	240
40-60	x	50	50x
60-80	12	70	840
80-100	7	90	630
100-120	8	110	880
Total	$\sum f_i=40+x$		$\sum f_i x_i=2640+50x$

Here, $\sum f_i x_i=2640+50x, \sum f_i=40+x, \overline{X}=62.8$

$\therefore \quad Mean\left(\overline{X}\right)=\frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow \quad 62.8=\frac{2640+50x}{40+x}$

$\Rightarrow \quad 2512 + 62.8x = 2640 + 50x$

$\Rightarrow \quad 62.8x - 50x = 2640 - 2512$

$\Rightarrow \quad 12.8x = 128$

$\therefore \ x = \frac{128}{12.8} = 10$

Hence, the missing frequency is 10.

Case Study Answers:

1. Answer:

i. (a) If x_i and f_i are sufficiently large, then direct method is appropriate choice for calculating mean.

Solution:

If f_i and x_i are very small, then direct method is appropriate method for calculating mean.

ii. (a) 8.15 litres

Solution:

The frequency distribution table from the given data can be drawn as:

Class	Class mark (x_i)	Frequency (f_i)	$f_i x_i$
3-5	4	5	20
5-7	6	10	60
7-9	8	10	80
9-11	10	7	70
11-13	12	8	96
Total		**40**	**326**

$\therefore \quad \text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{326}{40} = 8.15\ litres$

iii. (b) 16300 litres

Solution:

If 2000 vehicles comes daily and average quantity of diesel required for a vehicle is 8.15 liters, then total quantity of diesel required,

= 2000 × 8.15 = 16300 liters

iv. (c) 16

Solution:

Here, N = 40 and $\frac{N}{2}=20$

c.f. for the distribution are 5, 15, 25, 32, 40

Now, cf just greater than 20 is 25 which is corresponding to the class interval 7 - 9.

So median class is 7 - 9.

∴ Required sum of upper limit and lower limit = 7 + 9 = 16

v. (b) 7.7 litres

Solution:

We know, Mode = 3 Median - 2 Mean

= 3(8) - 2(8.15) = 24 - 16.3 = 7.7

2. Answer:

i. (d) 2m – b

Solution:

We know that,

$\text{Class mark} = \frac{\text{Lower limit} + \text{Upper limit}}{2}$

$\Rightarrow m = \frac{\text{Lower limit} + \text{b}}{2}$

⇒ Lower limit = 2m – b

ii. (a) 341hrs

Solution:

Lifetime (in hours)	Class mark (x_i)	(f_i)	$d_i = x_i - A$	$f_i d_i$
150-200	175	14	–150	–2100
200-250	225	56	–100	–5600
250-300	275	60	–50	–3000
300-350	325 = A	86	0	0
350-400	375	74	50	3700
400-450	425	62	100	6200
450-500	475	48	150	7200
Total			**50**	**6400**

∴ Average lifetime of a packet

$= A + \frac{\sum f_i d_i}{\sum f_i} = 325 + \frac{6400}{400} = 34$

iii. (b) 340hrs

Solution:

Here, N = 400 $\Rightarrow \frac{N}{2}=200$

Also, cumulative frequency for the given distribution are 14, 70, 130, 216, 290, 352, 400

∴ c.f just greater than 200 is 216, which is corresponding to the interval 300-350.

l = 300, f = 86, c.f. = 130, h = 50

∴ median

$=1+\left(\frac{\frac{N}{2}-c.f.}{f}\right)\times h = 300+\left(\frac{200-130}{86}\right)\times 50$

= 300 + 40.697 ≈ 340.697 ≈ 340 hrs. (approx)

iv. (a) 340 hrs.

Solution:

We know that Mode = 3 Median – 2 Mean

= 3(340.697) – 2(341)

= 1022.091 – 682 = 340.091 $\approx$ 340 hrs.

v. (c) 340 hrs.

Solution:

Since, minimum of mean, median and mode is approximately 340hrs. So, manufacturer should claim that lifetime of a packet is 340hrs.

Assertion Reason Answers:

1. (c) A is true but R is false.

2. (c) A is true but R is false.

Probability 14

1. **Probability** is a quantitative measure of uncertainty.
2. In the **experimental approach** to probability, we find the probability of the occurrence of an event by actually performing the experiment a number of times and adequate recording of the happening of event.
3. In the **theoretical approach** to probability, we try to predict what will happen without actually performing the experiment.
4. The experimental probability of an event approaches to its theoretical probability if the number of trials of an experiment is very large.
5. An **outcome** is a result of a single trial of an experiment.
6. The word **'experiment'** means an operation which can produce some well defined outcome(s). There are two types of experiments:
   i. **Deterministic experiments:** Experiments which are repeated under identical conditions produce the same results or outcomes are called deterministic experiments.
   ii. **Random or Probabilistic experiment**: If an experiment, when repeated under identical conditions, do not produce the same outcome every time but the outcome in a trial is one of the several possible outcomes, then it is known as a random or probabilistic experiment.

   In this chapter, the term experiment will stand for random experiment.
7. The collection of all possible outcomes is called the **sample space**.
8. An outcome of a random experiment is called an **elementary event**.
9. An event associated to a random experiment is a **compound event** if it is obtained by combining two or more elementary events associated to the random experiment.
10. An event A associated to a random experiment is said to occur if any one of the elementary events associated to the event A is an outcome.
11. An elementary event is said to be **favorable** to a compound event A, if it satisfies the definition of the compound event A. In other words, an elementary event E is favorable to a compound event A, if we say that the event A occurs when E is an outcome of a trial.
12. In an experiment, if two or more events have equal chances to occur or have equal probabilities, then they are called **equally likely events.**
13. The **theoretical probability (also called classical probability) of an event *E***, written as $P(E)$, is defined as

$$\frac{\textbf{Number of outcomes favourable to E}}{\textbf{Number of all possible outcomes of the experiment}}$$

14. For two events A and B of an experiment:

    If $P(A) > P(B)$ then event A is more likely to occur than event B.

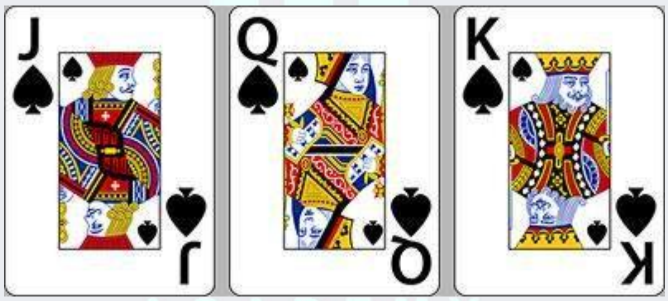
    If $P(A) = P(B)$ then events A and B are equally likely to occur.
15. An event is said to be **sure event** if it always occur whenever the experiment is performed. The probability of sure event is always one. In case of sure event elements are same as the sample space.

16. An event is said to be **impossible event** if it never occur whenever the experiment is performed. The probability of an impossible event is always zero. Also, the number of favorable outcome is zero for an impossible event.
17. Probability of an event lies between 0 and 1, both inclusive, i.e., $\mathbf{0 \le P\,(A) \le 1}$
18. If E is an event in a random experiment then the event 'not *E*' (denoted by $\overline{E}$) is called the **complementary event** corresponding to *E*.
19. The **sum of the probabilities** of all elementary events of an experiment is 1.
20. For an event *E*, $\mathbf{P(\overline{E}) = 1 - P(E)}$, where the event $\overline{E}$ representing 'not *E*" is the complement of event E.
21. **Suits of Playing Card**

    A pack of playing cards consist of 52 cards which are divided into 4 suits of 13 cards each. Each suit consists of one ace, one king, one queen, one jack and 9 other cards numbered from 2 to 10. Four suits are named as spades, hearts, diamonds and clubs.

22. **Face Cards**

    King, queen and jack are face cards.



The formula for finding the **geometric probability** of an event is given by:

$$P(E) = \frac{\textbf{Measure of the specified part of the region}}{\textbf{Measure of the whole region}}$$

Here, 'measure' may denote length, area or volume of the region or space.

Event and outcome

An Outcome is a result of a random experiment. For example, when we roll a dice getting six is an outcome.

An Event is a set of outcomes. For example when we roll dice the probability of getting a number less than five is an event.

Note: An Event can have a single outcome

Events and Types of Events in Probability

What are Events in Probability?

A probability event can be defined as a set of outcomes of an experiment. In other words, an event in probability is the subset of the respective sample space. So, what is sample space?

The entire possible set of outcomes of a random experiment is the sample space or the individual space of that experiment. The likelihood of occurrence of an event is known as probability. The probability of occurrence of any event lies between 0 and 1.

The sample space for the tossing of three coins simultaneously is given by:

S = {(T , T , T) , (T , T , H) , (T , H , T) , (T , H , H) , (H , T , T) , (H , T , H) , (H , H, T) ,(H , H , H)}

Suppose, if we want to find only the outcomes which have at least two heads; then the set of all such possibilities can be given as:

E = { (H , T , H) , (H , H ,T) , (H , H ,H) , (T , H , H)}

Thus, an event is a subset of the sample space, i.e., E is a subset of S.

There could be a lot of events associated with a given sample space. For any event to occur, the outcome of the experiment must be an element of the set of event E.

What is the Probability of Occurrence of an Event?

The number of favourable outcomes to the total number of outcomes is defined as the probability of occurrence of any event. So, the probability that an event will occur is given as:

Types of Events in Probability:

Some of the important probability events are:

- Impossible and Sure Events
- Simple Events
- Compound Events
- Independent and Dependent Events
- Mutually Exclusive Events
- Exhaustive Events
- Complementary Events
- Events Associated with "OR"
- Events Associated with "AND"
- Event E1 but not E2

Impossible and Sure Events

If the probability of occurrence of an event is 0, such an event is called an impossible event and if the probability of occurrence of an event is 1, it is called a sure event. In other words, the empty set ϕ is an impossible event and the sample space S is a sure event.

Simple Events

Any event consisting of a single point of the sample space is known as a simple event in probability. For example, if S = {56, 78 , 96 , 54 , 89} and E = {78} then E is a simple event.

Compound Events

Contrary to the simple event, if any event consists of more than one single point of the sample space then such an event is called a compound event. Considering the same example again, if S = {56 ,78 ,96 ,54 ,89}, E1 = {56 ,54 }, E2 = {78 ,56 ,89 } then, E_1 and E_2 represent two compound events.

Independent Events and Dependent Events

If the occurrence of any event is completely unaffected by the occurrence of any other event, such events are known as an independent event in probability and the events which are affected by other events are known as dependent events.

Mutually Exclusive Events

If the occurrence of one event excludes the occurrence of another event, such events are mutually exclusive events i.e. two events don't have any common point. For example, if S = {1, 2 , 3 , 4 , 5 , 6} and E1, E2 are two events such that E1 consists of numbers less than 3 and E2 consists of numbers greater than 4.

So, **E1 = {1,2} and E2 = {5,6}** .

Then, E1 and E2 are mutually exclusive.

Exhaustive Events

A set of events is called exhaustive if all the events together consume the entire sample space.

Complementary Events

For any event E1 there exists another event E1' which represents the remaining elements of the sample space S.

$$E1 = S - E1'$$

If a dice is rolled then the sample space S is given as S = {1 , 2 , 3 , 4 , 5 , 6 }. If event E1 represents all the outcomes which is greater than 4, then E1 = {5, 6} and E1' = {1, 2, 3, 4}.

Thus E1' is the complement of the event E1.

Similarly, the complement of $E_1, E_2, E_3 \ldots\ldots\ldots E_n$ will be represented as $E_1', E_2', E_3' \ldots\ldots\ldots E_n'$

Experimental Probability

Experimental probability can be applied to any event associated with an experiment that is repeated a large number of times.

A trial is when the experiment is performed once. It is also known as empirical probability.

$$\textbf{Experimental or empirical probability}: P(E) = \frac{\textbf{Number of trials where the event occurred}}{\textbf{Total Number of Trials}}$$

You and your 3 friends are playing a board game. It's your turn to roll the die and to win the game you need a 5 on the dice. Now, is it possible that upon rolling the die you will get an exact 5? No, it is a matter of chance. We face multiple situations in real life where we have to take a chance or risk. Based on certain conditions, the chance of occurrence of a certain event can be easily predicted. In our day to day life, we are more familiar with the word 'chance and probability'. In simple words, the chance of occurrence of a particular event is what we study in probability. In this article, we are going to discuss one of the types of probability called "Experimental Probability" in detail.

Theoretical Probability

Theoretical Probability, P(E) = Number of Outcomes Favourable to E / Number of all possible outcomes of the experiment

Here we assume that the outcomes of the experiment are equally likely.

Every one of us would have encountered multiple situations in life where we had to take a chance or risk. Depending on the situation, it can be predicted up to a certain extent if a particular event is going to take place or not. This chance of occurrence of a particular event is what we study in probability. In our everyday life, we are more accustomed to the word 'chance' as compared to the word 'probability'. Since Mathematics is all about quantifying things, the theory of probability basically quantifies these chances of occurrence or non-occurrence of certain events. In this article, we are going to discuss what is probability and its two different types of approaches with examples.

In Mathematics, the probability is a branch that deals with the likelihood of the occurrences of the given event. The probability value is expressed between the range of numbers from 0 to 1. The three basic rules connected with the probability are addition, multiplication, and complement rules.

Theoretical Probability Vs Experimental Probability

Probability theory can be studied using two different approaches:

Theoretical Probability

Experimental Probability

Theoretical Probability Definition

Theoretical probability is the theory behind probability. To find the probability of an event using theoretical probability, it is not required to conduct an experiment. Instead of that, we should know about the situation to find the probability of an event occurring. The theoretical probability is defined as the ratio of the number of favourable outcomes to the number of possible outcomes.

$$\text{Probability of Event P(E)} = \frac{\text{Number of Favourable outcomes}}{\text{Number of Possible outcomes}}$$

Experimental Probability Definition

The experimental probability also is known as an empirical probability, is an approach that relies upon actual experiments and adequate recordings of occurrence of certain events while the theoretical probability attempts to predict what will happen based upon the total number of outcomes possible. The experimental Probability is defined as the ratio of the number of times that event occurs to the total number of trials.

$$\text{Probability of Event P(E)} = \frac{\text{Number of times that event occurs}}{\text{Total number of trials}}$$

The basic difference between these two approaches is that in the experimental approach; the probability of an event is based on what has actually happened by conducting a series of actual experiments, while in theoretical approach; we attempt to predict what will occur without actually performing the experiments.

Elementary Event

An event having only one outcome of the experiment is called an elementary event.

Example: Take the experiment of tossing a coin n number of times. One trial of this experiment has two possible outcomes: Heads(H) or Tails(T). So for an individual toss, it has only one outcome, i.e Heads or Tails.

Sum of Probabilities

The sum of the probabilities of all the elementary events of an experiment is one.

Example: take the coin-tossing experiment. P(Heads) + P(Tails)

= (1/2)+ (1/2) =1

Impossible event

An event that has no chance of occurring is called an Impossible event, i.e. P(E) = 0.

E.g: Probability of getting a 7 on a roll of a die is 0. As 7 can never be an outcome of this trial.

Sure event

An event that has a 100% probability of occurrence is called a sure event. The probability of occurrence of a sure event is one.

E.g: What is the probability that a number obtained after throwing a die is less than 7?

So, P(E) = P(Getting a number less than 7) = 6/6= 1

Range of Probability of an event

The range of probability of an event lies between 0 and 1 inclusive of 0 and 1, i.e. $0 \le P(E) \le 1$.

Geometric Probability

Geometric probability is the calculation of the likelihood that one will hit a particular area of a figure. It is calculated by dividing the desired area by the total area. In the case of Geometrical probability, there are infinite outcomes.

Complementary Events

Complementary events are two outcomes of an event that are the only two possible outcomes. This is like flipping a coin and getting heads or tails.

$$P(E) + P(\overline{E}) = 1, \text{where E and } \overline{E}$$

are complementary events. The event $\overline{E}$, representing 'not E', is called the complement of the event E.

For any event A, there exists another event A' which shows the remaining elements of the sample space S. A' denotes complementary event of A.

A' = S – A.

Event A and A' are mutually exclusive and exhaustive.

Consider the example of tossing a coin. Let P(E) denote the probability of getting a tail when a coin is tossed. Then probability of getting a head is denoted by

$$P(\overline{E}).\, P(E) + P(\overline{E}) = 1$$

The event $P(\overline{E})$ means 'Not E'.

Example 1:

A bag contains only lemon-flavoured candies. Arjun takes out one candy without looking into the bag. What is the probability that he takes out an orange-flavoured candy?

Solution:

Let us take the number of candies in the bag to be 100.

Number of orange flavoured candies = 0 [since the bag contains only lemon-flavoured candies]

Hence, the probability that he takes out an orange-flavoured candy is:

P (Taking orange-flavoured candy) = Number of orange flavoured candies / Total number of candies.

= 0/100 = 0

Hence, the probability that Arjun takes out an orange-flavoured candy is 0.

This proves that the probability of an impossible event is 0.

Example 2:

A game of chance consists of spinning an arrow that comes to rest pointing at any one of the numbers such as 1, 2, 3, 4, 5, 6, 7, 8 and these are equally likely outcomes. What is the probability that it will point at (i)8, (ii) Number greater than 2 (iii) Odd numbers.

Solution:

Sample Space = {1, 2, 3, 4, 5, 6, 7, 8}

Total Numbers = 8

(i) Probability that the arrow will point at 8:

Number of times we can get 8 = 1

P (Getting 8) = 1/8.

(ii) Probability that the arrow will point at the number greater than 2:

Number greater than 2 = 3, 4, 5, 6, 7, 8.

No. of numbers greater than 2 = 6

P (Getting numbers greater than 2) = 6/8 = 3/4.

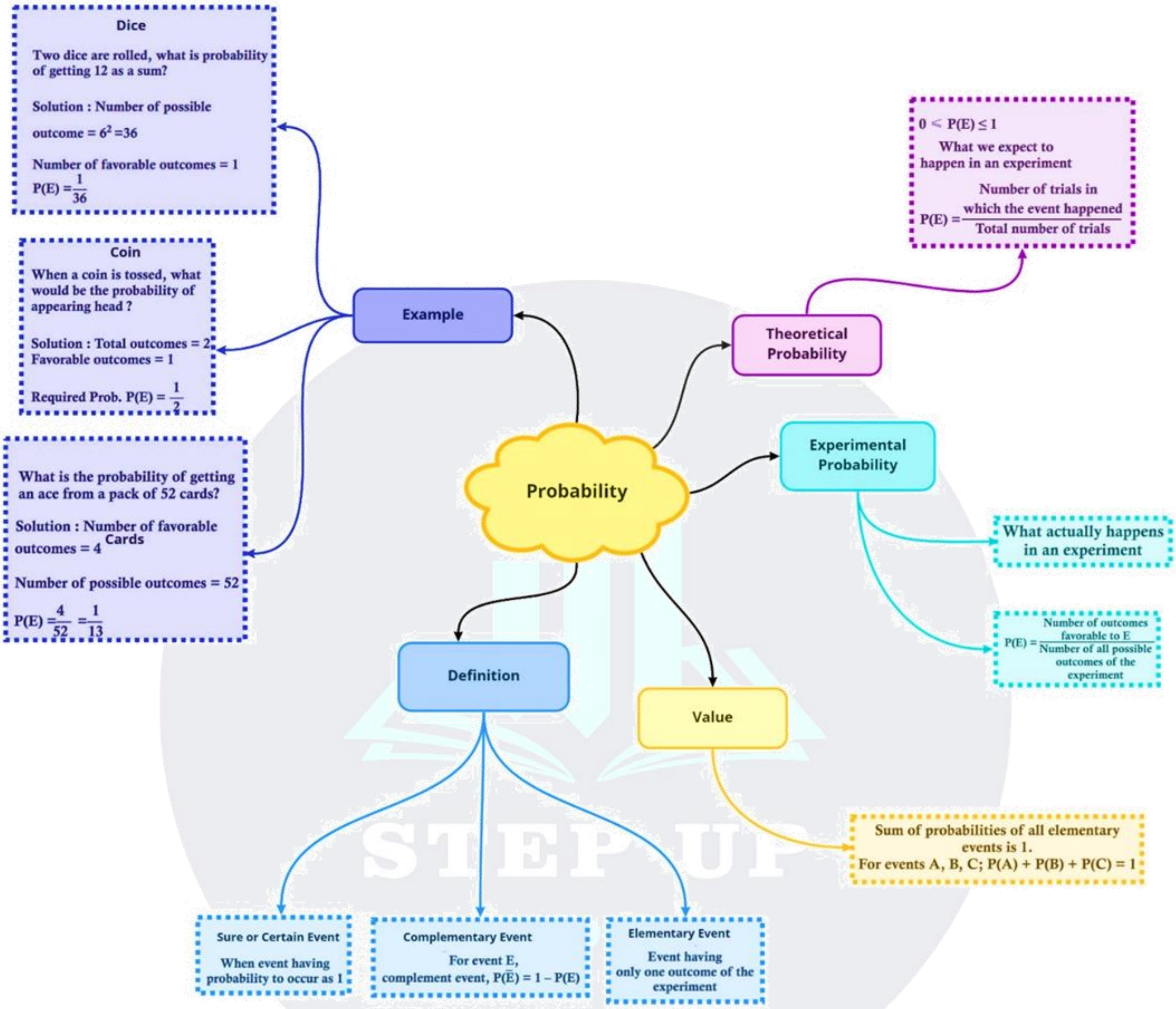
(iii) Probability that the arrow will point at the odd numbers:

Odd number of outcomes = 1, 3, 5, 7

Number of odd numbers = 4.

P (Getting odd numbers) = 4/8 = ½.

Class : 10th mathematics
Chapter- 15 :Probability



Important Questions

Multiple Choice Questions:

1. An event is very unlikely to happen. Its probability is closest to:
   (a) 0.0001
   (b) 0.001
   (c) 0.01
   (d) 0.1
2. If the probability of an event is P, the probability of its complementary event will be:
   (a) P – 1
   (b) P
   (c) 1 – p
   (d) $1 - \frac{1}{p}$
3. If P(A) denotes the probability of an event then:
   (a) $P(A) < 0$
   (b) $P(A) > 0$
   (c) $0 \le P(A) \le 1$
   (d) $-1 \le P(A) \le 0$
4. A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favourable to E is:
   (a) 4
   (b) 13
   (c) 48
   (d) 51
5. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is:
   (a) 7
   (b) 14
   (c) 21
   (d) 28
6. A girl calculate that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought?
   (a) 40
   (b) 240
   (c) 480
   (d) 750
7. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 5 is:
   (a) $\frac{1}{5}$
   (b) $\frac{3}{5}$
   (c) $\frac{4}{5}$
   (d) $\frac{1}{3}$
8. Someone is asked to take a number from 1 to 100. The probability that it is a prime is:
   (a) $\frac{1}{5}$
   (b) $\frac{6}{25}$
   (c) $\frac{1}{4}$
   (d) $\frac{13}{50}$
9. A school has five houses A, B, C, D and E. A class has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and rest from house E. A single student is selected at random to be the class monitor. The probability that the selected student is not from A, B and C is:
   (a) $\frac{4}{23}$
   (b) $\frac{6}{23}$
   (c) $\frac{8}{23}$
   (d) $\frac{17}{23}$
10. When a die is thrown, the probability of getting an odd number less them 3 is:
    (a) $\frac{1}{6}$
    (b) $\frac{1}{3}$
    (c) $\frac{1}{2}$
    (d) 0

Very Short Questions:

1. State true or false and give the reason. If I toss a coin 3 times and get head each tir ne, then I should expect a tail to have a higher chance in the 4th toss.
2. A bag contains slips numbered from 1 to 100. If Fatima chooses a slip at random from the bag, it will either be an odd number or an even number. Since, this situation has only two possible outcomes, so the probability of each is $\frac{1}{2}$ Justify.
3. In a family, having three children, there may be no girl, one girl, two girls or three girls. So, the probability of each is $\frac{1}{4}$. Is this correct? Justify your answer.
4. A game consists of spinning an arrow which comes to rest pointing at one of the regions (1, 2 or 3) Fig. Are the outcomes 1, 2 and 3 equally likely to occur? Give reason.
5. Two coins are tossed simultaneously. Find the probability of getting exactly one head.
6. From a well shuffled pack of cards, a card is drawn at random. Find the probability of getting a black queen.
7. If P (E) = 0.05, what is the probability of 'not E' ?
8. What is the probability of getting no head when two coins are tossed simultaneously?
9. In a single throw of a pair of dice, what is the probability of getting the sum a perfect square?
10. Someone is asked to choose a number from 1 to 100. What is the probability of it being a prime number?

Short Questions:

1. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is a prime number.
2. Find the probability that a number selected from the numbers 1 to 25 is not a prime number when each of the given numbers is equally likely to be selected.
3. One card is drawn at random from a pack of 52 cards. Find the probability that the card drawn is an ace and black.
4. A card is drawn at random from a pack of 52 playing cards. Find the probability that the card drawn is neither an ace nor a king.
5. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out (i) an orange flavoured candy? (ii) a lemon flavoured candy?
6. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.
7. Two players, Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta's winning the match is 0.62. What is the probability of Reshma's winning the match?
8. A child has a die whose six faces show the letters as given below:

   A B C D E A

   The die is thrown once. What is the probability of getting (i) A? (ii) D?
9. A card is drawn at random from a pack of 52 playing cards. Find the probability that the card drawn is neither a red card nor a black king.
10. Out of 400 bulbs in a box, 15 bulbs are defective. One bulb is taken out at random from the box. Find the probability that the drawn bulb is not defective.
11. Harpreet tosses two different coins simultaneously (say, one is of 1 and other of 2). What is the probability that she gets at least one head?
12. A game consists of tossing a one-rupee coin 3 times and noting the outcome each time. Ramesh wins the game if all the tosses give the same result (i.e. three heads or three tails) and loses otherwise. Find the probability of Ramesh losing the game.
13. Three unbiased coins are tossed together. Find the probability of getting:
    (i) all heads.
    (ii) exactly two heads.
    (iii) exactly one head.
    (iv) at least two heads.
    (v) at least two tails
14. A die is thrown once. Find the probability of getting:
    (i) a prime number.
    (ii) a number lying between 2 and 6.
    (iii) an odd number

15. Suppose we throw a die once.
    (i) What is the probability of getting a number greater than 4?
    (ii) What is the probability of getting a number less than or equal to 4?

Long Questions:

1. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting:
   (i) a king of red colour.
   (ii) a face card.
   (iii) a red face card.
   (iv) the jack of hearts.
   (v) a spade.
   (vi) the queen of diamonds.
2. One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to be drawn. Find the probability that the card drawn is:
   (i) an ace.
   (ii) red.
   (iii) either red or king.
   (iv) red and a king.
   (v) a face card.
   (vi) a red face card.
   (vii) "2' of spades.
   (viii) '10' of a black suit.
3. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1,2,3,4,5,6,7,8 see Fig, and these are equally likely outcomes. What is the probability that it will point at: (i) 8? (ii) an odd number? (iii) a number greater than 2? (iv) a number less than 9?
4. Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is: (i) 8? (ii) 13? (iii) less than or equal to 12?
5. A bag contains cards numbered from 1 to 49. A card is drawn from the bag at random, after mixing the cards thoroughly. Find the probability that the number on the drawn card is:
   (i) an odd number.
   (ii) a multiple of 5.
   (iii) a perfect square.
   (iv) an even prime number.

Case Study Questions:

1. In the month of May, the weather forecast department gives the prediction of weather for the month of June. The given table shows the probabilities of forecast of different days:


Sunny


Partially cloudy


Cloudy


Rainy

Days	Days	Cloudy	Partially Cloudy	Rainy
Probability	1/2	x	1/5	y

If the forecast is 100% correct for June, then answer the following questions.

i. The number of sunny days in June, is:
   a. 5
   b. 10
   c. 15
   d. 20

ii. If the number of cloudy days in June is 5, then x =
   a. $\frac{1}{4}$
   b. $\frac{1}{6}$
   c. $\frac{1}{8}$
   d. $\frac{1}{10}$

iii. The probability that the day is not rainy is:
   a. $\frac{13}{15}$
   b. $\frac{11}{15}$
   c. $\frac{1}{15}$
   d. None of these

iv. If the sum of x and y is $\frac{3}{10}$, then the number of rainy days in June is:
  a. 1
  b. 2
  c. 3
  d. 4

v. Find the number of partially cloudy days.
  a. 2
  b. 4
  c. 6
  d. 8

2. Vishal goes to a store to purchase juice cartons for his shop. The store has 80 cartons of orange juice, 90 cartons of apple juice, 38 cartons of mango juice and 42 cartons of guava juice. If Vishal chooses a carton at random, then answer the following questions.

i. The probability that the selected carton is of apple juice is:
  a. $\frac{1}{25}$
  b. $\frac{8}{25}$
  c. $\frac{13}{25}$
  d. $\frac{9}{25}$

ii. The probability that the selected carton is not of orange juice is:
  a. $\frac{14}{25}$
  b. $\frac{11}{25}$
  c. $\frac{17}{25}$
  d. $\frac{4}{125}$

iii. The probability of selecting a carton of guava juice is:
  a. $\frac{51}{125}$
  b. $\frac{16}{125}$
  c. 0
  d. $\frac{21}{125}$

iv. Vishal buys 4 cartons of apple juice, 3 cartons of orange juice and 3 cartons of guava juice. A customer comes to Vishal's shop and picks a tetrapack of juice at random. The probability that the customer picks a guava juice, if each carton has 10 tetrapacks of juice, is:
  a. $\frac{1}{10}$
  b. $\frac{2}{10}$
  c. $\frac{3}{10}$
  d. $\frac{2}{5}$

v. If the storekeeper bought 14 more cartons of apple juice, then the probability of selecting a tetra pack of apple juice from the store is:
  a. $\frac{25}{127}$
  b. $\frac{50}{127}$
  c. $\frac{75}{127}$
  d. $\frac{100}{127}$

Assertion Reason Questions:

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

Assertion: The probability of winning a game is 0.4, then the probability of losing it, is 0.6.

Reason: P (E) + P(not E) = 1

2. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

Assertion: If the probability of an event is P, then probability of its complementary event will be 1 × P.

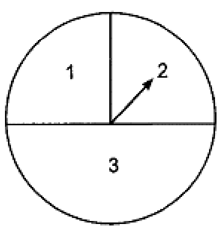
Reason: LCM × product of numbers = HCF

Answer Key

Multiple Choice Questions:

1. (a) 0.0001
2. (c) 1 – p
3. (c) $0 \le P(A) \le 1$
4. (d) 51
5. (b) 14
6. (c) 480
7. (a) $\frac{1}{5}$
8. (c) $\frac{1}{4}$
9. (b) $\frac{6}{23}$
10. (a) $\frac{1}{6}$

Very Short Answers:

1. False, because the outcomes 'head' and 'tail are equally likely. So, every time the probability of getting head or tail is $\frac{1}{2}$
2. True, because the outcomes odd number' and `even number' are equally likely here.
3. False, because the outcomes are not equally, likely. For no girl, outcome is bbb, for one girl, it is bgb, gbb, bbg, for two girls, it is bgg, ggb, gbg and for all girls, it is ggg.
4. 

False, because the outcome 3 is more likely than the other numbers.

5. Possible outcomes are {HH, HT, TH, TT}.

   (exactly one head) $= \frac{2}{4} = \frac{1}{2}$
6. Number of black queens in a pack of cards = 2

   ∴ P (black queen) $= \frac{25}{2} = \frac{1}{26}$
7. As we know that,

   P (E) + P (not E) = 1

   P (not E) = 1 – P (E) = 1 – 0.05 = 0.95
8. Favourable outcome is TT;

   ∴ P (no head) $= \frac{1}{4}$
9. Total outcomes = 36

   Favourable outcomes are {(1,3), (3, 1), (2, 2), (3, 6), (6,3), (4, 5), (5, 4)}

   ∴ Required probability $= \frac{7}{36}$
10. Total prime numbers between 1 to 100 = 25

    ∴ P (Prime number) $= \frac{25}{100} = 14$

Short Answers:

1. Product of the number on the dice is prime number, i.e., 2, 3, 5.

   The possible ways are, (1, 2), (2, 1), (1, 3), (3, 1), (5, 1), (1, 5)

   So, number of possible ways = 6

   ∴ Required probability $= \frac{6}{36} = \frac{1}{6}$
2. Total prime numbers from 1 to 25 = 9.

   ∴ Non-prime numbers from 1 to 25 = 25 – 9 = 16.

   ⇒ P (non-prime number) $= \frac{16}{25}$

3. Number of black aces in a pack of cards = 2

$\therefore$ P (an ace and black card) $= \frac{2}{52} = \frac{1}{26}$

4. Let E be the event card drawn is neither an ace nor a king.

Then, the number of outcomes favourable to the event E = 44 (4 kings and 4 aces are not there)

$\therefore\ P(E) = \frac{44}{52} = \frac{11}{13}$

5. (i) As the bag contains only lemon flavoured candies. So, the event related to the experiment of taking out an orange flavoured candy is an impossible event. So, its probability is 0.

(ii) As the bag contains only lemon flavoured candies. So, the event related to the experiment of taking out lemon flavoured candies is certain event. So, its probability is 1.

6. Here, total number of pens = 132 + 12 = 144

$\therefore$ Total number of elementary outcomes = 144

Now, favourable number of elementary events = 132

$\therefore$ Probability that a pen taken out is good one = $\frac{132}{144} = \frac{11}{12}$

7. Let S and R denote the events that Sangeeta and Reshma wins the match, respectively.

The probability of Sangeeta's winning = P(S) = 0.62

As the events R and S are complementary

$\therefore$ The probability of Reshma's winning = P(R) = 1 – P(S)

= 1 – 0.62 = 0.38.

8. The total number of elementary events associated with random experiment of throwing a die is 6.

(i) Let E be the event of getting a letter A.

$\therefore$ Favourable number of elementary events = 2

$\therefore$ P(E) $= \frac{2}{6} = \frac{1}{3}$

(ii) Let E be the event of getting a letter D.

$\therefore$ Favourable number of elementary events = 1

$\therefore$ P(E) $= \frac{1}{6}$

9. Let E be the event card drawn is neither a red card nor a black king'

The number of outcomes favourable to the event E = 24 (26 red cards and 2 black kings are not there, so 52 – 28 = 24)

$\therefore$ P(E) $= \frac{24}{52} = \frac{16}{13}$

10. Total number of bulbs in the box = 400

Total number of defective bulbs in the box = 15

Total number of non-defective bulbs in the box = 400 – 15 = 385

P (bulb is not defective)

$$= \frac{\text{Number of non-defective bulbs}}{\text{Total number of bulbs}}$$

$$= \frac{385}{400} = \frac{77}{80}$$

11. When two coins are tossed simultaneously, the possible outcomes are (H, H), (H, T), (T, H), (T, T) which are all equally likely. Here (H, H) means head up on the first coin (say on ₹ 1) and head up on the second coin (₹ 2). Similarly (H, T) means head up on the first coin and tail up on the second coin and so on.

The outcomes favourable to the event E, 'at least one head' are (H, H), (H, T) and (T, H). So, the number of outcomes favourable to E is 3.

Therefore, P(E) $= \frac{3}{4}$

i.e., the probability that Harpreet gets at least one head is $\frac{3}{4}$.

12. The outcomes associated with this experiment are given by

HHH, HHT, HTH, THH, TTH, THT, HTT, TTT

$\therefore$ Total number of possible outcomes = 8

Now, Ramesh will lose the game if he gets

HHT, HTH, THH, TTH, THT, HTT

$\therefore$ Favourable number of events = 6

$\therefore$ Probability that he lose the game $= \frac{6}{8} = \frac{3}{4}$

13. Elementary events associated to random experiment of tossing three coins are

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

$\therefore$ Total number of elementary events = 8

(i) The event "getting all heads" is said to occur, if the elementary event HHH occurs,

i.e., HHH is an outcome.

$\therefore$ Favourable number of elementary events = 1

Hence, required probability $= \frac{1}{8}$

(ii) The event "getting two heads" will occur, if one of the elementary events HHT, THH, HTH occurs.

∴ Favourable number of elementary events = 3

Hence, required probability $= \frac{3}{8}$

(iii) The event of "getting one head", when three coins are tossed together, occurs if one of the elementary events HTT, THT, TTH, occurs.

Favourable number of elementary events = 3

Hence, required probability $= \frac{3}{8}$

(iv) If any of the elementary events HHH, HHT, HTH, and THH is an outcome, then we say that the event "getting at least two heads" occurs.

∴ Favourable number of elementary events = 4

honom 4 Hence, required probability $= \frac{4}{8} = \frac{1}{2}$

(v) Similar as (iv) P (getting at least two tails) $= \frac{4}{8} = \frac{1}{2}$

14. We have, the total number of possible outcomes associated with the random experiment of throwing a die is 6 (i.e., 1, 2, 3, 4, 5, 6).

(i) Let E denotes the event of getting a prime number.

So, favourable number of outcomes = 3 (i.e., 2, 3, 5)

∴ $P(E) = \frac{3}{6} = \frac{1}{2}$

(ii) Let E be the event of getting a number lying between 2 and 6.

∴ Favourable number of elementary events (outcomes) = 3 (i.e., 3, 4, 5)

∴ $P(E) = \frac{3}{6} = \frac{1}{2}$

(iii) Let E be the event of getting an odd number.

∴ Favourable number of elementary events = 3 (i.e., 1, 3, 5)

∴ $P(E) = \frac{3}{6} = \frac{1}{2}$

15. (i) Here, let E be the event getting a number greater than 4′. The number of possible outcomes are six : 1, 2, 3, 4, 5 and 6, and the outcomes favourable to E are 5 and 6. Therefore, the number of outcomes favourable to E is 2. So,

P(E) = P (number greater than 4) $= \frac{2}{6} = \frac{1}{3}$

(ii) Let F be the event 'getting a number less than or equal to 4'.

Number of possible outcomes = 6

Outcomes favourable to the event F are 1, 2, 3, 4.

So, the number of outcomes favourable to F is 4.

Therefore, $P(F) = \frac{4}{6} = \frac{2}{3}$

Long Answers:

1. Here, total number of possible outcomes = 52

(i) As we know that there are two suits of red card, i.e., diamond and heart and each suit contains one king.

∴ Favourable number of outcomes = 2

∴ Probability of getting a king of red colour $= \frac{2}{52} = \frac{1}{26}$

(ii) As we know that kings, queens and jacks are called face cards. Therefore, there are 12 face cards.

∴ Favourable number of elementary events = 12

∴ Probability of getting a face card $= \frac{12}{52} = \frac{3}{13}$

(iii) As we know there are two suits of red cards, i.e., diamond and heart and each suit contains 3 face cards.

∴ Favourable number of elementary events = 2 × 3 = 6

∴ Probability of getting red face card $= \frac{6}{52} = \frac{3}{26}$

(iv) Since, there is only one jack of hearts.

∴ Favourable number of elementary events = 1

∴ Probability of getting the jack of heart $= \frac{1}{52}$

(v) Since, there are 13 cards of spade.

∴ Favourable number of elementary events = 13

∴ Probability of getting a spade $= \frac{13}{52} = \frac{1}{4}$

(vi) Since, there is only one queen of diamonds.

∴ Favourable number of outcomes (elementary events) = 1

∴ Probability of getting a queen of diamond $= \frac{1}{52}$

2. Out of 52 cards, one card can be drawn in 52 ways.

So, total number of elementary events = 52

(i) There are four ace cards in a pack of 52 cards. So, one ace can be chosen in 4 ways.

∴ Favourable number of elementary events = 4

Hence, required probability $= \frac{4}{52} = \frac{1}{13}$

(ii) There are 26 red cards in a pack of 52 cards. Out of 26 red cards, one card can be chosen in 26 ways.

∴ Favourable number of elementary events = 26

Hence, required probability $= \frac{26}{52} = \frac{1}{2}$

(iii) There are 26 red cards, including two red kings, in a pack of 52 playing cards. Also, there are 4 kings, two red and two black. Therefore, card drawn will be a red card or a king if it is any one of 28 cards (26 red cards and 2 black kings).

∴ Favourable number of elementary events = 28

Hence, required probability $= \frac{28}{52} = \frac{7}{13}$

(iv) A card drawn will be red as well as king, if it is a red king. There are 2 red kings in a pack of 52 playing cards.

∴ Favourable number of elementary events = 2

Hence, required probability $= \frac{2}{52} = \frac{1}{26}$

(v) In a deck of 52 cards: kings, queens, and jacks are called face cards. Thus, there are 12 face cards. So, one face card can be chosen in 12 ways.

Favourable number of elementary events = 12

Hence, required probability $= \frac{12}{52} = \frac{3}{13}$

(vi) There are 6 red face cards 3 each from diamonds and hearts. Out of these 6 red face cards, one card can be chosen in 6 ways.

∴ Favourable number of elementary events = 6

Hence, required probability $= \frac{6}{52} = \frac{3}{26}$

(vii) There is only one '2' of spades.

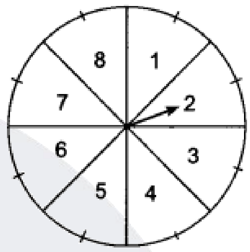
∴ Favourable number of elementary events = 1 Hence, required probability = 2

(viii) There are two suits of black cards viz. spades and clubs. Each suit contains one card bearing number 10.

∴ Favourable number of elementary events = 2

Hence, required probability $= \frac{2}{52} = \frac{1}{26}$

3.



Here, total number of elementary events (possible outcomes) = 8

(i) We have only one 'P' on the spining plant.

∴ Favourable number of outcomes = 1

Hence, the probability that arrow points at 8 $= \frac{1}{26}$.

(ii) We have four odd points (i.e., 1, 3, 5 and 7)

∴ Favourable number of outcomes = 4

∴ Probability that arrow points at an odd number $= \frac{4}{8} = \frac{1}{2}$

(iii) We have 6 numbers greater than 2, i.e., 3, 4, 5, 6, 7 and 8.

Therefore, favourable number of outcomes = 6

∴ Probability that arrow points at a number greater than 2 $= \frac{6}{8} = \frac{3}{4}$

(iv) We have 8 numbers less than 9, i.e, 1, 2, 3, ... 8.

∴ Favourable number of outcomes = 8

∴ Probability that arrow points at a number less than 9 $= \frac{8}{8} = 1$

4. When the blue dice shows 'l', the grey die could show any one of the numbers 1, 2, 3, 4, 5, 6.

The same is true when the blue dice shows '2', '3', '4', '5' or '6'. The possible outcomes of the experiment are listed in the table below; the first number in each ordered pair is the number appearing on the blue dice and the second number is that on the grey dice. So, the number of possible outcomes = 6 × 6 = 36.

(i) The outcomes favourable to the event the sum of the two numbers is 8′ denoted by E, are :

(2, 6), (3, 5), (4, 4), (5, 3), (6, 2) (see figure)

i.e., the number of outcomes favourable to E = 5.

Hence, P(E) = $\frac{5}{36}$

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

(ii) As you can see from figure, there is no outcome favourable to the event F, ‘the sum of two numbers is 13’.

So, P(F) = $\frac{0}{36}$ = 0

(iii) As you can see from figure, all the outcomes are favourable to the event G, ‘sum of two numbers ≤ 12.

So, P(G) = $\frac{36}{36}$ = 1.

5. Total number of cards = 49

Total number of outcomes = 49

(i) Odd number

Favourable outcomes : 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49

Number of favourable outcomes = 25

$$\text{Probability (E)} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{25}{49}$$

(ii) A multiple of 5

Favourable outcomes:

5, 10, 15, 20, 25, 30, 35, 40

Number of favourable outcomes = 9

$$\text{Probability (E)} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{9}{49}$$

(iii) A perfect square

Fabourable outcomes: 1, 4, 9, 16, 25, 36, 49

Number of favourable outcomes = 7

$$\text{Probability (E)} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{7}{49} = \frac{1}{7}$$

(iv) An even prime number

Favourable outcome = 2

Number of favourable outcome = 1

$$\text{Probability (E)} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{1}{49}$$

Case Study Answers:

1. Answer:

Total number of days in June = 30

i. (c) 15

Solution:

Number of sunny days =

$$\text{P(sunny day)} \times 30 = \frac{1}{2} \times 30 = 15$$

ii. (b) $\frac{1}{6}$

Solution:

Number of cloudy days in June = 5

$$\therefore x = \frac{5}{30} = \frac{1}{6}$$

iii. (a) $\frac{13}{15}$

Solution:

$$\text{Required probalility} = \frac{1}{2} + \frac{1}{6} + \frac{1}{5} = \frac{13}{15}$$

iv. (d) 4

Solution:

$$\text{We have, } x + y = \frac{3}{10} \Rightarrow y = \frac{3}{10} - \frac{1}{6} = \frac{2}{15}$$

So, number of rainy days $=\frac{2}{15}\times 30=4$

v. (c) 6

Solution:

Number of partially cloudy days =

p(partially cloudy days) $\times 30=\frac{1}{5}\times 30=6$

2. Answer:

Total number of cartons in the store = 80 + 90 + 38 + 42 = 250

i. (d) $\frac{9}{25}$

Solution:

P(choosing an apple juice carton $=\frac{90}{250}=\frac{9}{25}$

ii. (c) $\frac{17}{25}$

Solution:

P(choosing an orange juice carton) $=\frac{80}{250}=\frac{8}{25}$

$\therefore$ p(choosing not an orange juice carton)

$=1-\frac{8}{25}=\frac{17}{25}$

iii. (d) $\frac{21}{125}$

Solution:

P(choosing a guava juice carton) $=\frac{42}{250}=\frac{21}{125}$

iv. (c) $\frac{3}{10}$

Solution:

Total number of cartons Vishal bought = 4 + 3 = 10

Number of tetra packs in I carton = 10

$\therefore$ Total number of tetra packs Vishal has = 100

So, p(customer picks a guava juice tetra pack) $=\frac{3\times 10}{100}=\frac{3}{10}$

v. (b) $\frac{50}{127}$

Solution:

Number of cartons left with storekeeper = 250 – 10 = 240

Number of cartons he bought = 14

$\therefore$ Total number of cartons are with storekeeper now = 240 + 14 = 254

So, p(selecting a tetra pack of apple juice from store)

$=\frac{(90-4+14)\times 10}{254\times 10}=\frac{100}{254}=\frac{50}{127}$

Assertion Reason Answers:

1. (a)Both A and R are true and R is the correct explanation of A.
2. (a) Both A and R are true and R is the correct explanation of A.